Complexity

Reference:

Nam P. Suh, *Complexity: Theory and Applications*, Oxford University Press, 2005

Why complexity?

- In many fields (engineering, economics, politics, academic administration, etc.), one of the major goals is to reduce complexity.
- In many fields, we want to understand the causes for complexity.
- There is a great deal of confusion what we mean by complexity.
- Computer scientists and mathematicians have considered complexity, which may not be applicable to design and engineering fields.

(David P. Feldman and Jim Crutchfield, A Survey of “Complexity Measures”, 1998 Complex Systems Summer School, Santa Fe Institute, 11 June 1998.

Issues related to complexity

- Why is "something" complex?
- Why is "something" that appeared to be complex is not complex once we understand it?
- What is complexity?
- Is it better to reduce complexity of an engineered system?
- How do we reduce complexity?
- Is the complexity of engineering systems different from that of socio-economic-political systems?
- Is the complexity of natural systems different from engineered systems?

Consider the task of cutting a rod?

- Is this a complex task?
- Suppose we must cut it to 1 m +/- 1 cm. Is this a complex task?
- Suppose we must cut the rod to 1 m +/- 1 micron?

Consider the task of controlling the *flow rate* and the *temperature* of water using the faucet shown below?

Figure removed for copyright reasons.
See Figure E3.3a in Suh, N. P.
Axiomatic Design: Advances and Applications. New York, NY: Oxford University Press, 2001. ISBN: 0195134664.

Is this a complex task?

- Suppose the task is to control the flow rate 1 +/- 0.3 gallons per minute and the temperature within 90 +/- 25 F.
- Is it a complex task? Why? Why not?
- If the flow rate has to be controlled to 1 gallon +/- 0.01 gallons/minute and the temperature within 60 +/- 0.2 F, it appears to be complex.
- The task is more complex when these two FRs are coupled by the design!

International Space Station Beta Gimbal Assembly Failure

Other basic questions:

"How do we guarantee the long-term stability of engineered systems?"

"Why is there so much wasted effort in developing new products?"

“What is complexity?”

Departure from Conventional View of Complexity

- Most people have been trying to understand complexity in terms of physical things.

Many Different Definitions of Complexity

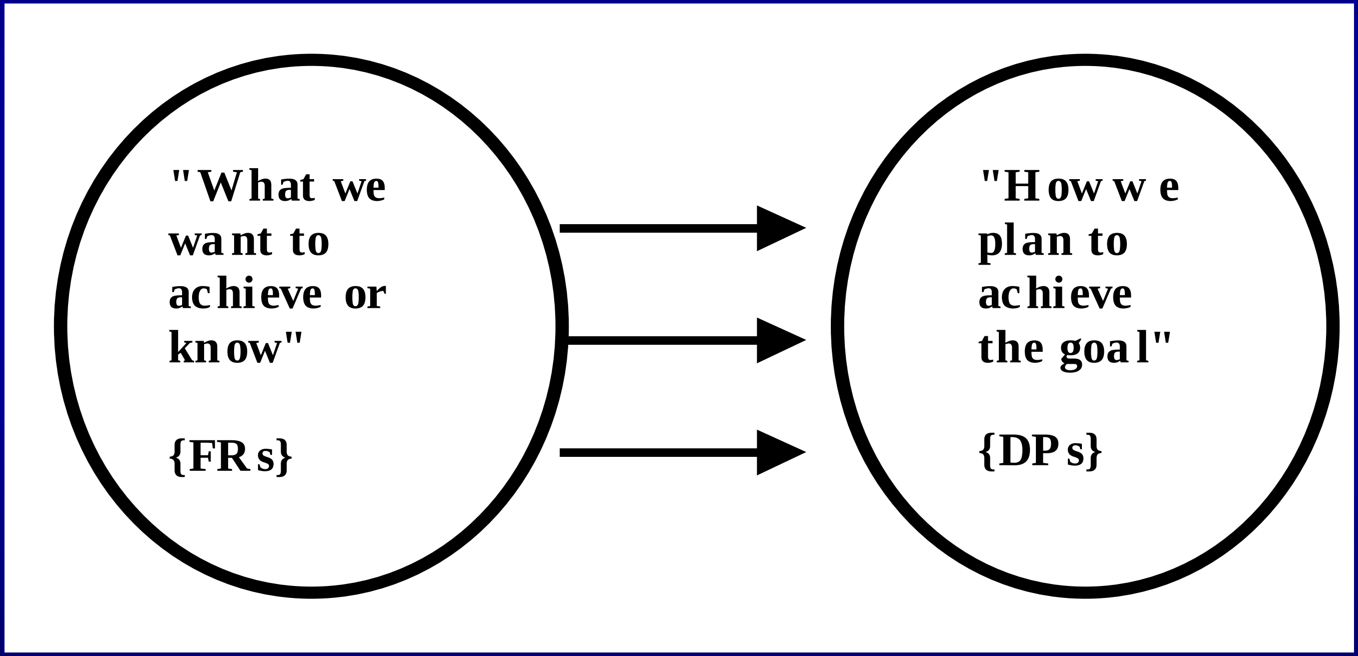
- Computational complexity
- Algorithmic complexity
- Probabilistic complexity
- ……
- ……

Departure from Conventional View of Complexity

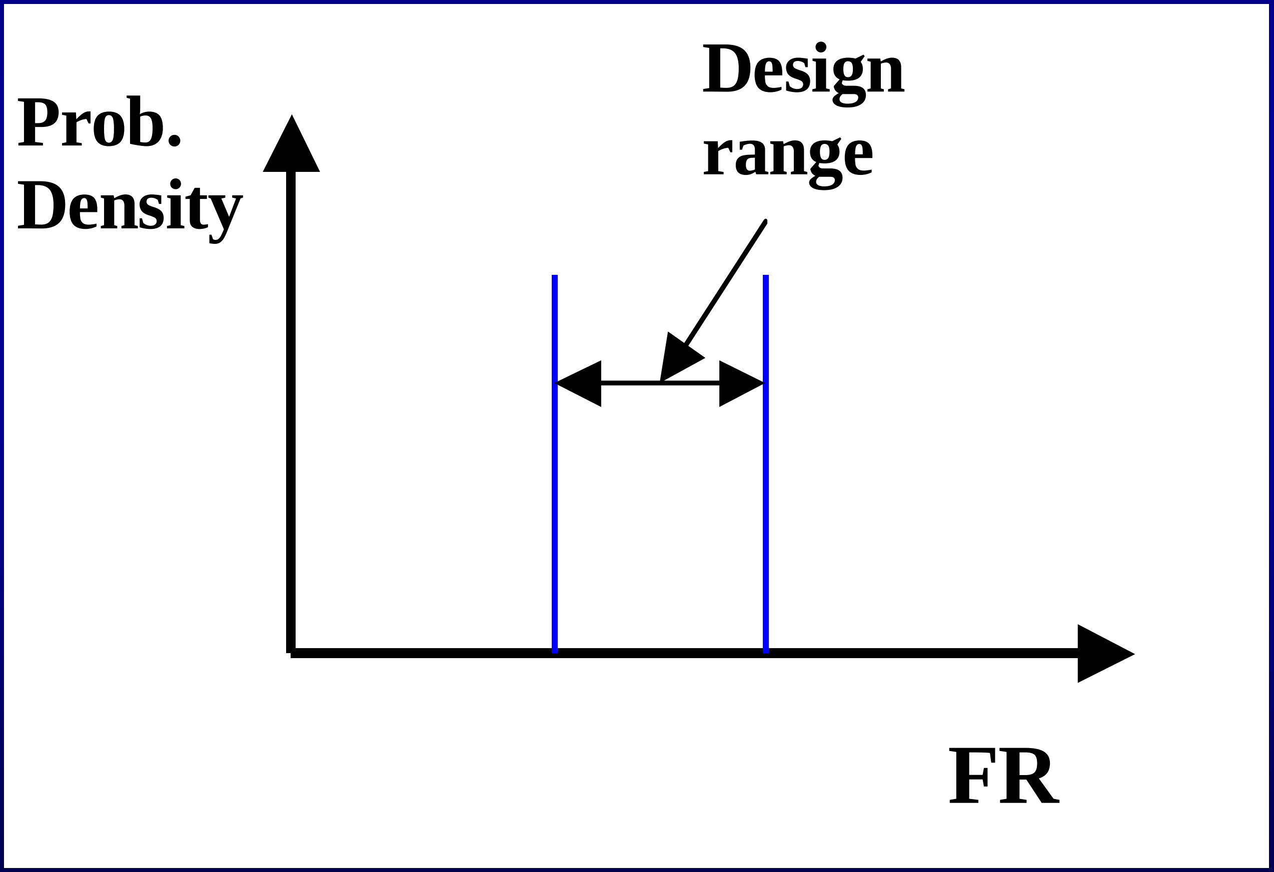
- Most people have been trying to understand complexity in terms of physical things.

