2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

# Control of Manufacturing Processes

**Subject 2.830/6.780/ESD.63**

**Spring 2008**

**Lecture #16**

## Process Robustness

**April 10, 2008**

# Outline

- Last Time
  - Optimization Basics
  - Empirical Response Surface Methods
    - Steepest Ascent - Hill Climbing Approach
- Today
  - Process Robustness
    - Minimizing Sensitivity
    - Maximizing Process Capability
  - Variation Modeling
    - Noise Inputs as Random Factors
  - Taguchi Approach
    - Inner - Outer Arrays

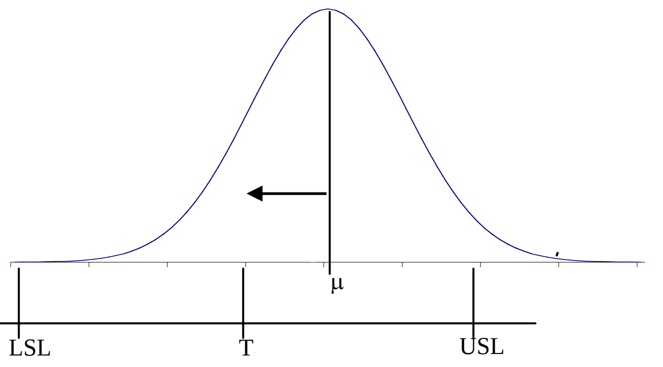
# What to Optimize?

- Process Goals
  - Cost (Minimize)
  - Quality (Maximize Cpk or Minimize E(L))
  - Rate (Maximize)
  - Flexibility (N/A for now)

