

# **Toward a 1D Device Model Part 1: Device Fundamentals**

Lecture 7 – 9/29/2011

MIT Fundamentals of Photovoltaics  
2.626/2.627 – Fall 2011

Prof. Tonio Buonassisi

# Learning Objectives: Toward a 1D Device Model

1. Describe the difference between “Energy Conversion Efficiency” and “Quantum Efficiency.”
2. Describe common factors that cause solar cell IV curves to deviate from an ideal diode model: shunt & series resistance, recombination currents, and current crowding.
3. Calculate series resistance for a solar cell.
4. Calculate the Fermi Energy of a solar cell as a function of dopant concentration, illumination condition, and temperature.
5. Calculate carrier generation as a function of depth in a solar cell.
6. Calculate how material quality (minority carrier diffusion length) affects QE and solar cell performance.
7. Create a 1D model for solar cell performance based on diffusion length, optical absorption coefficient, surface reflectivity, and series & shunt resistances.

## Key Concept:

“Energy conversion efficiency” is not the same thing as “quantum efficiency”.

“Quantum efficiency (QE)” is defined as the number of electrons out per incident photon. Note that QE is simply a census: it does not take into consideration the energy of the electron or photon.

QE is generally reported as a function of wavelength. QE is a useful troubleshooting tool to identify why a device is underperforming.

QE values can be quite high (between 60 and 99% for certain wavelengths), and thus can be used by devious individuals to misrepresent the conversion efficiency of their solar cell device.

# A Note about “Efficiency”

## LETTERS

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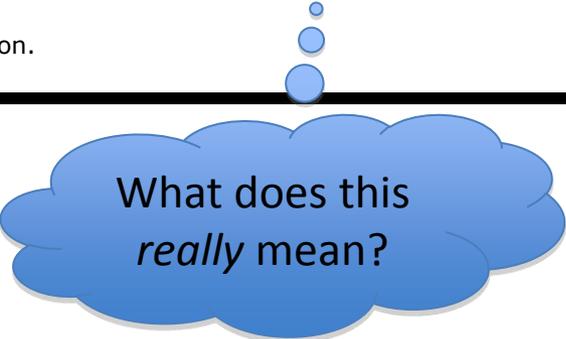
### Solution-processed PbS quantum dot infrared photodetectors and photovoltaics

Under  $-5$  V bias and illumination from a 975 nm laser, our detectors show an internal quantum efficiency of 3%, a ratio of photocurrent to dark current of 630, and a maximum responsivity of  $3.1 \times 10^{-3}$  A W $^{-1}$ . The photovoltaic response under 975 nm excitation results in a maximum open-circuit voltage of 0.36 V, short-circuit current of 350 nA, and short-circuit internal quantum efficiency of 0.006%.

Courtesy of Edward H. Sargent. Used with permission.

#### Technical Terms:

- Solar Conversion Efficiency
- External Quantum Efficiency
- Internal Quantum Efficiency



What does this  
*really* mean?

# Solar Conversion Efficiency: Defined in Previous Slides

$$\eta = \frac{\text{Power Out}}{\text{Power In}} = \frac{I_{\text{mp}} \cdot V_{\text{mp}}}{\Phi} = \frac{\text{FF} \cdot I_{\text{sc}} \cdot V_{\text{oc}}}{\Phi}$$

Typical values are 12–20% for established technologies, <10% for most emerging technologies.

$\eta$  and  $\Phi_{\text{F}}$ : Vary with illumination intensity (e.g., 1 Sun)

# External Quantum Efficiency

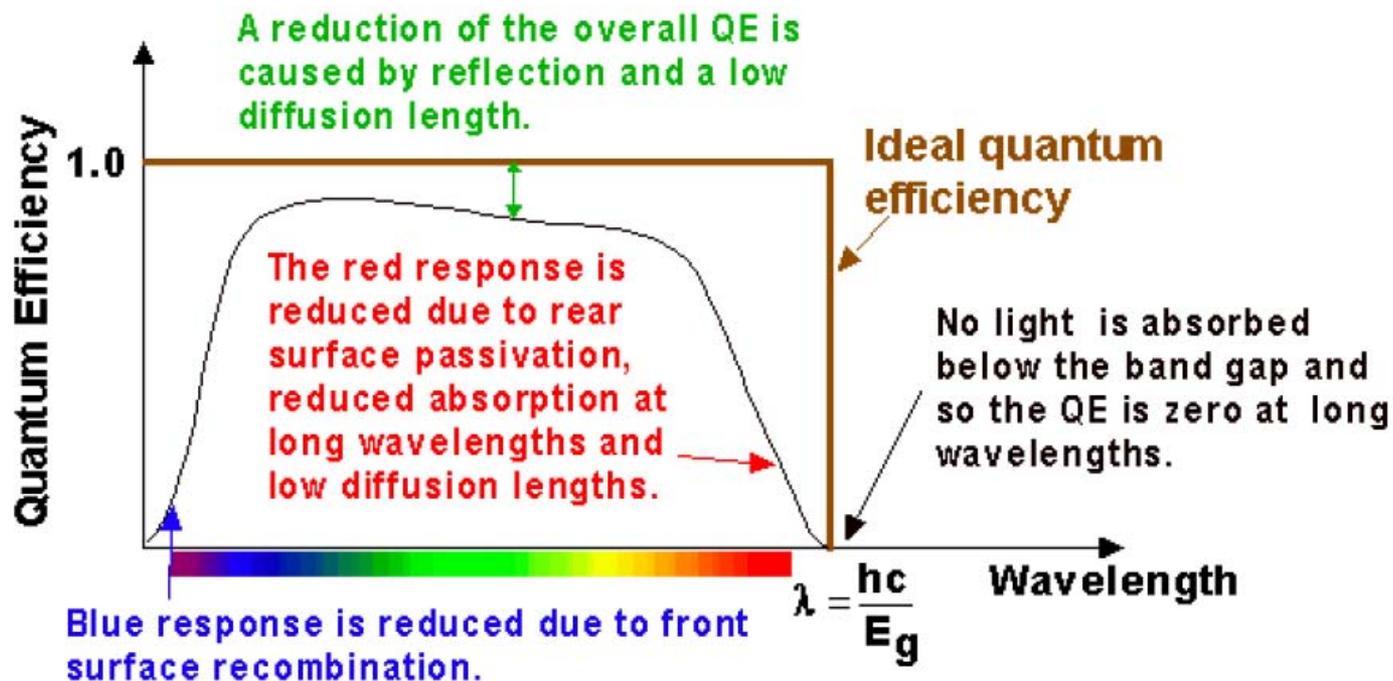
$$\text{EQE} = \frac{\text{Electrons Out}}{\text{Photons In}}$$