- Complexity must be viewed in the functional domain.

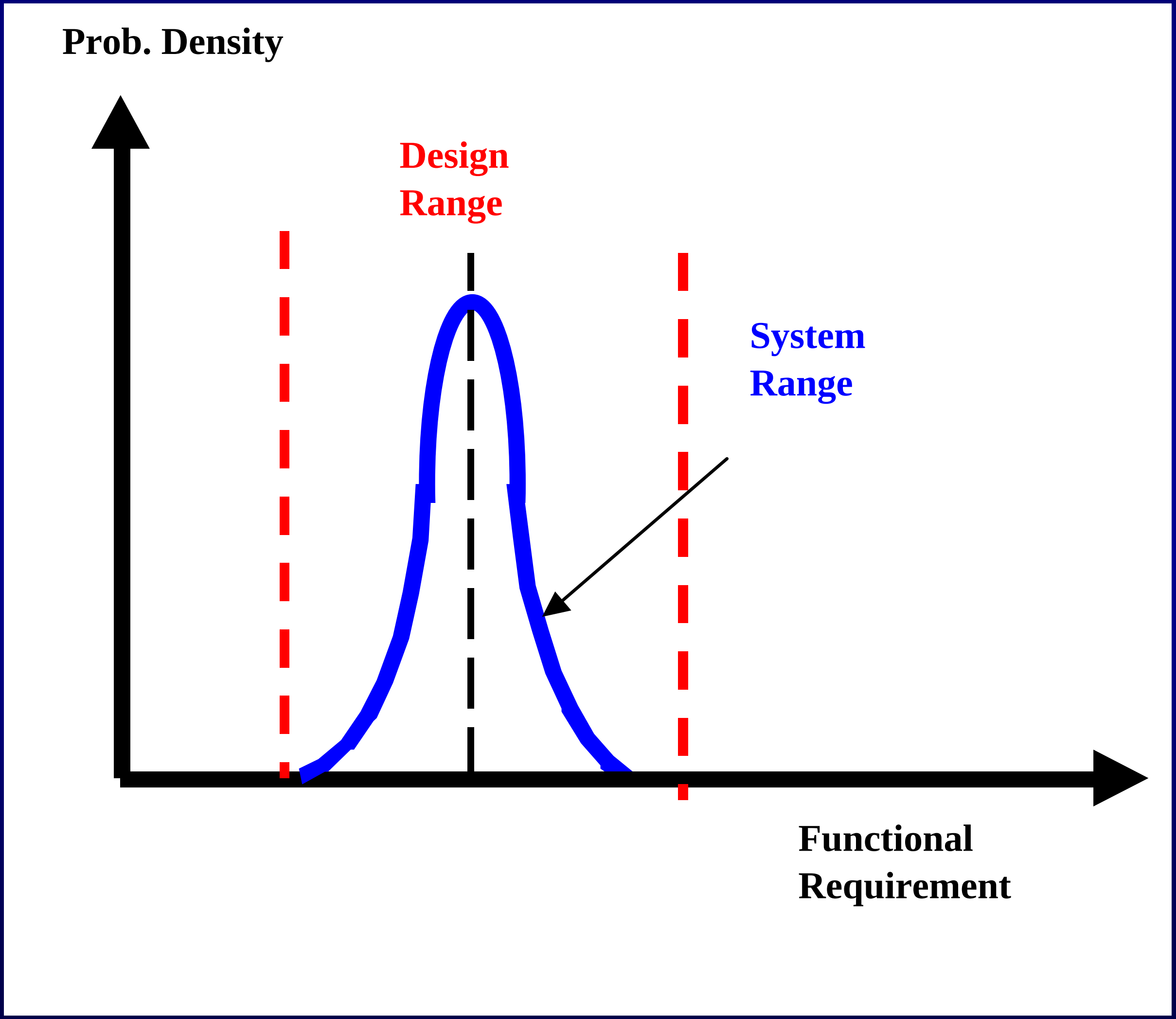
Functional Domain vs Physical Domain



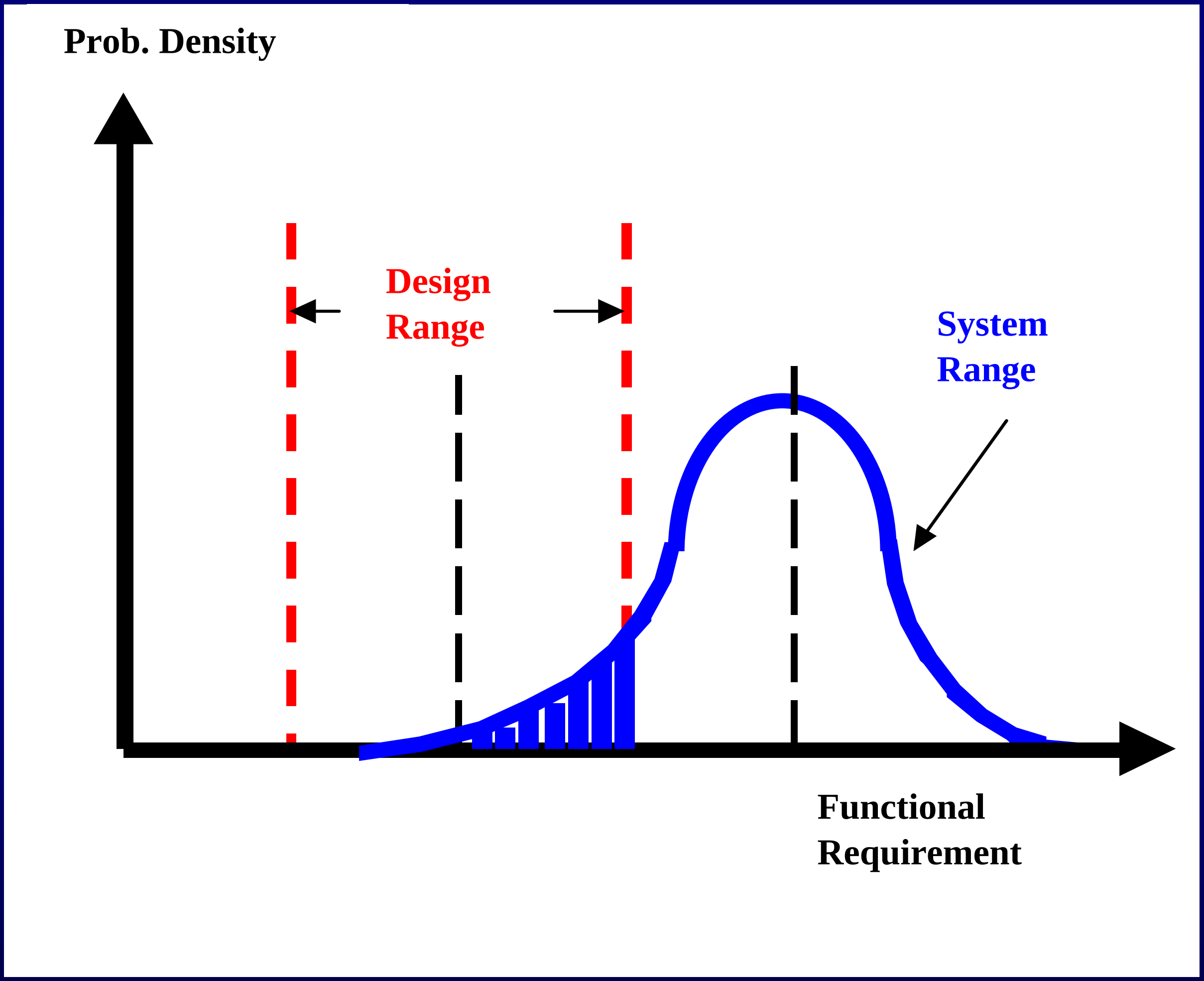
Consider the problem associated with satisfying an FR.



If the system range is inside the design range, FR can easily be achieved. Therefore, it is not complex.



In this case, FR cannot always be satisfied. There is a finite uncertainty. Therefore, the task of achieving the FR appears to be complex.