# Simple Problem: Minimum Cost

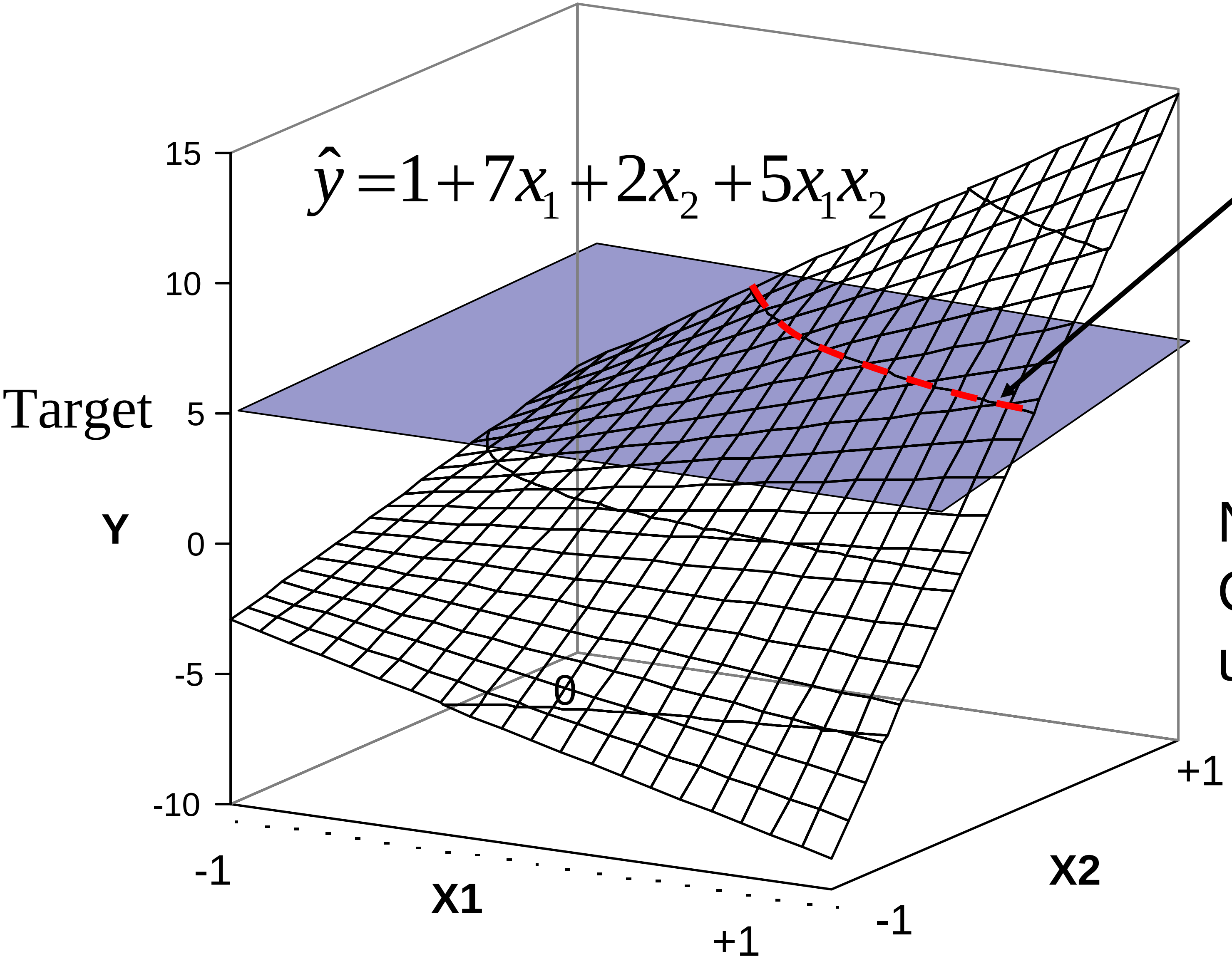
- Must Hit Target

$$\bar{x} = T$$




- Multiple Input Factors
  - Contours of constant output
  - Match to Target
  - Assume constant output variance
- Choose Operating Point to
  - Minimize Cost (e.g. material usage; tool wear, etc)
  - Minimize Cycle Time

# Linear Model with Constraint




Need Second Criterion to select unique $x_1$ and $x_2$

- cost
- rate

# Quality: Minimum Variation

$$\Delta Y = \frac{\partial Y}{\partial \alpha}\Delta \alpha + \frac{\partial Y}{\partial u}\Delta u$$

- Minimize Sensitivity to $\Delta\alpha$
  - Process Robustness
- Maximize $C_{pk}$
- Minimize expected quality loss: E{L(x))}

# Maximizing Cpk

$$C_{pk} = \min\left(\frac{(USL - \mu)}{3\sigma}, \frac{(LSL - \mu)}{3\sigma}\right)$$

Measure using estimates of response of *y* and *s*:

$$C_{pk} = \min\left(\frac{(USL - \hat{y})}{3\hat{s}}, \frac{(LSL - \hat{y})}{3\hat{s}}\right)$$

Or create a new response variable from the raw data

$$\eta_j = \min\left(\frac{(USL - \bar{y}_j)}{3s_j}, \frac{(LSL - \bar{y}_j)}{3s_j}\right)$$

- Single variable that combines *y* and *s*
- Could be discontinuous

# Variance Dependence on Operating Point

- We often assume that $\sigma^2$ is constant throughout the operating space