Typical peak values are 60–90%, depending on reflectivity, for moderate-efficiency devices.

EQE highly wavelength- and illumination-dependent!

# External Quantum Efficiency

Here's an example of a QE spectrum for a solar cell. Note the near-unity (i.e., 100%) QE in the visible wavelengths.



Courtesy of [PVCDROM](#). Used with permission.

from PVCDROM

# Internal Quantum Efficiency

$$\text{IQE} = \frac{\text{EQE}}{(1-R)} = \frac{\text{Electrons Out}}{(\text{Photons In}) \cdot (1-R)}$$

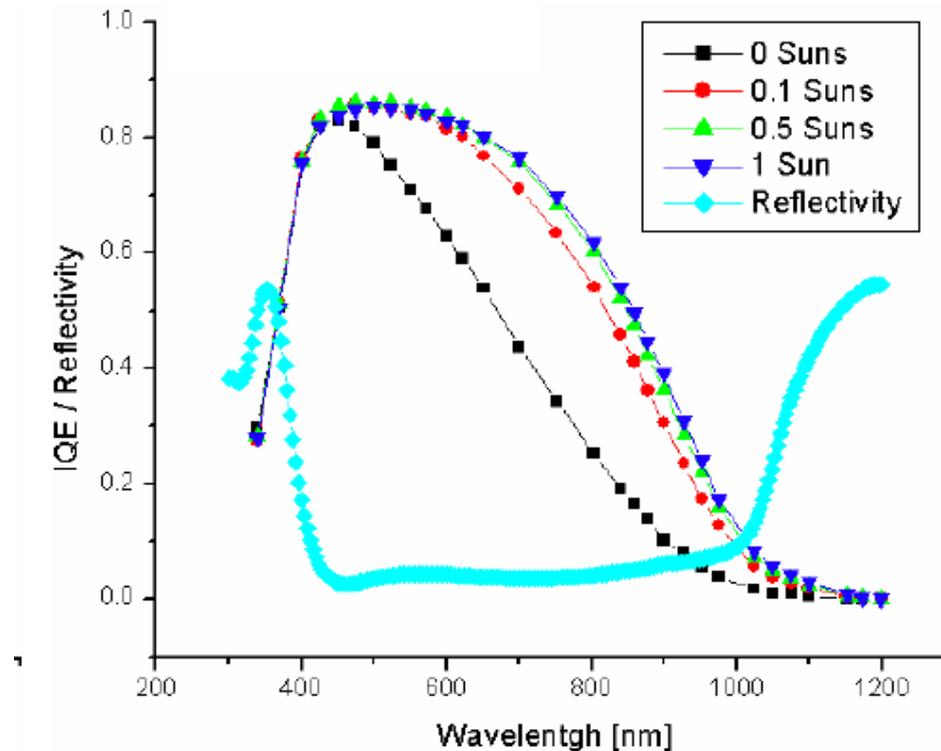
... where R = Reflectivity

Typical peak values between 80–98% for moderate-efficiency devices.

IQE highly wavelength- and illumination-dependent!

# Internal Quantum Efficiency

Reflectivity and IQE  
(measured with different bias illumination)



Examples of illumination-dependent IQE measurements for a defect-rich multicrystalline silicon solar cell. Minority carrier trapping results in low IQE with low bias illumination.

## Approach “efficiency” with a grain of salt:

When an efficiency is quoted, think about:

- What “efficiency” is being measured?
- What is the nature of the light being used?
  - What spectrometer to simulate solar spectrum?
  - If monochromatic, what wavelength?
  - What intensity (photon flux)?

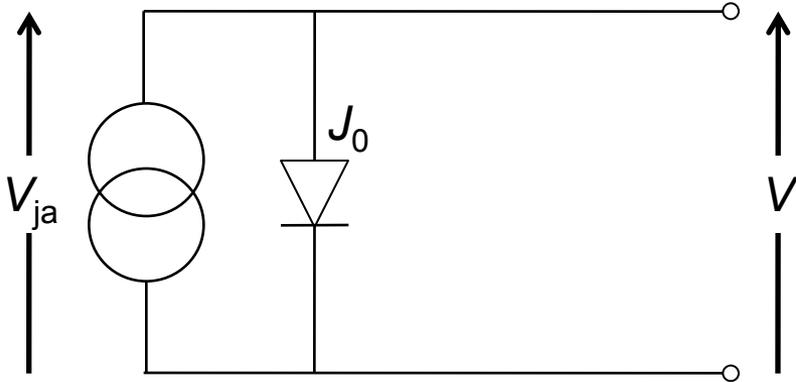
# An example of honest efficiency reporting

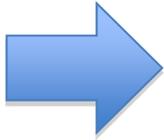
Please see the abstract from Huynh, W., J. Dittmer et al. "[Hybrid Nanorod-Polymer Solar Cells](#)." *Science* 295, no. 5564 (2002): 2425-7.

