Lecture 19: Quantum Games

1) Classical games
2) PQ Penny Flipover
3) Q. Prisoner's Dilemma
4) Tragedy of the Commons
5) Q. Public Goods Game

Games field founded by von Neumann, Modgenstern (1950s)

Multiperson decision theory
- Analysis of decision making process, assuming rational behavior (i.e. each person maximizes rewards, profits, incomes, subjective benefits)

Example 1: Prisoner's Dilemma (Albert Tucker, 1950)

2 burglars caught, separated by police (no communication)
Each can either cooperate w/ partner (stay silent) or defect (confess)

Rewarded $r=1$
Punished $p=1$
Tempted $t=5$
Sucker's payoff $s=0$

Payoff:

		Bob C	Bob D
Al	C	3,3	0,5
	D	5,0	1,1

(A, B)

Goal: Maximize payoff
Clear optimum choice: D, D

Def: <u>Dominant strategy</u> earns a player a larger payoff than any other strategy, regardless of what other players do

Example 2: Price Wars

Chuang's Gadgets \ Shon's Widgets	\$1	\$2	\$3
\$1	0, 0	50, -10	40, -20
\$2	-10, 50	20, 20	90, 10
\$3	-20, 40	10, 90	50, 50

(C, S)

Trajectory always leads to 0,0
No dominant strategy here!
(Player's choices depend very much on other's)

∃ local max: At \$1, \$1, neither wants to change

Def: <u>Nash Equilibrium</u> is a set of strategies (one for each player) s.t. no player has incentive to change his/her action (partial deriv = 0)

Example 3: $\boxed{\text{Lefty-Righty, Uppy-Downy}}$

	L	R
U	8, 8	0, 6
D	6, 0	7, 7

2 Nash equil: {LU, RD}

(Can move along straight line in table, but not diagonally)

Example 4: $\boxed{\text{3-Player Lefty-Righty}}$

	L	R
U	0, 0, 10	-5, -5, 10
D	-5, -5, 0	1, 1, -5

Matrix A

	L	R
U	-2, -2, 0	-5, -5, 0
D	-5, -5, 0	-1, -1, 5

Matrix B

(3 players: Col, Row, Matrix)

Nash equil: ULA

Vulnerable to coalition → DRB

Nash equil: Unilateraly optimal

II $\boxed{\text{PQ Penny Flipover}}$

$$\text{Coin Tails} \longrightarrow \underset{\boxed{\text{flip/not flip}}}{Q} \longrightarrow \underset{\boxed{\text{flip/not flip}}}{P} \longrightarrow \underset{\boxed{\text{flip/not flip}}}{Q} \longrightarrow \begin{cases} \text{heads, } Q \text{ wins} & +1 \\ \text{tails, } Q \text{ loses} & -1 \end{cases}$$

		$\boxed{Q}$			
		NN	NF	FN	FF
$\boxed{P}$	N	-1	1	1	-1
	F	1	-1	-1	1

No Nash equil

Best strategy: mixed

$P = \frac{1}{2}$ N, $\frac{1}{2}$ F

$Q = \frac{1}{4} \{NN, FN, NF, FF\}$

Quantum Version:

$$|\text{tail}\rangle \longrightarrow \underset{\mathcal{H}}{Q} \longrightarrow \underset{X,\,I}{P} \longrightarrow \underset{X\mathcal{H}}{Q} \longrightarrow |\text{heads}\rangle$$

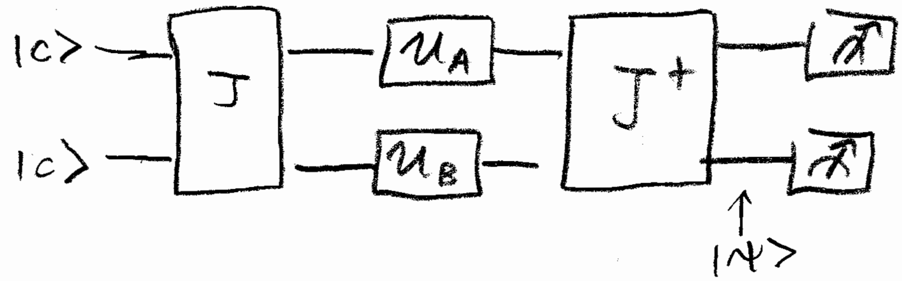
Q always wins! (P = Picard)