  - Implicit in simple ANOVA, most regression fits
  - Process optimization might also assume this
    - E.g. $C_{pk}$, E(L), sensitivity to $\alpha$ independent of *u*

- Reality: process variation may be different at different operating points!
  - Imperfect control of *u* implies $\delta Y/\delta u$ can vary, if model/dependence is nonlinear
  - Presence or sensitivity to noise may depend on *u*

# Process Output Variance

- We can define the response variable as $\eta = \sigma_j$ and solve for $\underline{\eta} = \mathbf{X}\underline{\beta} + \underline{\varepsilon}$

| | Input and Levels | | | Response Replicates | | | Within Test mean | Within Test std.dev. |
|---|---|---|---|---|---|---|---|---|
| Test | x 1 | x 2 | | $\eta_{i1}$ | $\eta_{i2}$ | $\eta_{i3}$ | ybar $_i$ | $s_i$ |
| 1 | - | - | | $\eta_{11}$ | … | … | y bar $_1$ | $s_1$ |
| 2 | + | - | | … | … | … | y bar $_2$ | $s_2$ |
| 3 | - | + | | … | … | $\eta_{33}$ | y bar $_3$ | $s_3$ |
| 4 | + | + | | | | $\eta_{43}$ | y bar $_4$ | $s_4$ |

New Response Variable

# Process Output Variance

- Solve for $\underline{\eta} = \mathbf{X}\underline{\beta} + \underline{\varepsilon}$ using the same **X** matrix as with y.