# Learning Objectives: Toward a 1D Device Model

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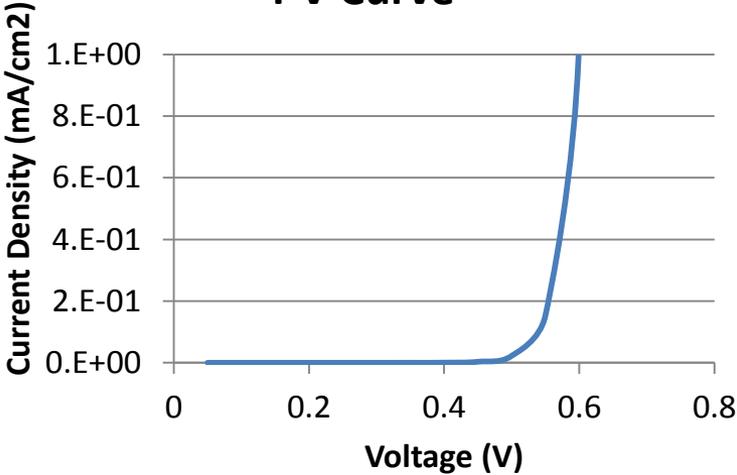
# Equivalent Circuit: Simple Case



$$J = J_0 \left( \exp\left(\frac{qV}{kT}\right) - 1 \right) - J_L$$


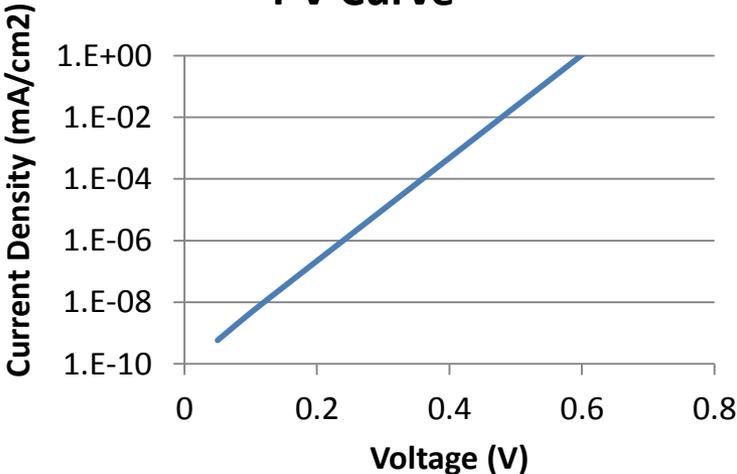
Lin Scale

I-V Curve

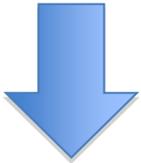
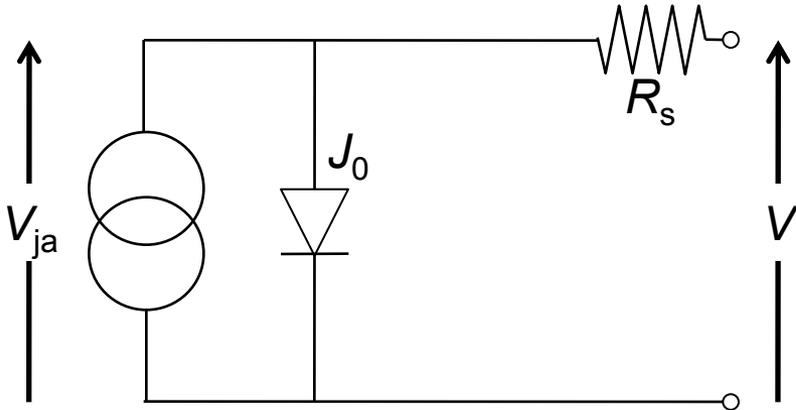