Definition of Complexity

Complexity is defined as a measure of uncertainty in satisfying the functional requirements.

Definition of Complexity as applied to natural science

Complexity is defined as a measure of uncertainty in our ability to predict a certain natural phenomenon to the desired accuracy.

Definition of Complexity

Complexity,

which is defined as a measure of uncertainty in satisfying the FRs,

is a *relative* quantity.

Four Different Kinds of Complexity

- **Time-Independent *Real* Complexity**
- **Time-Independent *Imaginary* Complexity**

- **Time-Dependent *Combinatorial* Complexity**
- **Time-Dependent *Periodic* Complexity**

“Complexity” can be reduced by taking the following actions:

- Reduce Time-Independent Real Complexity
- Eliminate Time-Independent Imaginary Complexity
- Transform Time-Dependent Combinatorial Complexity into Time-Dependent Periodic Complexity

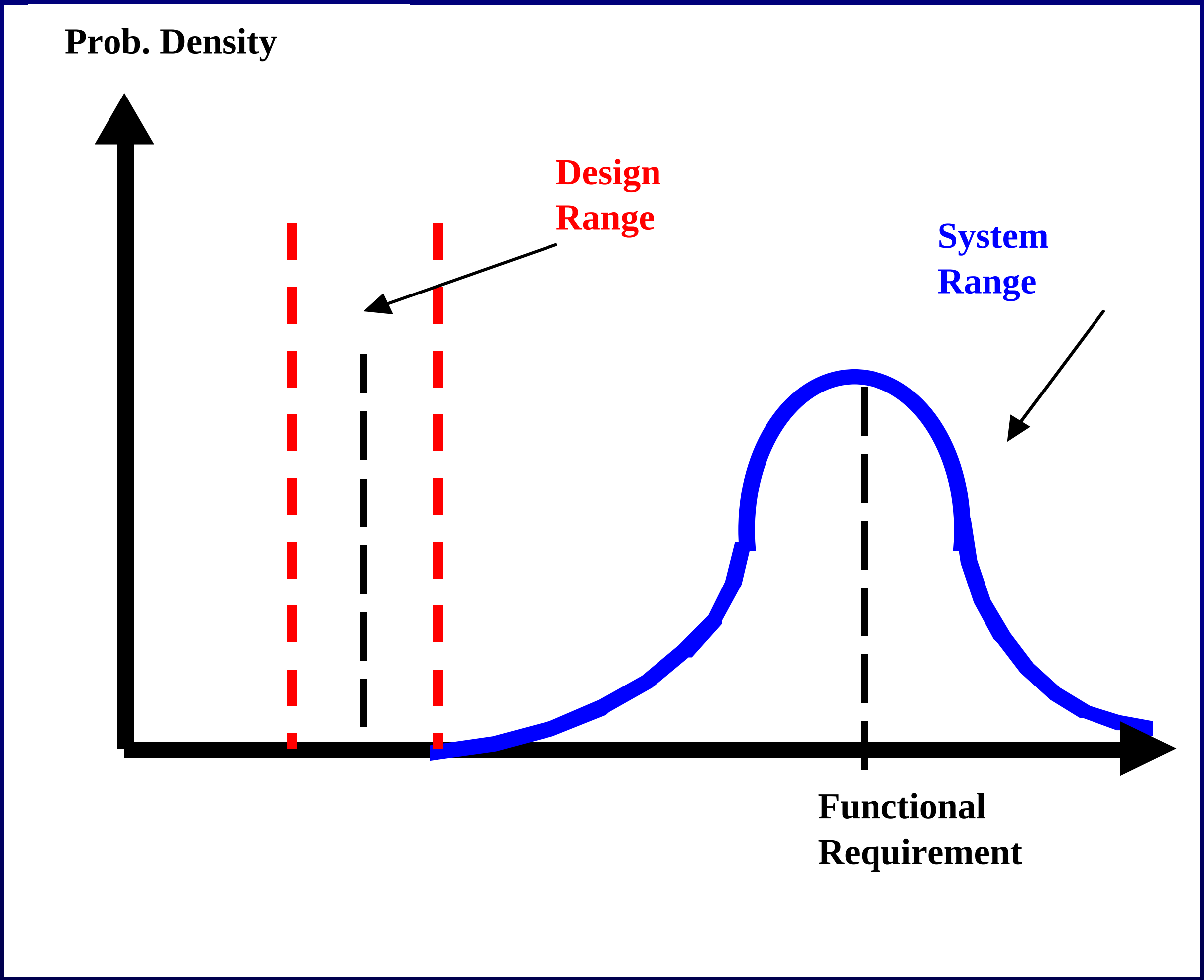
Important concept: Functional Periodicity

Functional Periodicity

- Temporal periodicity
- Geometric periodicity
- Biological periodicity
- Manufacturing process periodicity
- Chemical periodicity
- Thermal periodicity
- Information process periodicity
- Electrical periodicity
- Circadian periodicity
- Material periodicity

Time-Independent Real Complexity

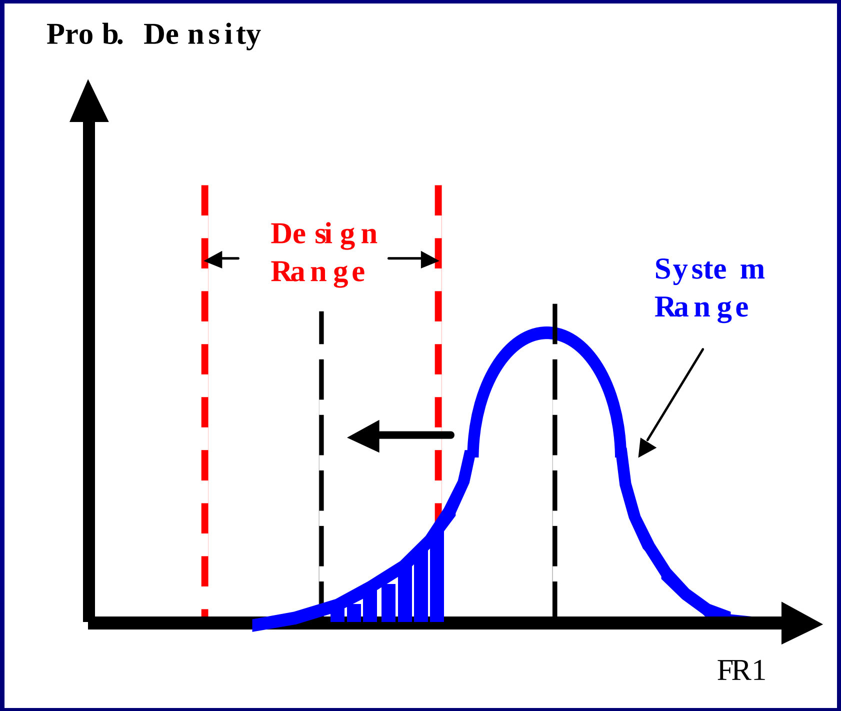
Time-Independent Real Complexity --> Infinity.

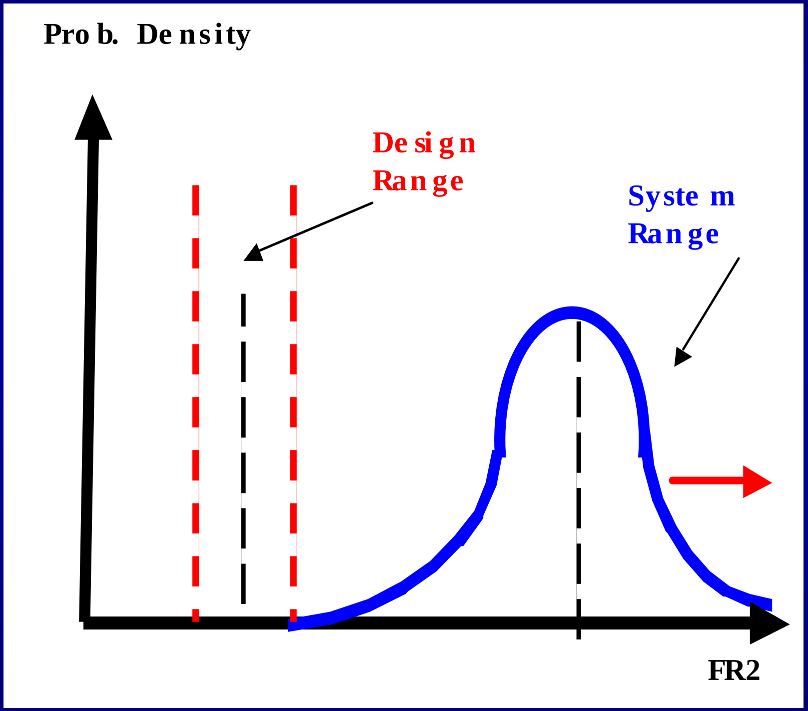


What happens when there are many FRs?