- This will yield a “variance response surface”

- Linear model: minimum at the boundary

# Combining Mean and Variance:

- Find the line (or general function) defining minimum error from the *y* response surface
- Find the minimum variance using those constrained $x_1$ and $x_2$ values

# Combining Mean and Variance: Direct Method

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$$

y surface

$$y^* = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 = \text{target}$$

solve for $x_1$

$$x_1 = \frac{(y^* - \beta_0 - \beta_1 x_2)}{\beta_1 + \beta_{12} x_2} x_2$$

(line on surface)

$$\hat{s} = \beta'_0 + \beta'_1 x_1 + \beta'_2 x_2 + \beta'_{12} x_1 x_2$$

s surface

# Combining Mean and Variance: Direct Method

Substitute for $x_1$ in the $s$ equation and find minimum

$$x_1 = \frac{(y^* - \beta_0 - \beta_1 x_2)}{\beta_1 + \beta_{12} x_2} x_2$$

$$\hat{s} = 1 + \beta'_1 x_1 + \beta'_2 x_2 + \beta'_{12} x_1 x_2$$

s surface

$$\hat{s} = f(x_2) \text{ it will be non} - \text{linear in general}$$

$$\frac{\partial \hat{s}}{\partial x_2} = 0 \quad \text{solve for } x_2$$

# Minimizing E(L)

$$E\{L(x)\} = k\sigma_x^2 + k(\mu_x - x^*)^2$$

define a new response variable:

$$\eta_j = ks_{\bar{y}_j}^2 + k(\bar{y}_j - y^*)^2$$

and find min($\eta$)

NOTE: Since the response variable is *quadratic* in *y* and *s*, the new estimation model should be *quadratic* as well

# Problems?

- With Variance Varying?
- What Caused Non-Constant Variance?
- Can We Assess “Robustness”?

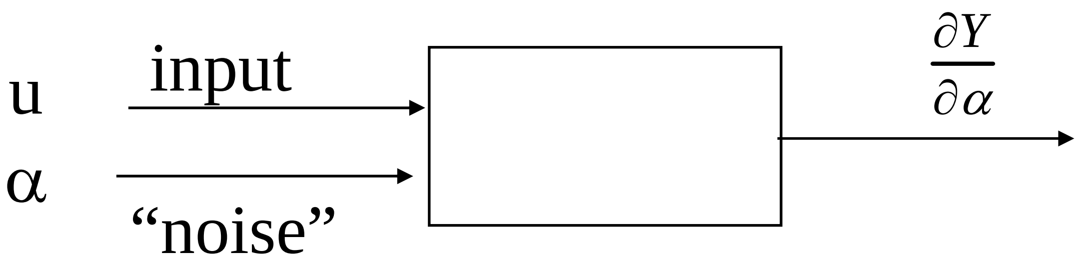
# Use of the Variation Model

Recall: $$\Delta Y = \frac{\partial Y}{\partial \alpha}\Delta \alpha + \frac{\partial Y}{\partial u}\Delta u$$

Disturbance Sensitivity

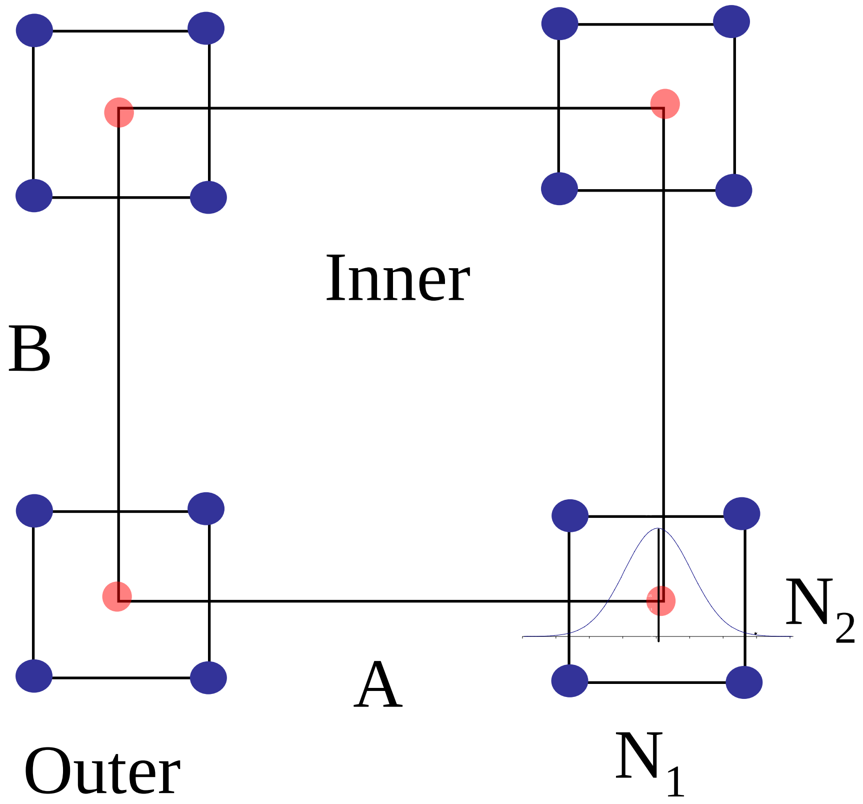
$$\Rightarrow \quad \frac{\partial Y}{\partial \alpha} = f(u, \alpha)$$