III $\boxed{\text{Quantum Prisoner's Dilemma}}$

Rule: No communication btwn players

Def: $|C\rangle$, $|D\rangle$ Hilbert space basis

Game:



$$J = 1/\sqrt{2}\,(JJ + iXX)$$
$$= \exp(i\,\pi/4\; XX)$$

Choose $\left.\begin{array}{l}\hat{C} := I \\ \hat{D} := X\end{array}\right\}$ Choices for U_A, U_B

$\Rightarrow$ Game reduces to classical game:

$$0 = [J, \hat{C}\hat{C}] = [J, \hat{C}\hat{D}] = [J, \hat{D}\hat{C}] = [J, \hat{D}\hat{D}]$$

Measure prob(CC, CD, DC, DD)

Payoff:
$$\begin{cases} \$_A = \underbrace{r P_{CC}}_{\text{reward}} + \underbrace{p P_{DD}}_{\text{punishment}} + \underbrace{t P_{DC}}_{\text{temptation}} + \underbrace{s P_{CD}}_{\text{sucker's payoff}} \\ \$_B = r P_{CC} + p P_{DD} + t P_{CD} + s P_{DC} \end{cases}$$

let $r = 3$, $p = 1$, $t = 5$, $s = 0$ as before

New quantum strategy: $\hat{Q} = Z$

u_A \ u_B	C	D	Q
C	3,3	0,5	1,1
D	5,0	1,1	0,5
Q	1,1	5,0	3,3

[Shaded: New! (See below)]

$\boxed{u_A \, u_B}$

$XZX = -Z$
$ZX = iY$

$\underline{\hat{Q}\ \hat{C}}$:

$$
\begin{aligned}
|\psi\rangle &= J^{+}(Z \otimes I) J |CC\rangle \\
&= (II - iXX)(ZI)(II + iXX)|CC\rangle \\
&= (ZI - YX - YX - ZI)|CC\rangle \\
&= -2YX|CC\rangle \\
&\approx |DD\rangle \text{ (when normalized)}
\end{aligned}
$$

$\underline{\hat{Q}\hat{Q}}$:

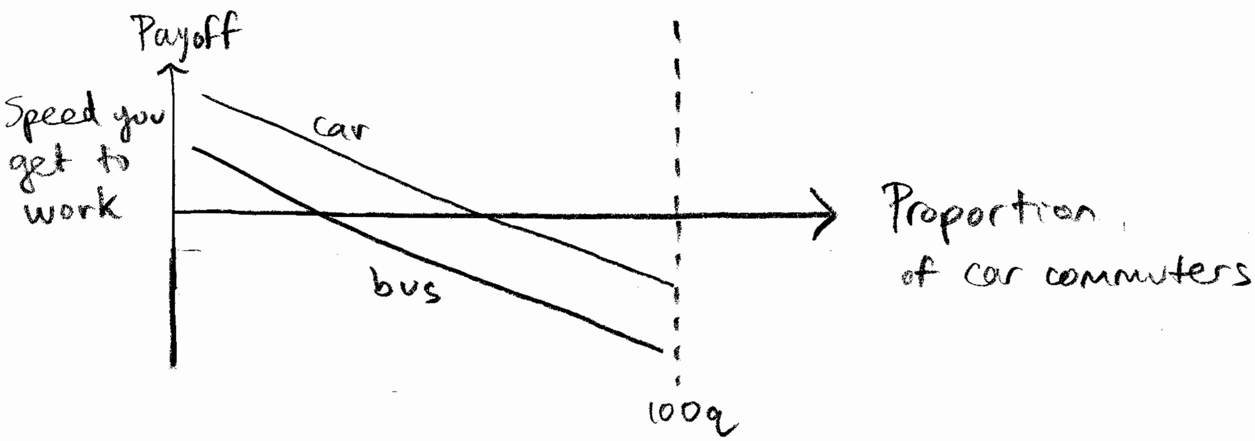
$$
\begin{aligned}
|\psi\rangle &= (II - iXX)(ZZ)(II + iXX)|CC\rangle \\
&= |CC\rangle
\end{aligned}
$$

Old Nash equil: DD
New Nash equil: QQ (better!)

<u>Problems</u>: - Why help prisoners by offering φ choice
- Who enforces I?
- What about other rotations?