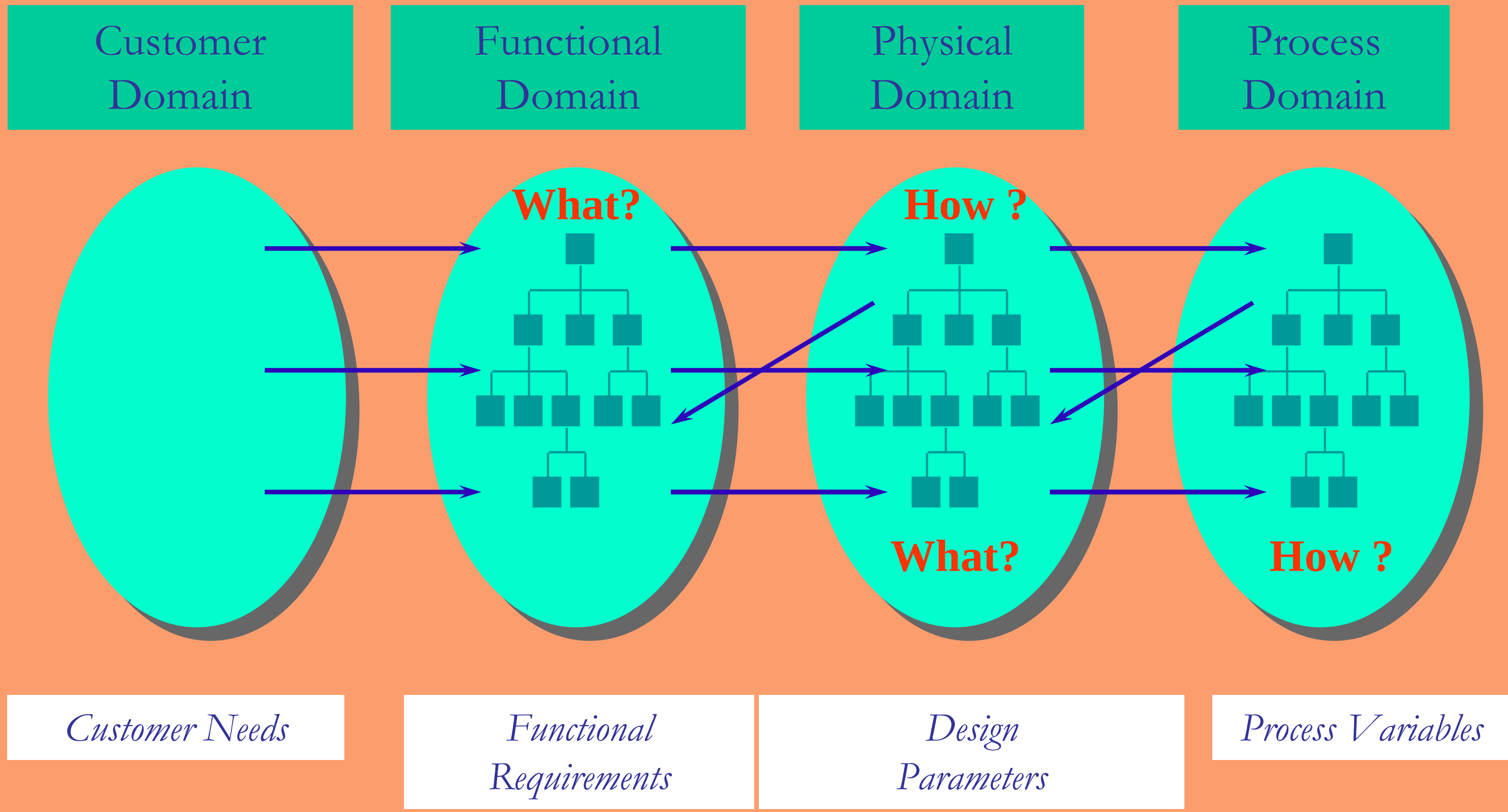
- Most engineered systems must satisfy many FRs at each level of the system hierarchy.
- The relationship between the FRs determine how difficult it will be to satisfy the FRs within the desired certainty and thus complexity.

If FRs are not independent from each other, the following situation may exist.





Axiomatic design:
Mapping, hierarchies, and zigzagging



Design Axioms

- **Axiom 1 The Independence Axiom**

  Maintain the independence of functional requirements.

- **Axiom 2 The Information Axiom**

  Minimize the information content.

Design Equation and Matrix

The relationship between {FRs} and {DPs} can be written as

$$\{FRs\} = [A]\ \{DPs\}$$

When the above equation is written in a differential form as

$$\{dFRs\} = [A]\ \{dDPs\}$$

[A] is defined as the Design Matrix given by elements :

$$Aij = \partial FRi / \partial DPi$$

Uncoupled, Decoupled, and Coupled Design

- **Uncoupled Design**

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix}$$

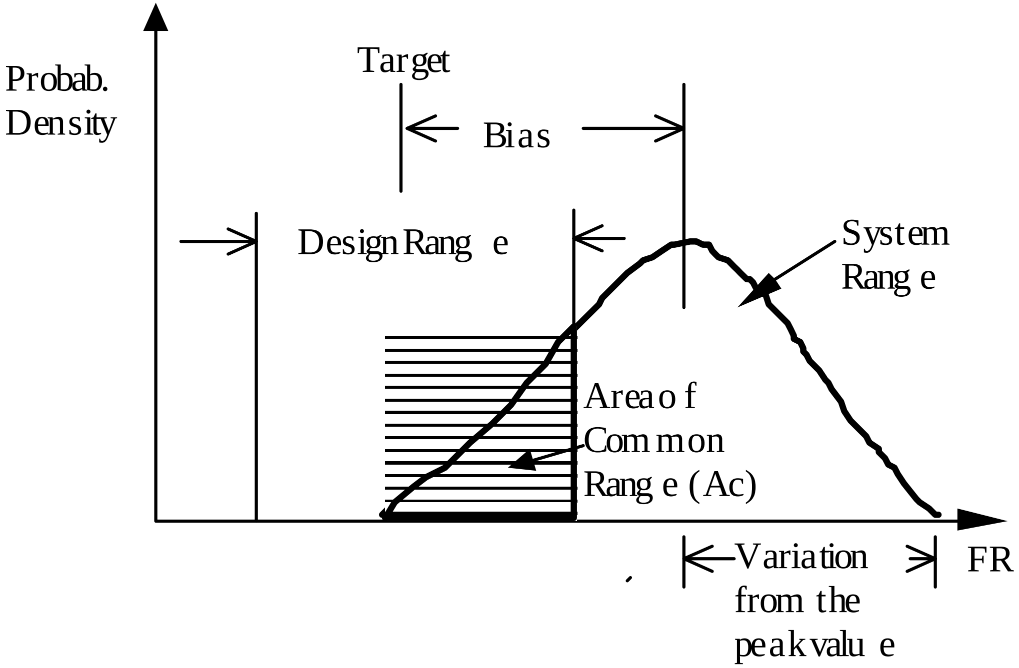
- **Decoupled Design**

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix}$$

- **Coupled Design**

All other design matrices

Design Range, System Range, and Common Range



Coupling Increases Time-Independent Real Complexity!

Real complexity of a decoupled system is, in general, larger than that of a uncoupled design.