How would we minimize $\frac{\partial Y}{\partial \alpha}$ ?



# Robustness to Noise Factors – Inner and Outer Factors

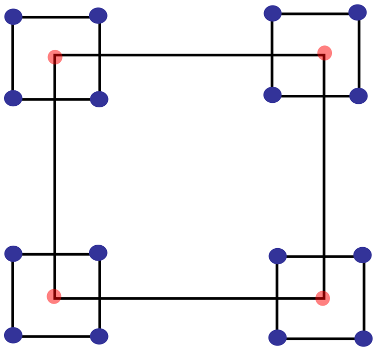
- Find Control Factors that Minimize Effect of Noise
- Taguchi Approach: Varying Noise Factors at Each Level of Control Factors




$k_n$ = # noise factors
$k_c$ = # control factors

Each corner gives a measure of $\frac{\partial Y}{\partial \alpha}$

# Robustness to Noise Factors



## Crossed Array Design

Variance caused by Outer array variations

**Outer Array**

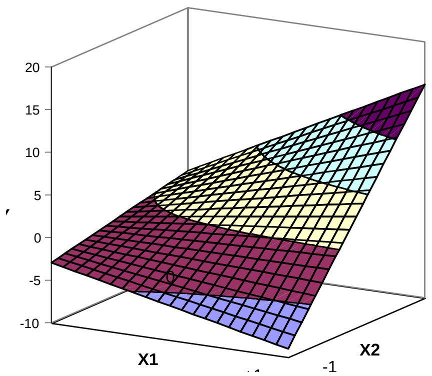
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | N1 | -1 | 1 | -1 | 1 | | |
| | N2 | -1 | -1 | 1 | 1 | | |
| A | B | | | | | Average | Variance |
| -1 | -1 | Yij | Yij | Yij | Yij | Ybar1 | s21 |
| 1 | -1 | Yij | Yij | Yij | Yij | Ybar2 | s22 |
| -1 | 1 | Yij | Yij | Yij | Yij | Ybar3 | s23 |
| 1 | 1 | Yij | Yij | Yij | Yij | Ybar4 | s24 |

Inner Array

Number of Tests?

Taguchi S/N = $-10\log\frac{\bar{y}_i^{\,2}}{S_i^{\,2}}$

| S/N |
|---|
| SN1 |
| SN2 |
| SN3 |
| SN4 |



# Taguchi – Signal-to-Noise Ratios

- Nominal the best:

$$SN_N = 10 \log(\bar{y}/s)$$

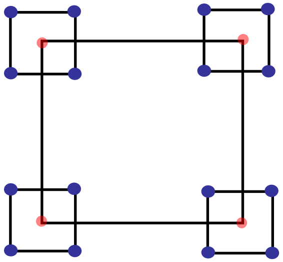
- Larger the better:

$$SN_L = -10 \log(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2})$$

- Smaller the better:

$$SN_S = -10 \log(\frac{1}{n} \sum_{i=1}^{n} y_i^2)$$

# Crossed Array Method



- Number of tests
  - Control Factor tests * Noise Factor tests
  - Linear model leads to linear response surface for S/N
  - True Optimum requires Quadratic test on inner array
    - # Tests = $3^{kc}$ $2^{kn}$ unless interaction ignored
    - $3^{kc}$ requirement can be reduced with use of central composite or related designs