Log Scale

I-V Curve

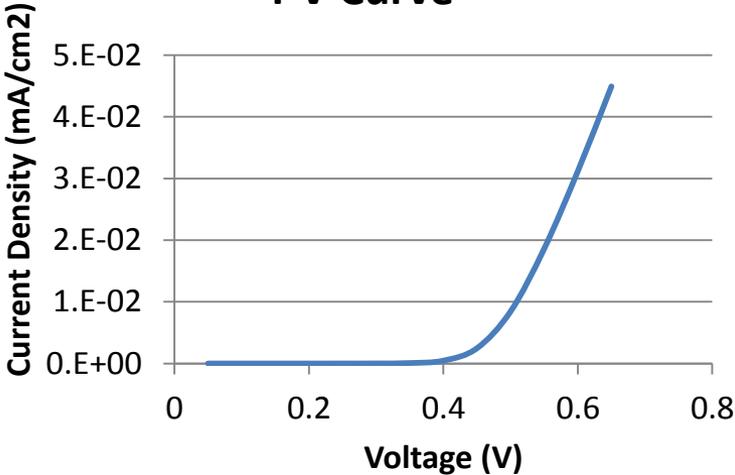


# Equivalent Circuit: Simple Case

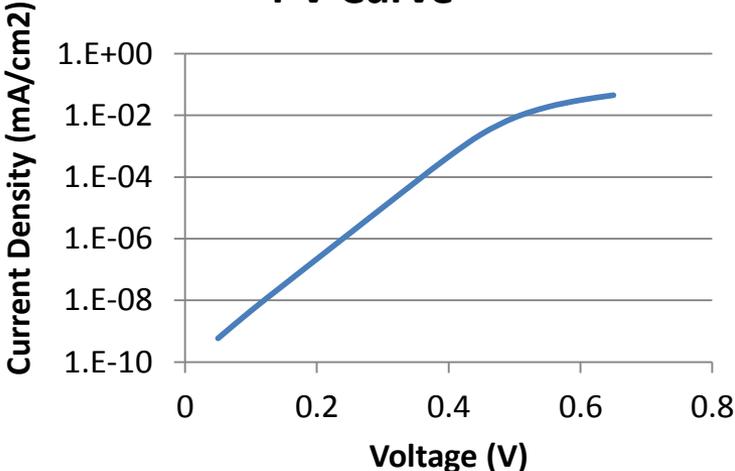


$$J = J_0 \left( \exp \left( \frac{q(V - JR_s)}{kT} \right) - 1 \right) - J_L$$

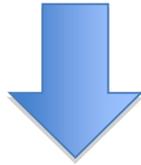
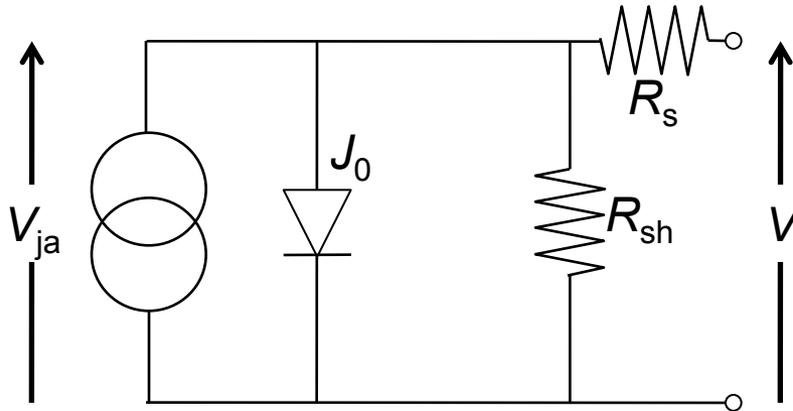
I-V Curve

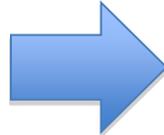


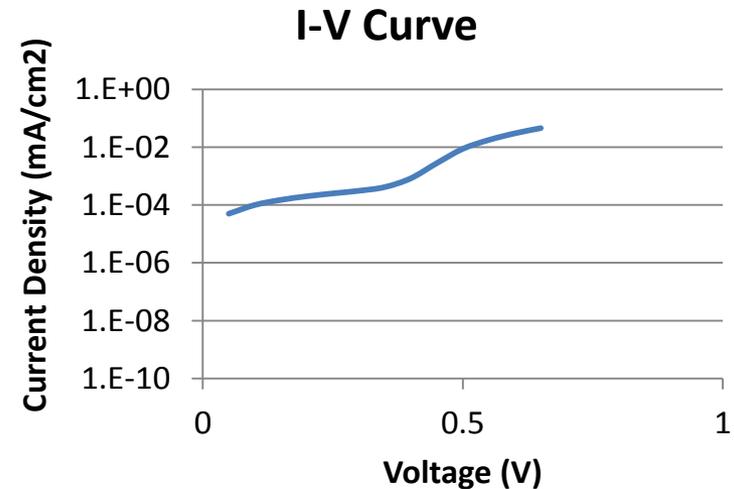
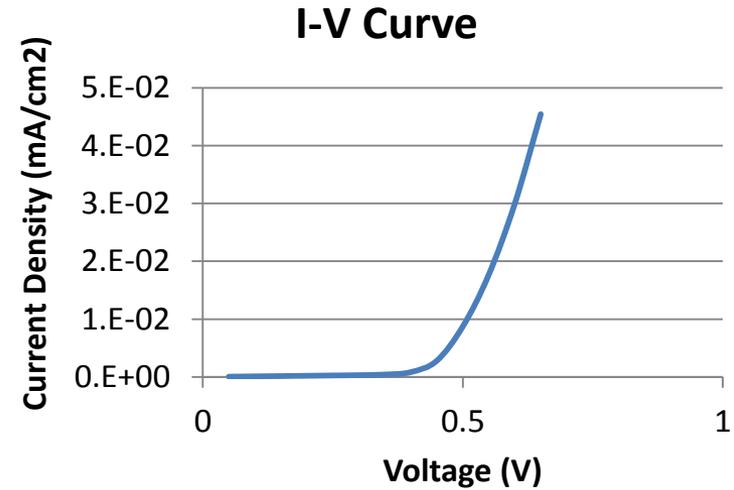
I-V Curve



# Equivalent Circuit: Simple Case



$$J = J_0 \left( \exp \left( \frac{q(V - JR_s)}{kT} \right) - 1 \right) + \frac{V - JR_s}{R_{sh}} - J_L$$




## Key Concepts:

The ideal diode equation can be enhanced in two key ways:

- 1) We can add the effects of parallel resistance and series resistance.
- 2) Advanced Concept: Instead of one saturation current  $I_0$ , there are usually two saturation currents contributing to most solar cell devices: (a) one resulting from carrier recombination in the space-charge region (dominant at lower forward bias voltages) and (b) one resulting from carrier recombination in the bulk (dominant at higher forward bias voltages). The “two-diode model” takes both saturation currents into account.

$$J = J_{01} \exp\left(\frac{q(V - JR_s)}{n_1 kT}\right) + J_{02} \exp\left(\frac{q(V - JR_s)}{n_2 kT}\right) + \frac{V - JR_s}{R_{sh}} - J_L$$