Uncoupled

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2}{A22}$$

$$\Delta DP3 = \frac{\Delta FR3}{A33}$$

Decoupled

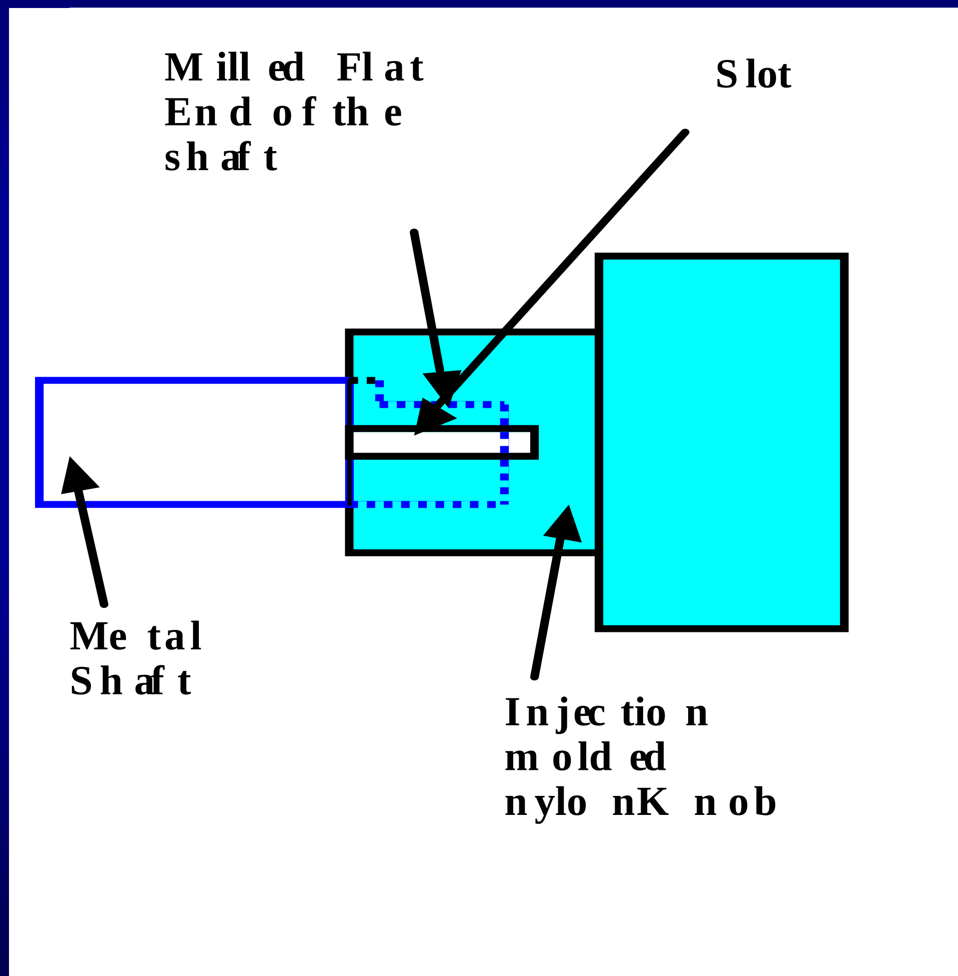
$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ A12 & A22 & 0 \\ A13 & A23 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$

$$\Delta DP2 = \frac{\Delta FR2 - \left| A21 \cdot \Delta DP1 \right|}{A22}$$

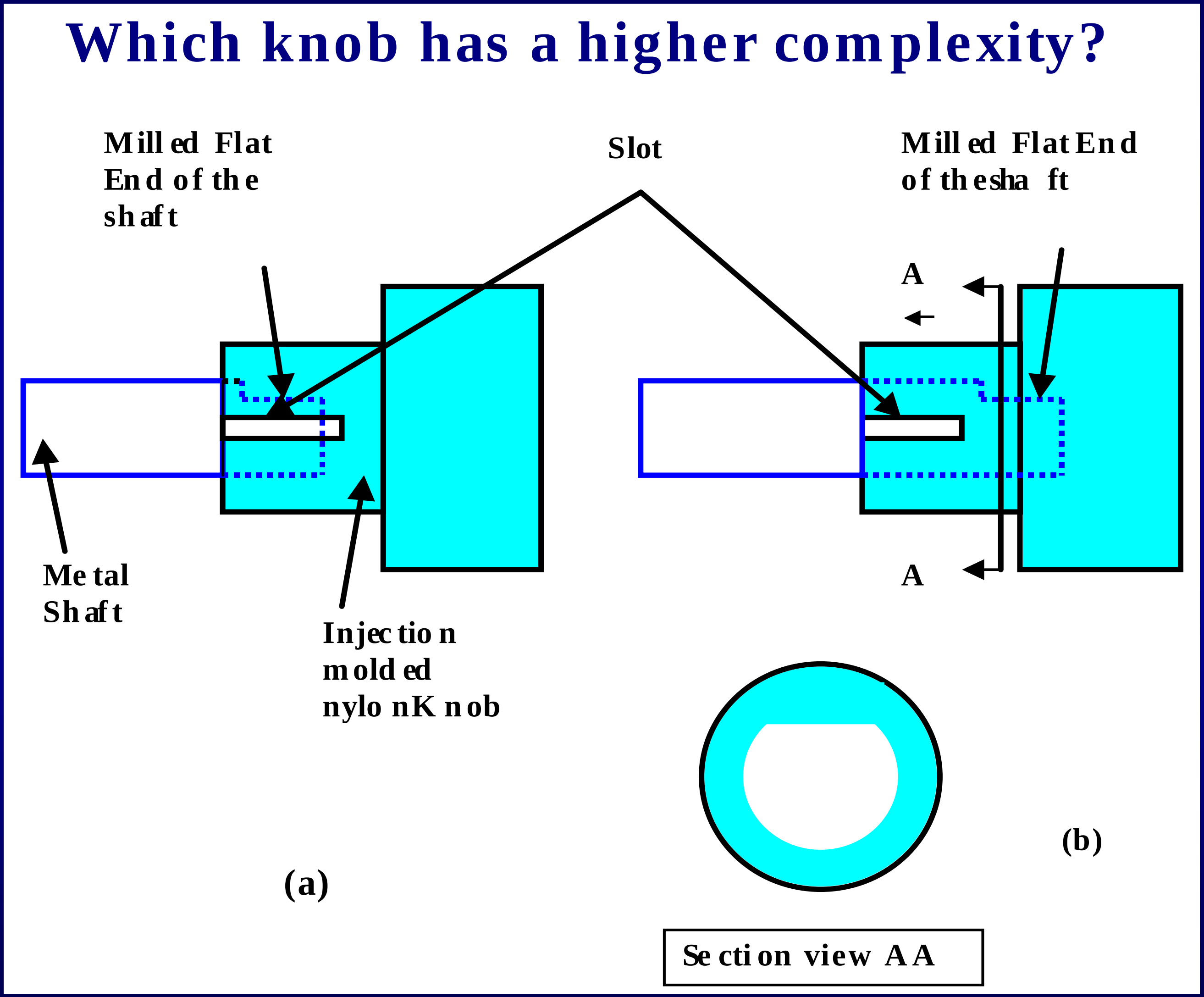
$$\Delta DP3 = \frac{\Delta FR3 - \left| A31 \cdot \Delta DP1 \right| - \left| A32 \cdot \Delta DP2 \right|}{A33}$$

Complexity of a Knob Design



Knob designs

Which is more complex?



Knob designs

Conventional Engine: Real Complexity

Schematic removed for copyright reasons.

A New Engine

Schematic removed for copyright reasons.

Simulation Results of the New Engine (?)

New Engine

What is Time-Independent Imaginary Complexity?

Imaginary complexity is defined as:

Uncertainty that is not real uncertainty, but arises because of the "designer's" lack of knowledge and understanding of a specific "design" itself.

What is Time-Independent Imaginary Complexity?

Example:

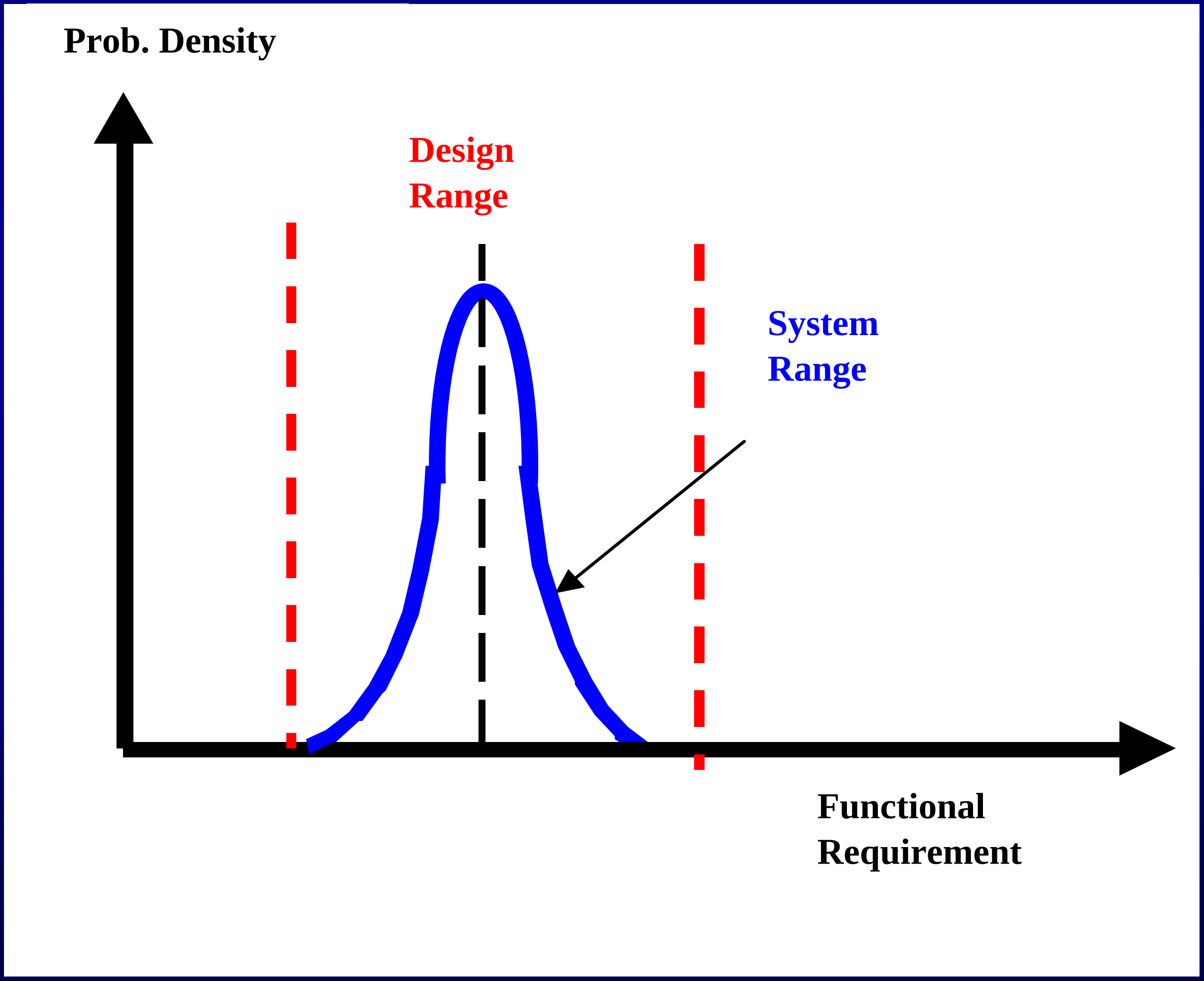
Combination Lock

Imaginary complexity changes with a priori knowledge:

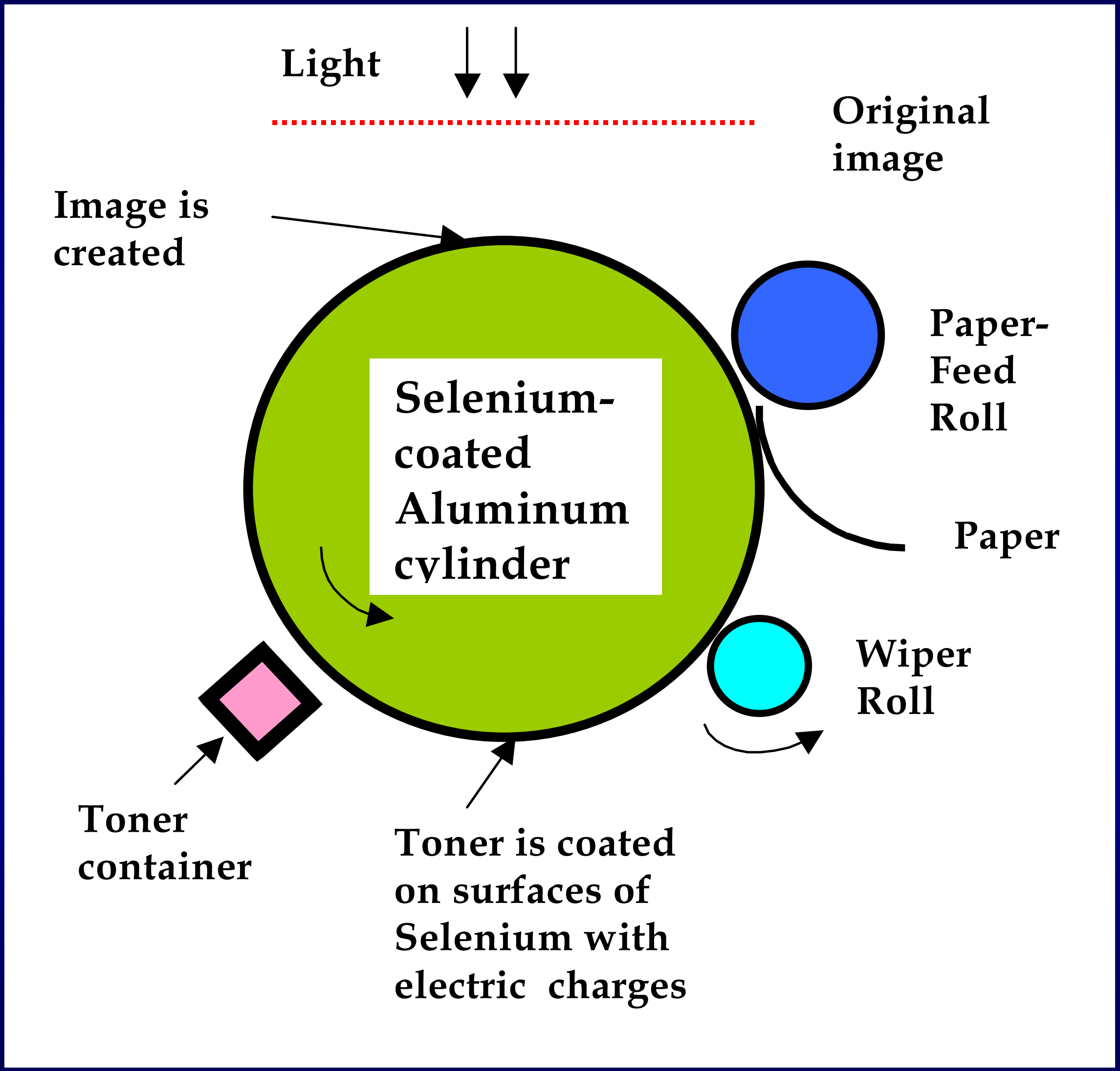
(a) Without knowing any of the numbers
(b) Know the numbers but the sequence is not given
(c) Know both the numbers and the sequence

Time-Independent Imaginary Complexity

What is Imaginary Complexity ?



Example of Imaginary Complexity



Schematic drawing of the xerography based printing machine

Time-Independent Imaginary Complexity

Consider a triangular design matrix for a decoupled design:

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

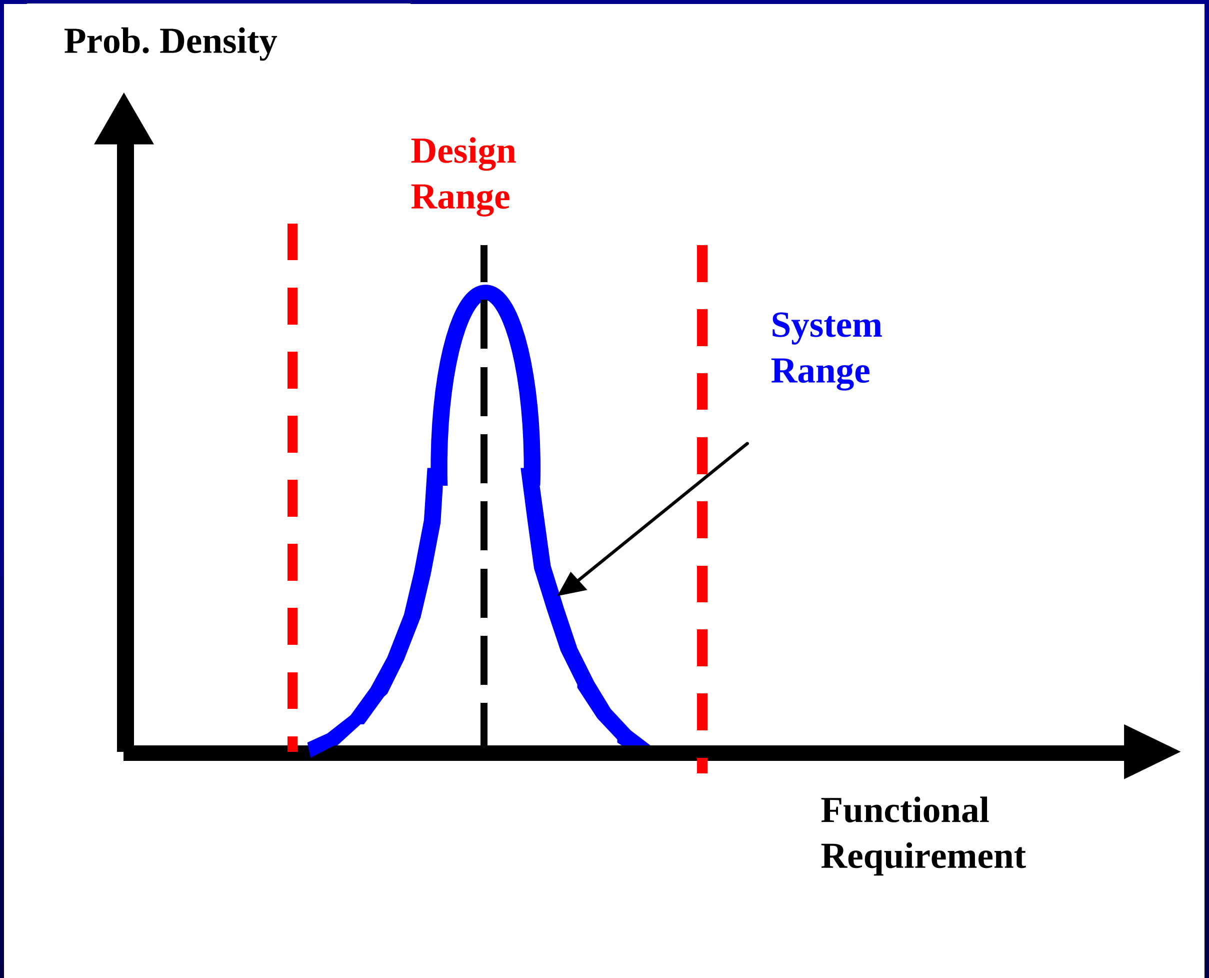
$$C_I = -\log_2 P = \log_2 n!$$

n	n!	p = 1/ n!
1	1	1
2	2	0.5
3	3	0.1667
4	24	0.04167
5	120	0.8333×10^{-2}
6	720	0.1389×10^{-2}
7	5,040	0.1984×10^{-3}
8	40,320	0.2480×10^{-4}

Time-Independent Imaginary Complexity

$$C_R = 0$$

$$(C_I)_{max} = \log_2 m!$$



CMP Machine Designed and Built by Four New Graduate Students

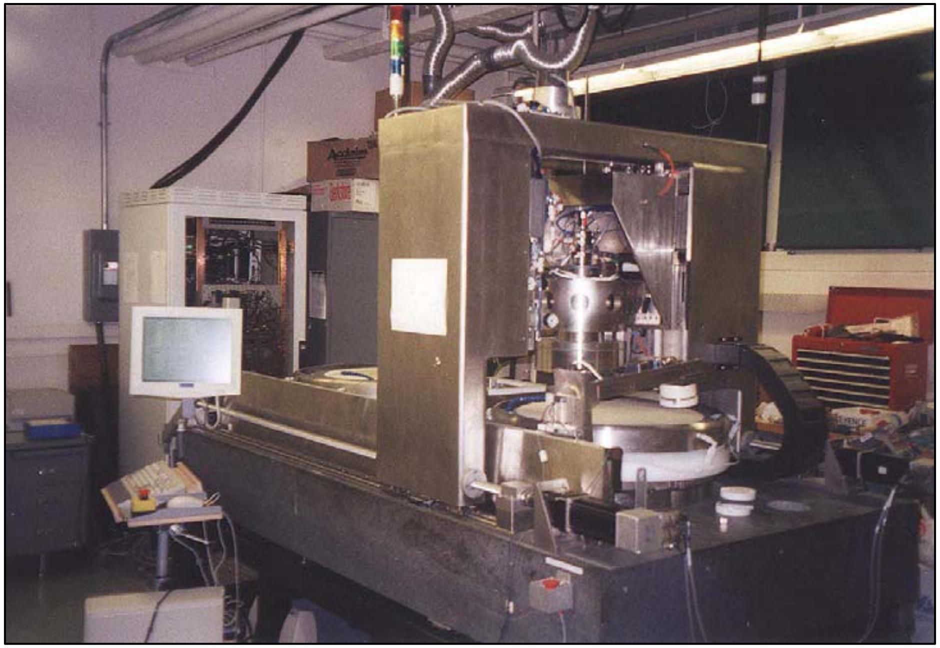
CAD model

Fabricated machine

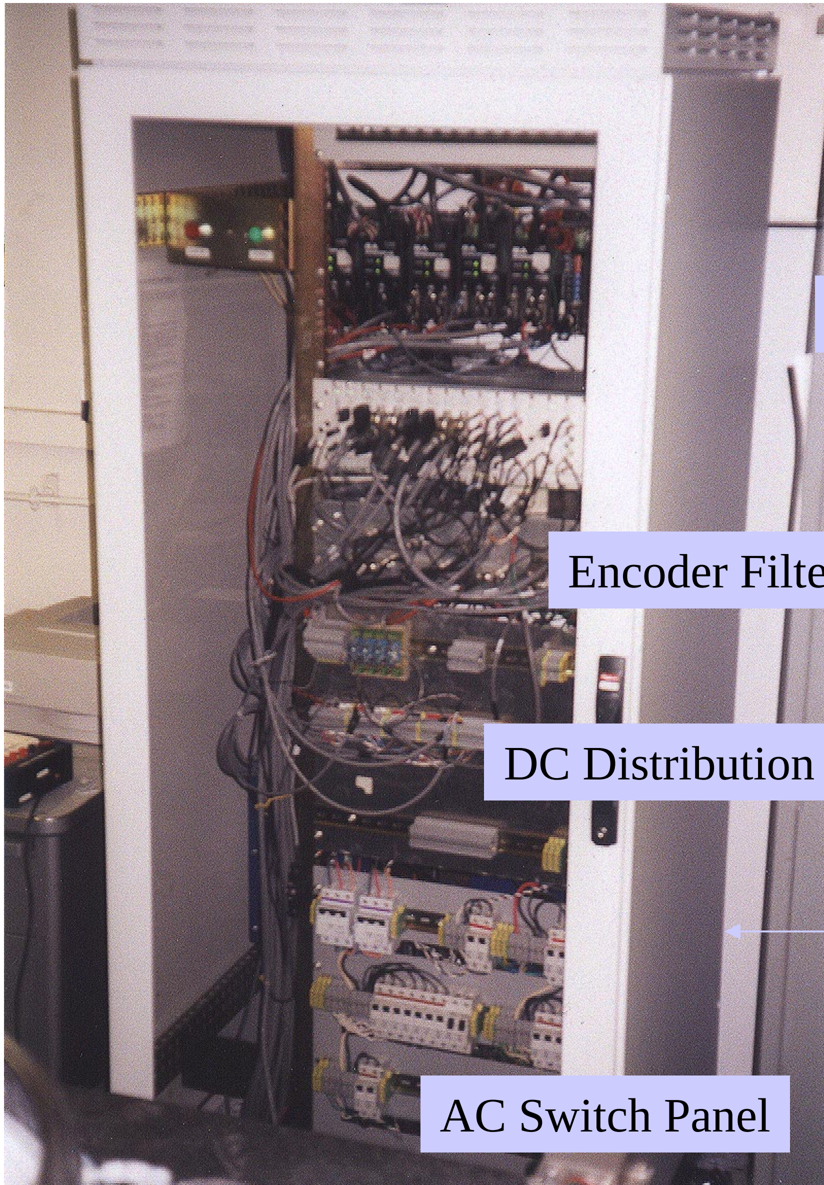
Figures removed for copyright reasons.
See Figure 8.20 and 8.21 in Suh, N. P.
Axiomatic Design: Advances and Applications. New York, NY:
Oxford University Press, 2001. ISBN: 0195134664.