# Robustness Using Noise Response

$$\Delta Y = \frac{\partial Y}{\partial \alpha}\Delta\alpha + \frac{\partial Y}{\partial u}\Delta u$$

*u:* • control factors

*α:* • some can be manipulated if desired (Noise Factors)

• some cannot (Pure error)

Treat *control and noise* as factors for experiments:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1$$

$x_i$ are control factors, $z_j$ are noise factors

# Noise Response Surface Approach

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \varepsilon$$

Assume $z_1 \sim N(0, \sigma_z)$

$\varepsilon \sim N(0, \sigma)$

Full factorial in $x_1$ and $x_2$ (Control Factors)

Interaction terms for $z_1$ (Noise Factors)

- Why are they vital?

# Noise Response Models

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \varepsilon$$

Assume $z_1 : N(0, \sigma_z)$

$\varepsilon : N(0, \sigma)$

Mean of Response:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2$$

Variance of Response:

$$V(y) = (\gamma_1 + \delta_{11} x_1 + \delta_{21} x_2)^2 \sigma_z^2 + \sigma^2$$

<u>Now variance is a function of control factors</u>

# Variance Models

$$V(y) = (\gamma_1 + \delta_{11}x_1 + \delta_{21}x_2)^2 \sigma_z^2 + \sigma^2$$

$$V(x_1, x_2) = (\gamma_1{}^2 + 2\gamma_1\delta_{12}x_1 + 2\gamma_1\delta_{21}x_2 + \delta_{12}{}^2 x_1{}^2 + \delta_{21}{}^2 x_2{}^2 + 2\delta_{12}\delta_{21}x_1\, x_2)\sigma_z^2$$

Quadratic in $x_1$, $x_2$

# Example:  Robust Bending

- Control Factors
  - Depth of Punch  $x_1$
  - Width of Die $x_2$
- Noise Factors
  - Yield Point of Material $z_1$
  - Thickness of Material $z_2$

# Example: Robust Bending

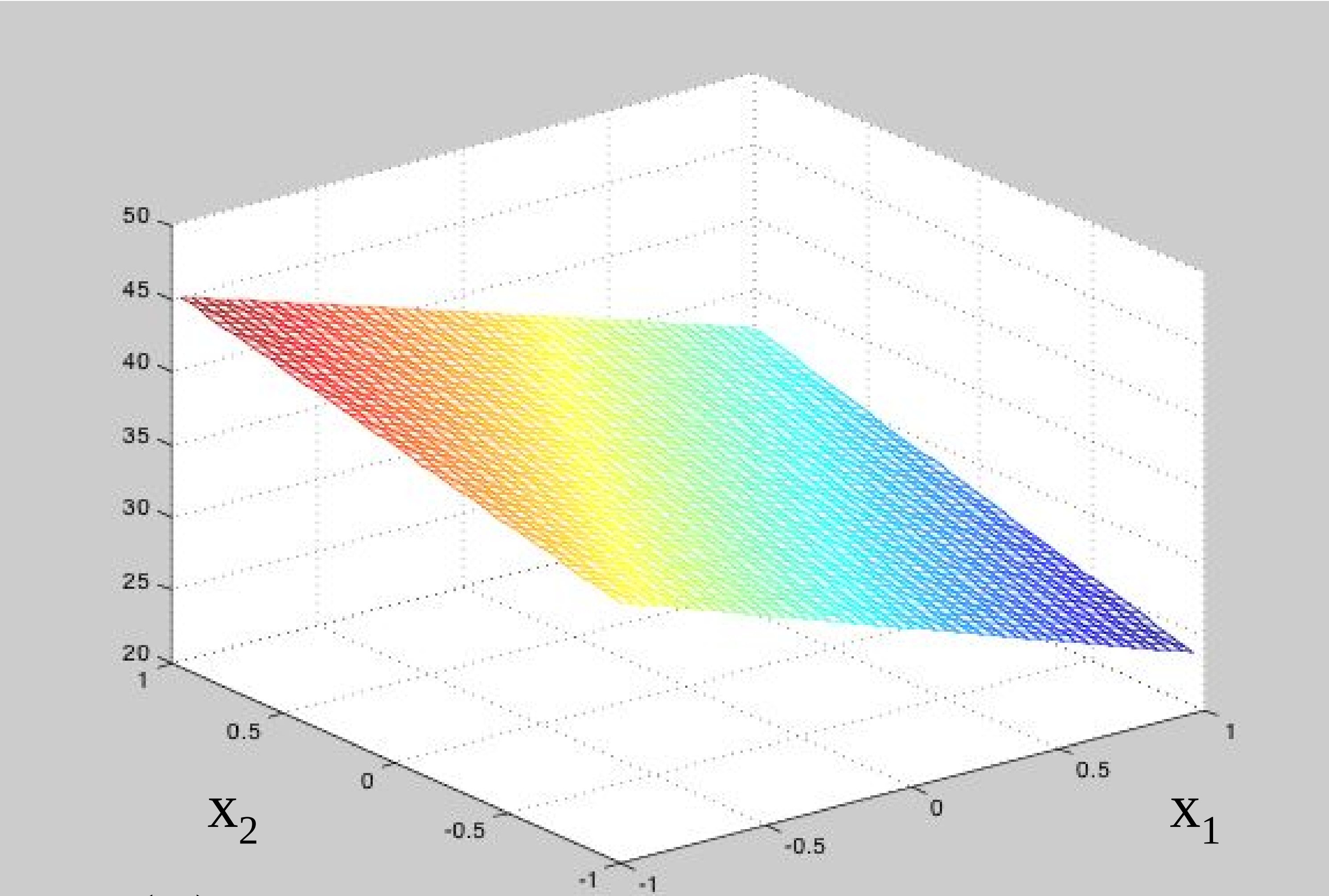
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \varepsilon$$