## Further Reading:

Green, Chapter 5

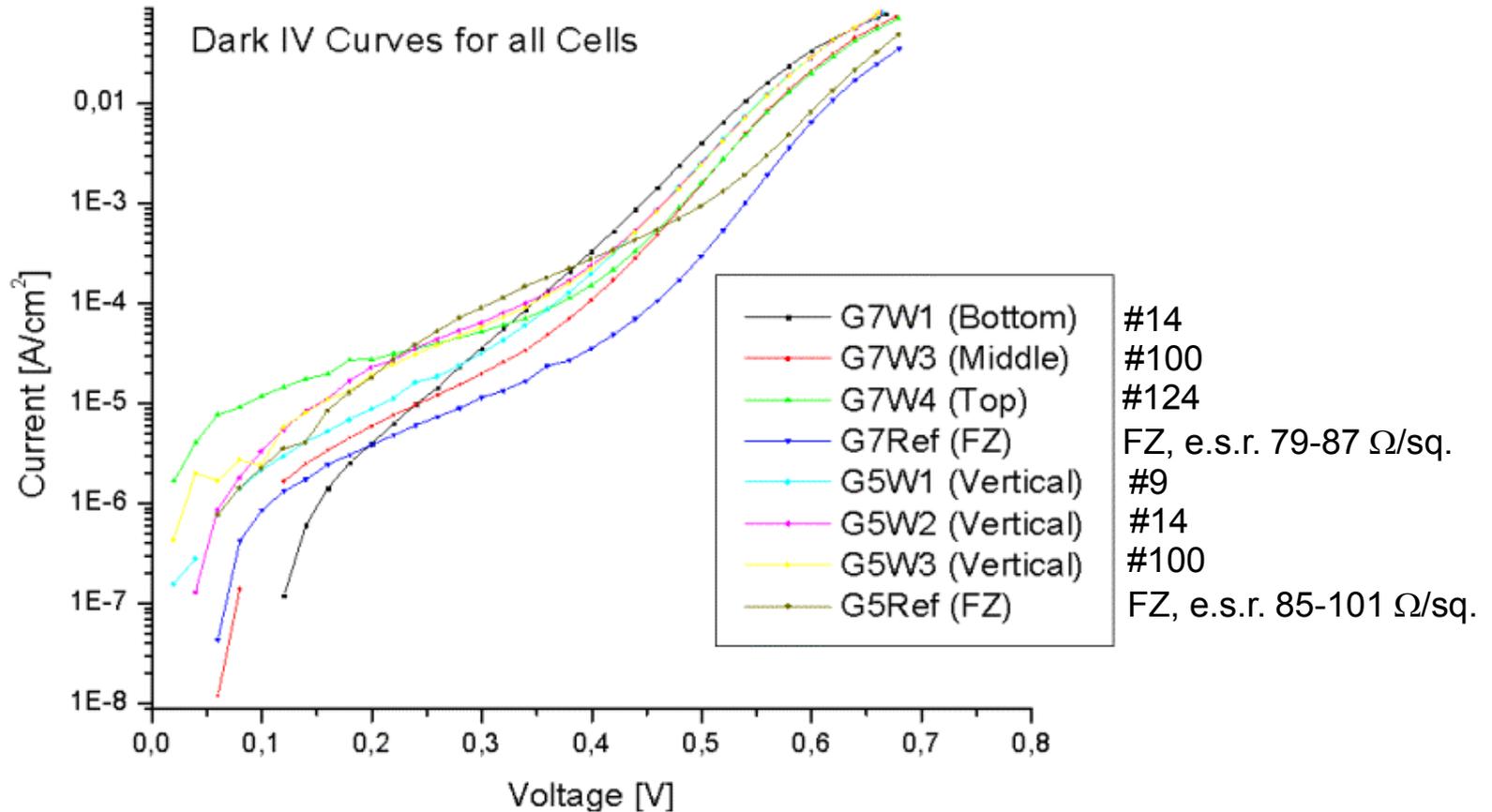
PVCDROM, Chapter 4: *Solar Cell*

*Operation* <http://www.pveducation.org/pvcdrom/solar-cell-operation/solar-cell-structure>

K. McIntosh: “Lumps, Humps and Bumps: Three Detrimental Effects in the Current-Voltage Curve of Silicon Solar Cells,” Ph.D. Thesis, UNSW, Sydney, 2001

# IV Curve Measurements

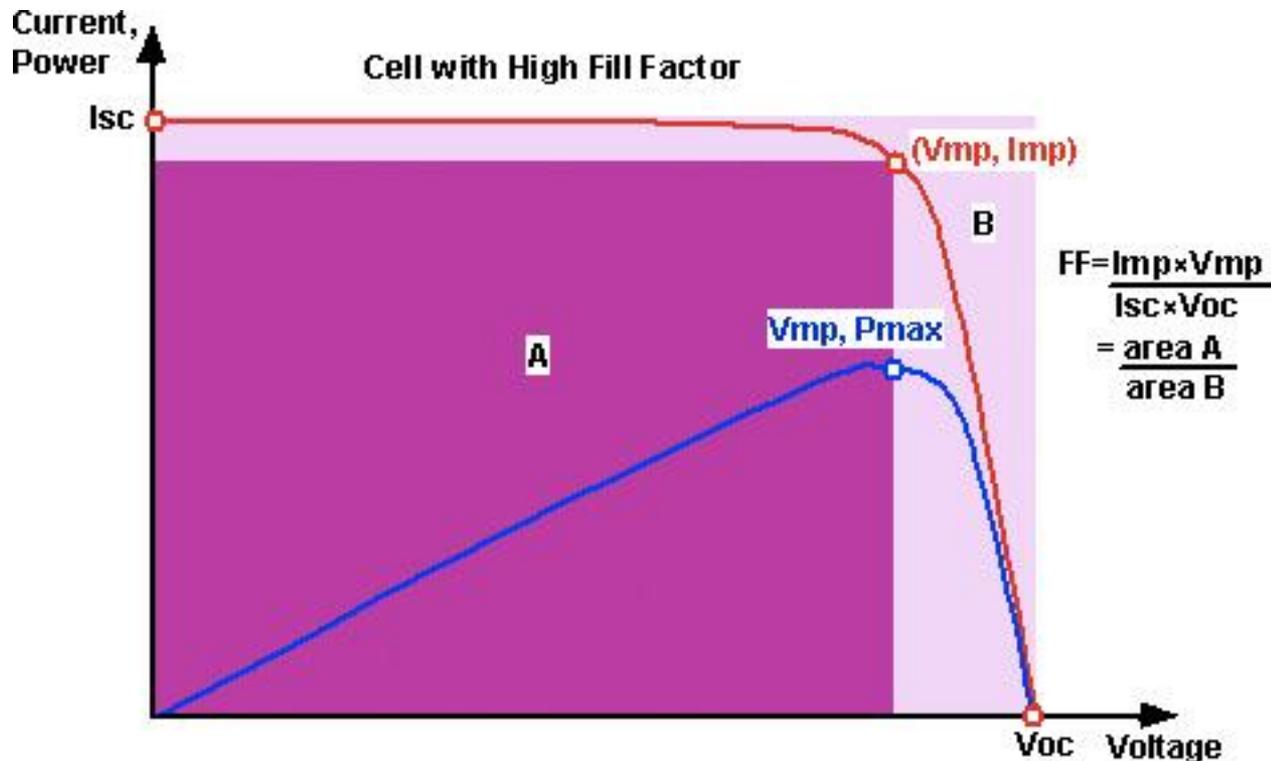
Several IV curves for real solar cells, illustrating a variety of IV responses!



