13.002
Introduction to Numerical Methods for Engineers
Problem set 4
Issued: Mar. 3, 2005
Due: Mar. 10, 2005

Problem 1.

Certain wave propagation problems lead to systems of linear equations of the form,

$$\begin{bmatrix} e^{-\alpha} & 1 & 0 \\ -1 & e^{-\alpha} & -1 \\ 1 & e^{-\alpha} & -2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ e^{-\alpha} \\ e^{-\alpha} \end{Bmatrix} \tag{1}$$

1. Determine the solution in the limits $\alpha = 0$ and $\alpha \to \infty$.
2. Make a set of subroutines (or matlab functions) for the following subtasks associated with solving a general $n \times n$ system of equations:
   - Gaussian elimination without pivoting
   - Back-substitution
3. Make a program (C, Fortran or Matlab) using these subroutines to solve Eq. (1) for $\alpha = [0,\ 5,\ 10,\ 20,\ 40]$ and discuss the behavior of the solution for large α.
4. Modify your subroutines to use partial pivoting and redo the solution for the above series of values of α. Check the solution with the limits determined in Question 1.
5. Suggest a rearrangement of the unknowns which yields a stable solution with your original solver routines without pivoting. Demonstrate the stability using the series of α used above.

Amendment to Problem Set 4

The previous equation has been changed to be:

$$\begin{bmatrix} e^{-\alpha} & 1 & 0 \\ -1 & e^{-\alpha} & -1 \\ 1 & -2 & e^{-\alpha} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ e^{-\alpha} \\ e^{-\alpha} \end{Bmatrix}$$

1. $\alpha = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$x_1 = 3$

$x_2 = -2$

$x_3 = -6$

$\alpha = \infty$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 1 & -2 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$x_1 = 2$

$x_2 = 1$

$x_3 = -2$

2.

```
clear
clc

alfa=100;
A=[exp(-alfa) 1 0; -1 exp(-alfa) -1; 1 -2 exp(-alfa)];
%A=[1 exp(-alfa) 0; exp(-alfa) -1 -1; -2 1 exp(-alfa)];
oA=A;
B=[1; exp(-alfa); exp(-alfa)];
oB=B;
```

```
[mm,n]=size(A);
A=[A B];
L=zeros(mm,n);

for i=1:mm-1
    for j=i+1:mm
        m(j,i)=A(j,i)/A(i,i);
        L(j,i)=m(j,i);
        for k=i:n+1
            A(j,k)=A(j,k)-m(j,i)*A(i,k);
        end
    end
end
U=A(:,1:n);
L=L+eye(mm,n);
B=A(:,n+1);

x=zeros(n,1);
for j=mm:-1:1
    x(j)=(B(j)-A(j,j+1:n)*x(j+1:n))/A(j,j);
end
x
```

oB-oA*x

3.

$\alpha = 0$

$x_1 = 3$

$x_2 = -2$

$x_3 = -6$

$B - Ax = 0$ The solutions are exactly correct.

$\alpha = 5$

$x_1 = 1.9933$

$x_2 = 0.9866$

$x_3 = -1.9934$

$B - Ax \approx 0$ Solutions are very good approximation.

$\alpha = 10$

$x_1 = 2$

$x_2 = 0.9999$

$x_3 = -2$

$B - Ax \approx 0$ Solutions are very good approximation

$\alpha = 20$

$x_1 = 2$

$x_2 = 1$

$x_3 = -2$

$B - Ax \approx 0$ Solutions are very good approximation

$\alpha = 40$

$x_1 = 0$

$x_2 = 1$

$x_3 = 0$

$$B - Ax = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$ Big errors exist. Without pivoting, it’s not stable now.

4.

```
clear
clc

alfa=40;
%A=[1 exp(-alfa) 0; exp(-alfa) -1 -1; -2 1
exp(-alfa)];
A=[exp(-alfa) 1 0; -1 exp(-alfa) -1; 1 -2
exp(-alfa)]; %original one.
oA=A;
B=[1; exp(-alfa); exp(-alfa)];
oB=B;

[mm,n]=size(A);
A=[A B];
L=zeros(mm,n);
```

```
for i=1:mm-1

    %pivoting begins from here.
    [Y I]=max(abs(A(i:mm,i)));
    temp_store1=A(I,:);
    temp_store2=B(I);
    A(I,:)=A(i,:);
    B(I)=B(i);
    A(i,:)=temp_store1;
    B(i)=temp_store2;
    %pivoting ends from here.

    for j=i+1:mm
        m(j,i)=A(j,i)/A(i,i);
        L(j,i)=m(j,i);
        for k=i:n+1
            A(j,k)=A(j,k)-m(j,i)*A(i,k);
        end
    end
end
U=A(:,1:n);
L=L+eye(mm,n);
```

```
B=A(:,n+1);

x=zeros(n,1);
for j=mm:-1:1
    x(j)=(B(j)-A(j,j+1:n)*x(j+1:n))/A(j,j);
end
x
oB-oA*x
```

5. switch x_1 and x_2