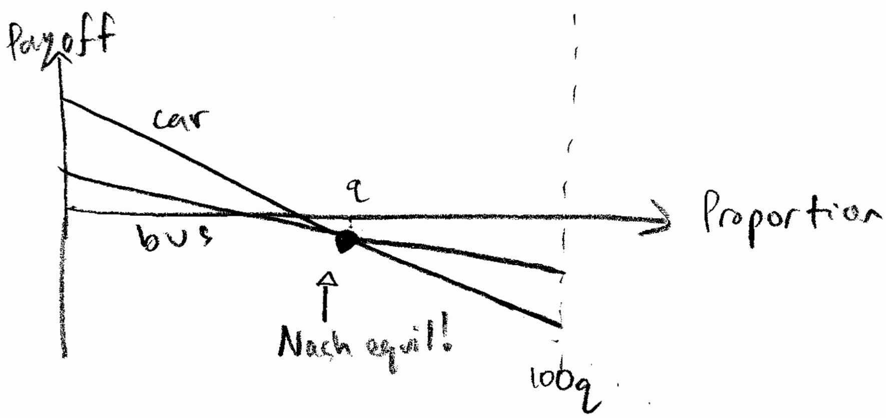
IV Tragedy of the Commons

<u>Proportion game model:</u>



Dominant strategy: Take car. Sucks!

<u>New model:</u>

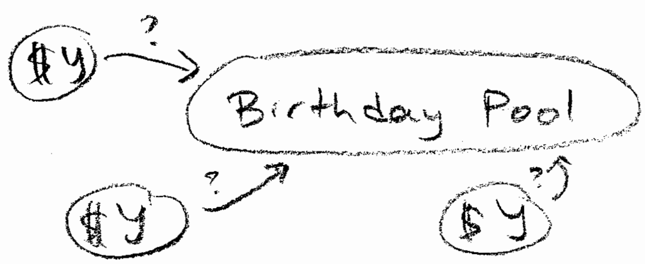


e.g. Gov't provides incentives to take bus till get intersection

Tragedy: All common property resources tend to be overexploited & thus degraded.

Traditional sol^n: 3rd-party regulator (e.g. gov't)

IV $\boxed{\text{Q Public Goods Game}}$



n = # players

y = initial endowment

C_k = contribution from player k

a = public gain, $1 < a < n$

$$\$_k = y - C_k + \frac{a}{n} \sum_j C_j$$

Claim: Nash equil is $\sum_j C_j = 0$

let $y = 1$, $n = 2$:

		Alice C	Alice D
Alice	C	a, a	$a/2,\ 1+a/2$
	D	$1+\frac{1}{2}a,\ a$	$1, 1$

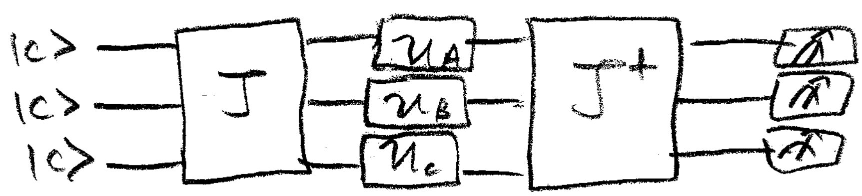
Same matrix as prisoner's dilemma! Oh no!
No one gives to birthday pool, though it'd be awesome if everyone did

Let $n=3$, 0 = defect, 1 = contribute

	ABC	\$A	\$B	\$C	$a \sum C_i$
No one contributes	000	1	1	1	0
Only 1 contributes	001 010 011	$1+a/3$	$1+a/3$	$a/3$	a
2 contribute	100 101 110	$1+2a/3$	$2a/3$	$2a/3$	$2a$
Everyone contributes	111	a	a	a	$3a$

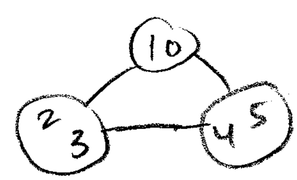
Nash equil: 000 (sad!)

$\boxed{\text{Quantum Game}}$ Chen, Hogg, ... quant-ph/0301013



Assume J & J^+ performed by trusted authority (e.g. bank)

- let $J = 1/\sqrt{2}\,(III + iXXX)$
  Claim: Expected payoff $= 1/2\,(1+a)$



- Let J have pairwise entanglement
  Claim: Expected payoff $= a - 2^{-(n-1)}(n-1) \approx a$

Problems: Will people trust J? Trust quantum mech?
Will people collude?
Will people swap in their own qstates?