# Fill Factor

# Why Fill Factor (FF) Matters:

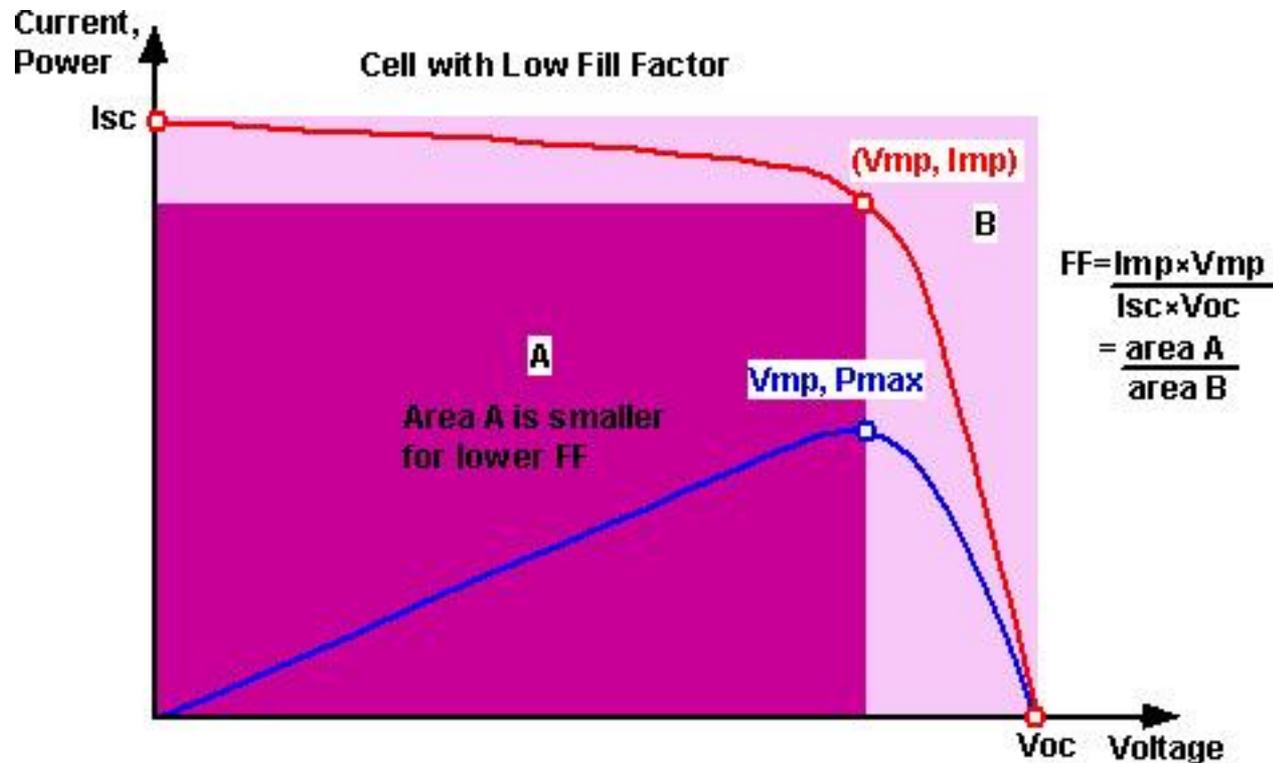
This is a sample IV curve for a high-efficiency solar cell: High FF.



Courtesy of [PVCDROM](#). Used with permission.

# Why Fill Factor (FF) Matters:

This is a sample IV curve for a low-efficiency solar cell (same  $I_{sc}$  and  $V_{oc}$ , but lower FF).