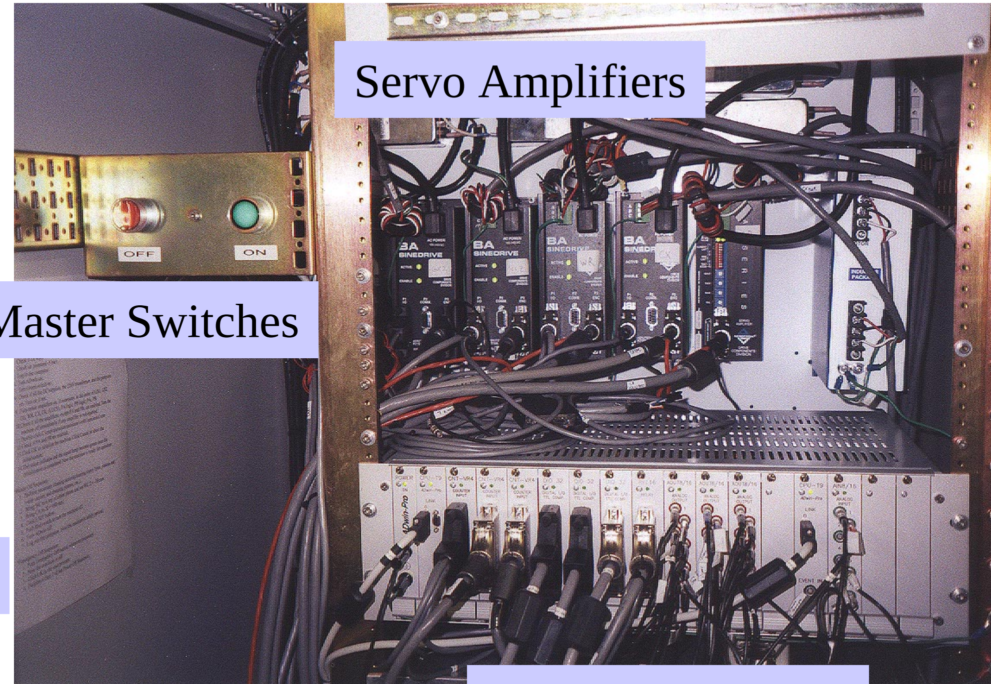
CMP machine

- **2 platen, single head (200 mm)/Multi-step wafer polishing.**
- **Developed at MIT to meet the research needs.**
- **9 servo motors/4 pumps/8 pressure regulators/60 on-off controllers.**



Control hardware



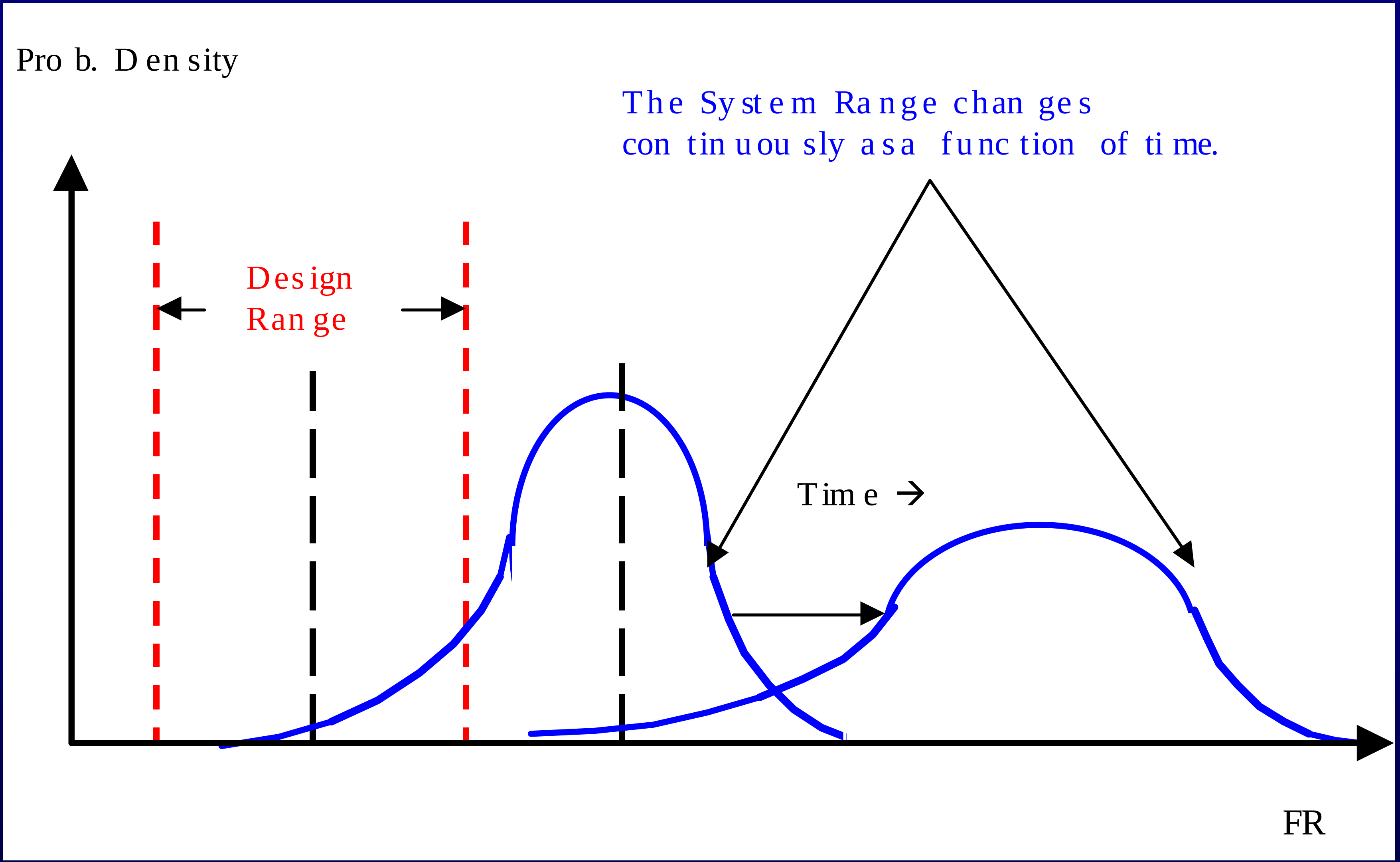


Time-Dependent Combinatorial Complexity

Time-dependent combinatorial complexity arises because the future events occur in unpredictable ways and thus cannot be predicted.

For example, it occurs when the system range moves away from the design range as a function of time.

Time-Dependent Combinatorial Complexity



Example

Airline Schedule

3. Time-Dependent Combinatorial Complexity

The combinatorial complexity is *defined* as the complexity that increases as a function of time due to a continued expansion in the number of possible combinations with time, which may eventually lead to a chaotic state or a system failure.

Example: Job shop scheduling

Future scheduling is affected by the decisions made earlier and it's complexity is a function of the decisions made over its past history

4. Time-Dependent Periodic Complexity

The periodic complexity is *defined* as the complexity that only exists in a finite period, resulting in a finite and limited number of probable combinations.

Example: Airline flight scheduling

All of the uncertainties introduced during the course of a day terminate at the end of a 24-hour cycle, and hence the complexity does not extend to the following day

Reduction of Time-Dependent Complexity

Transform

a design with time-dependent *combinatorial* complexity

to

a design with time-dependent *periodic* complexity

“Complexity” can be reduced by taking the following actions:

- Reduce Time-Independent Real Complexity
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Reduction of Combinatorial Complexity

How?

Through
"*Re-initialization*" of the system
by defining
a "*Functional Period*".

Note: Functional period is defined by a repeating set of functions, not by time period, unless "time" is a set of functions.

Transformation of Time-Dependent Combinatorial Complexity

- **Basic Idea**
  1. Make sure that the design satisfies the Independence Axiom.
  2. Identify a set of FRs that undergoes a cyclic change and has a functional period.
  3. Identify the functional requirement that may undergo a combinatorial process
  4. $$T\left\langle C_{com}(FR_i)\right\rangle \Rightarrow \left\langle C_{per}(FR_i)\right\rangle$$
  6. "Reinitialization"
  5. Set the beginning of the cycle as t=0

Functional Periodicity

- The functional periodicity are the following types:

(1) Temporal periodicity
(2) Geometric periodicity
(3) Biological periodicity
(4) Manufacturing process periodicity
(5) Chemical periodicity
(6) Thermal periodicity
(7) Information process periodicity
(8) Electrical periodicity
(9) Circadian periodicity
(10) Materials periodicity

Example: Productivity of A Manufacturing System that Consists of Many Sub-Systems

- **How do we maximize the productivity of a manufacturing system?**
- **Example: Semiconductor Manufacturing**