| yp<br>x1 | w<br>x2 | sigma<br>z | Mean<br>Angle |
|---|---|---|---|
| -1 | -1 | -1 | 38.1 |
| 1 | -1 | -1 | 45.9 |
| -1 | 1 | -1 | 24.1 |
| 1 | 1 | -1 | 33.2 |
| -1 | -1 | 1 | 37.8 |
| 1 | -1 | 1 | 45.3 |
| -1 | 1 | 1 | 23.7 |
| 1 | 1 | 1 | 32.6 |

| | |
|---|---|
| β0 | 35.09 |
| β1 | 4.16 |
| β2 | -6.69 |
| γ1 | -0.24 |
| β12 | 0.34 |
| δ11 | -0.06 |
| δ21 | -0.01 |
| β123 | 0.01 |

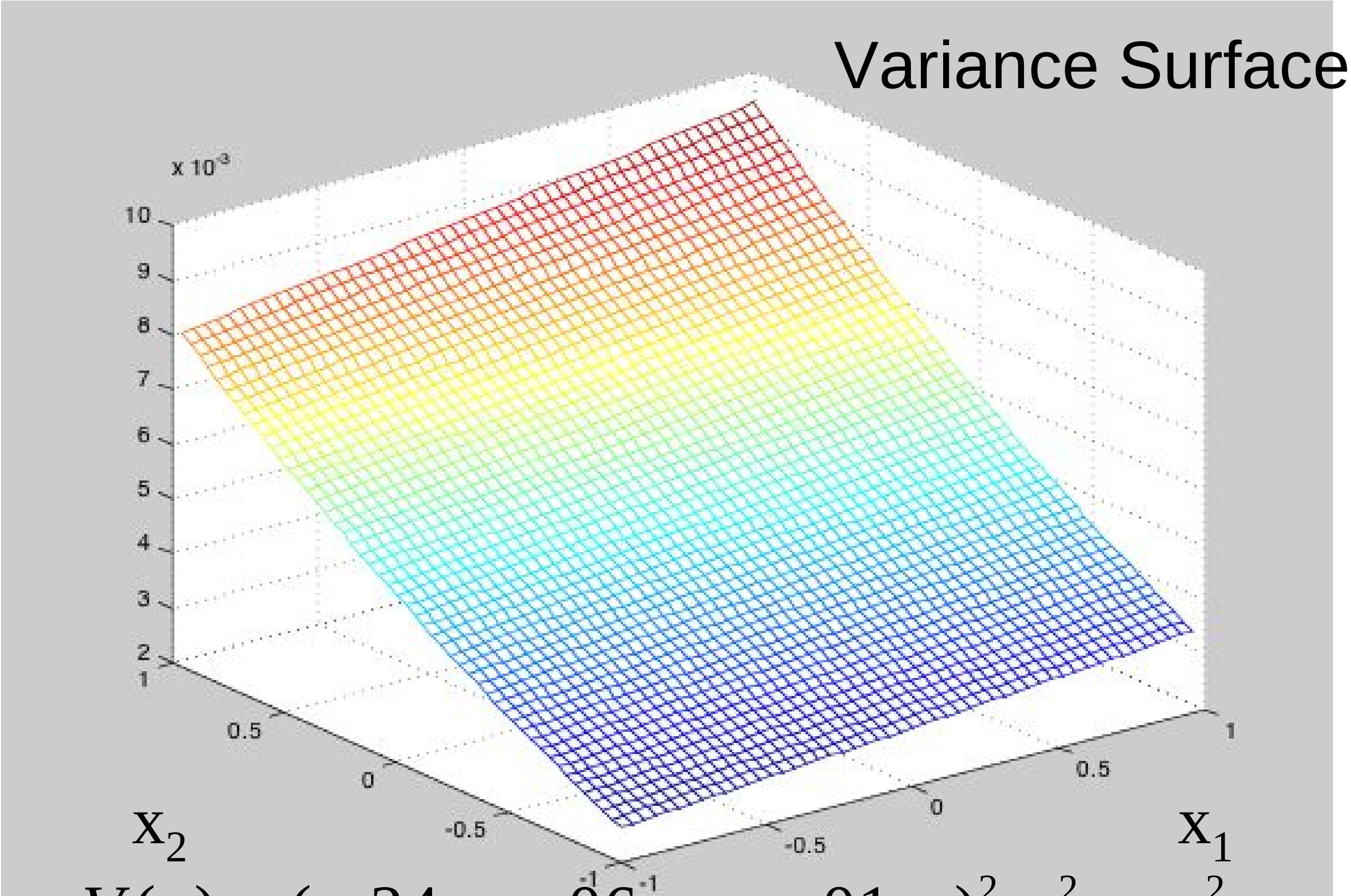
$$y = 35.1 + 4.2x_1 - 6.7x_2 + 0.34x_1x_2$$
$$- .24z_1 - .06x_1z_1 + -0.013x_2z_1 + \varepsilon$$