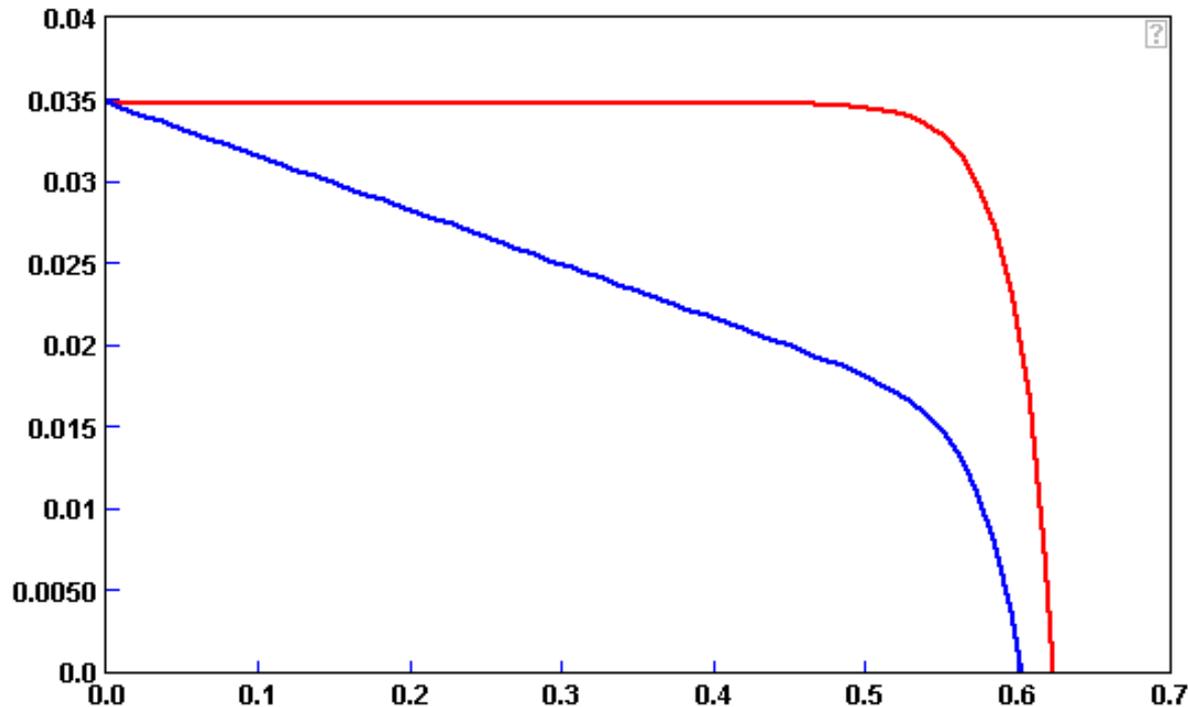


Courtesy of [PVCDROM](#). Used with permission.

# Effect of Low Shunt Resistance ( $R_{sh}$ )

$$J = J_L - J_0 \exp\left(\frac{q(V + JR_s)}{nkT}\right) - \frac{V + JR_s}{R_{shunt}}$$

Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/shunt-resistance>



Ideal Solar Cell:  $I_{sc} = 0.035$   $V_{oc} = 0.622$   $FF = 0.83$

Real Solar Cell:  $I_{sc} = 0.035$   $V_{oc} = 0.601$   $FF = 0.43$

Shunt R: 1

# Physical Causes of Shunt Resistance ( $R_{sh}$ )

Paths for electrons to flow from the emitter into the base. Can be caused by physical defects (scratches), improper emitter formation, metallization over-firing, or material defects (esp. those that traverse the space-charge region).

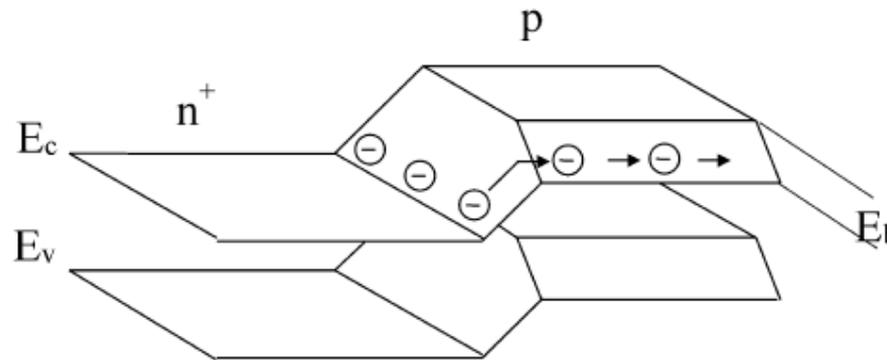


Fig. 6. Schematic 2-dimensional potential distribution on a positively charged surface (in front) crossing an  $n^+p$ -junction.  $E_c$ : conduction band edge,  $E_v$ : valence band edge,  $E_b$ : surface potential barrier height.

Courtesy of Elsevier, Inc., <http://www.sciencedirect.com>. Used with permission.