# Example:  Robust Bending



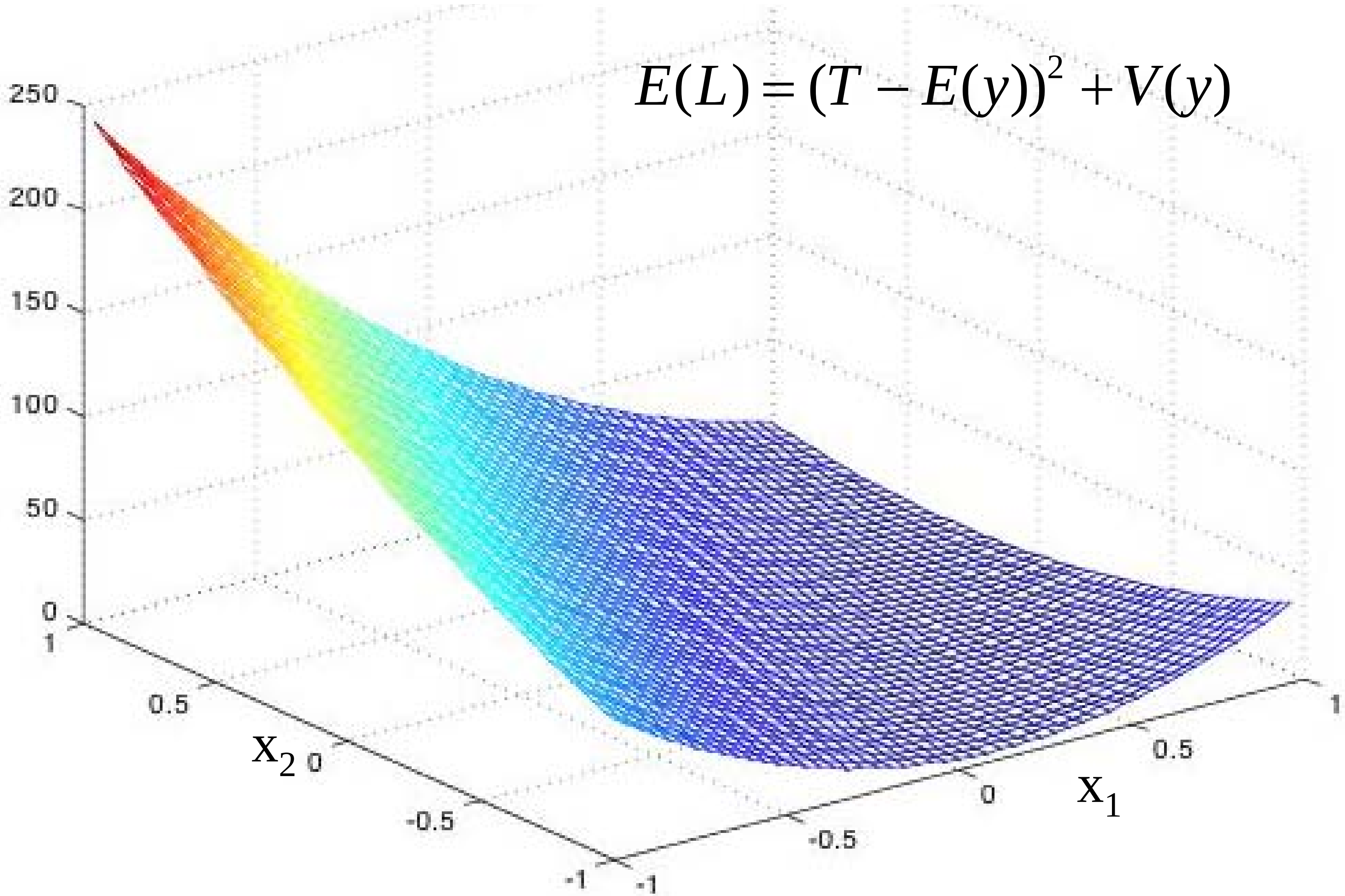

$$E(y) = 35.1 + 4.2x_1 - 6.7x_2 + 0.34x_1x_2$$

# Example: Robust Bending




$$V(y) = (-.24 + -.06x_1 + -.01x_2)^2 \sigma_z^2 + \sigma^2 \qquad (\sigma_z^2 = 0.1)$$

# Example: Robust Bending



# Response Model Method

- Define Control and Noise Factors
- Perform Appropriate Linear Experiment
- If possible scale noise factor changes to $\pm 1\sigma_z$
  - *(Assumes we know noise factor statistics)*
- Define Response Surface for V
- Optimize V, Subject to desired E(y)
- Number of Tests?
  - Full factorial with center point: $(2^{kc}+1)(2^{kn})$
  - Quadratic in control: $3^{kc}\ 2^{kn}$
  - RSM, full factorial with center point: $(2^{kc+kn} + 1)$
  - RSM, central composite: $2^{kc+kn}+2(kc+kn)+1$
- Taguchi orthogonal arrays: fractional factorials ignoring noise factor interactions

# Comparison

| Control Factors | Noise Factors | Crossed Array Lin. | Crossed Array (quad) | Response Surface |
|---|---|---|---|---|
| 2 | 1 | 8 | 18 | 6 |
| 2 | 2 | 16 | 36 | 13 |
| 3 | 3 | 64 | 216 | 60 |
| 4 | 3 | 128 | 648 | 124 |

- Crossed array with S/N does not adjust mean
- Size of Experiments is Large vs. RSM
- Forces use of Fractional Factorial DOE
  - Assumes little or no Interaction

# Conclusions: Process Optimization

- Cost, Rate, Quality
- Quality:
  - Min E(L)
  - Max $C_{pk}$
  - Max S/N
- All depend on variation equation:

$$\Delta Y = \frac{\partial Y}{\partial \alpha} \Delta \alpha + \frac{\partial Y}{\partial u} \Delta u \quad \Rightarrow \quad \frac{\partial Y}{\partial \alpha} = f(u, \alpha)$$