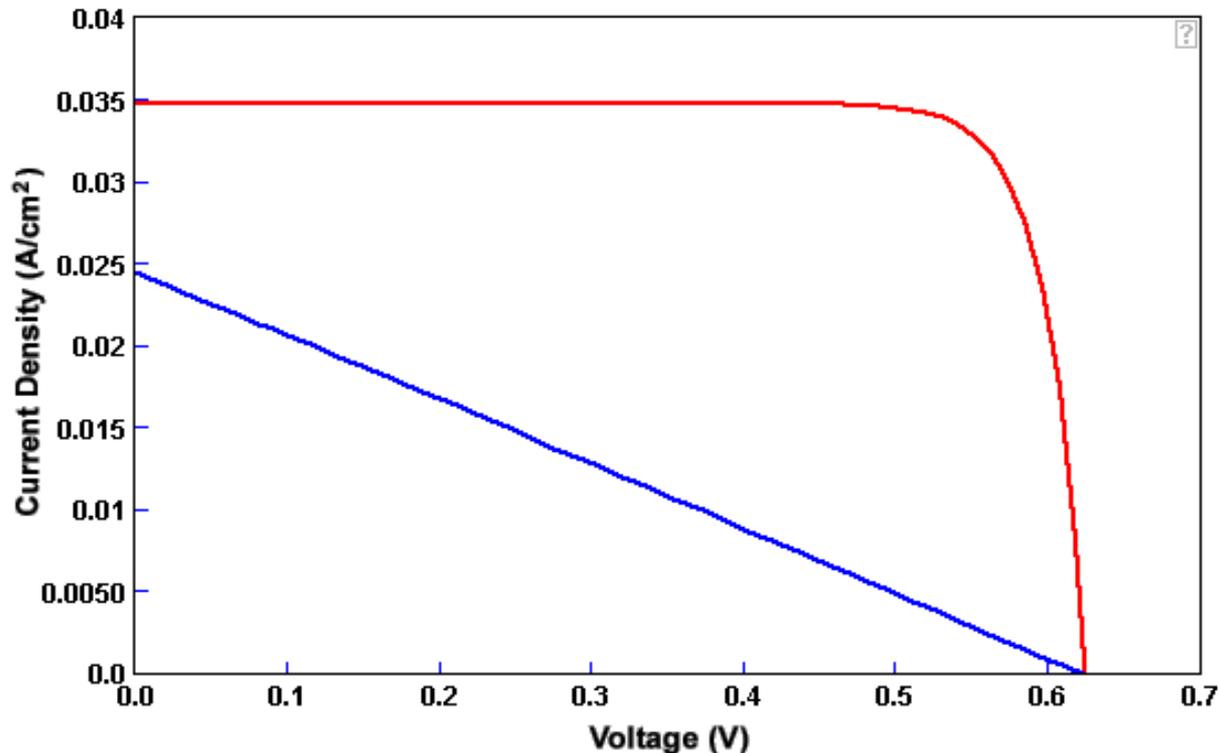
For more information: See publications by Dr. Otwin Breitenstein (Max-Planck Institute in Halle, Germany) on use of lock-in thermography for shunt detection and classification in solar cells.

# Effect of High Series Resistance ( $R_s$ )

$$J = J_L - J_0 \exp\left(\frac{q(V + JR_s)}{nkT}\right) - \frac{V + JR_s}{R_{shunt}}$$

*NB: IV curve flipped!*

Source: <http://www.pveducation.org/pvcdrom/solar-cell-operation/series-resistance>



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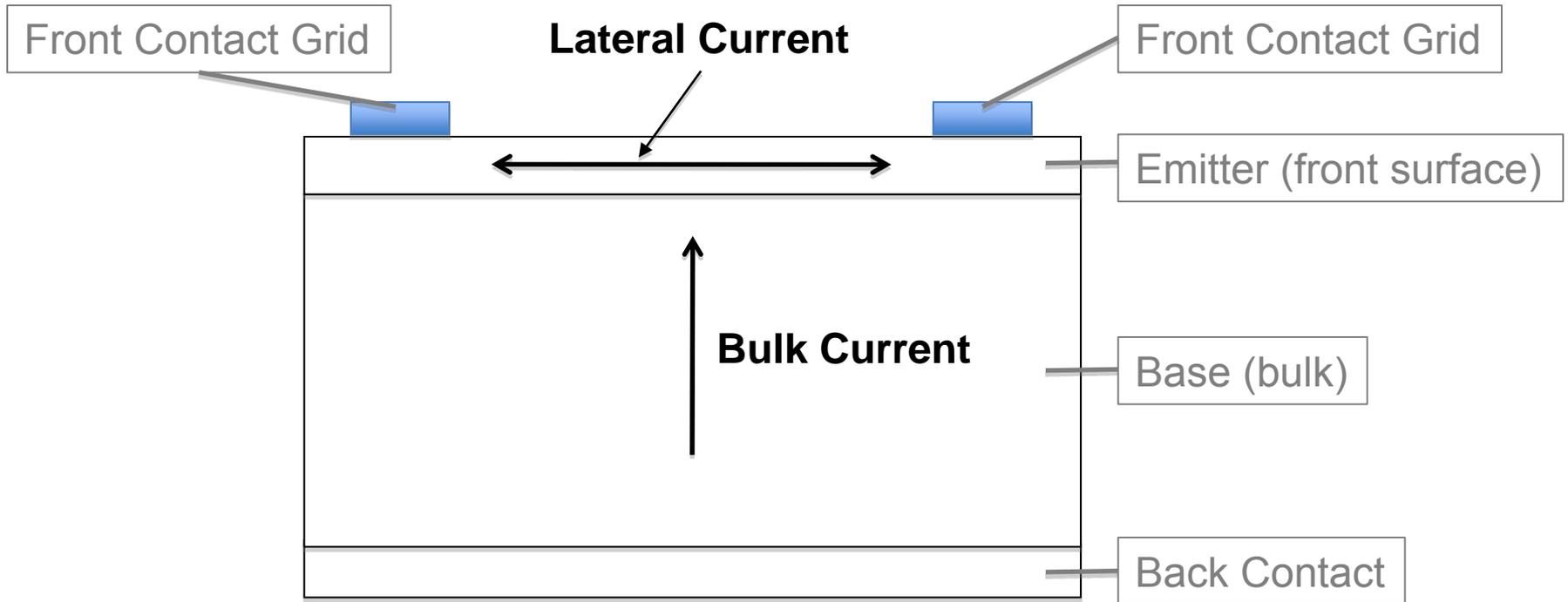
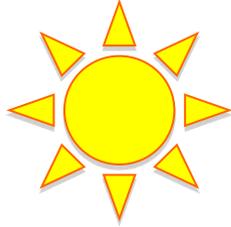
**Real Solar Cell:  $I_{sc} = 0.024$   $V_{oc} = 0.625$   $FF = 0.25$**

Series R: .01      24.14      10

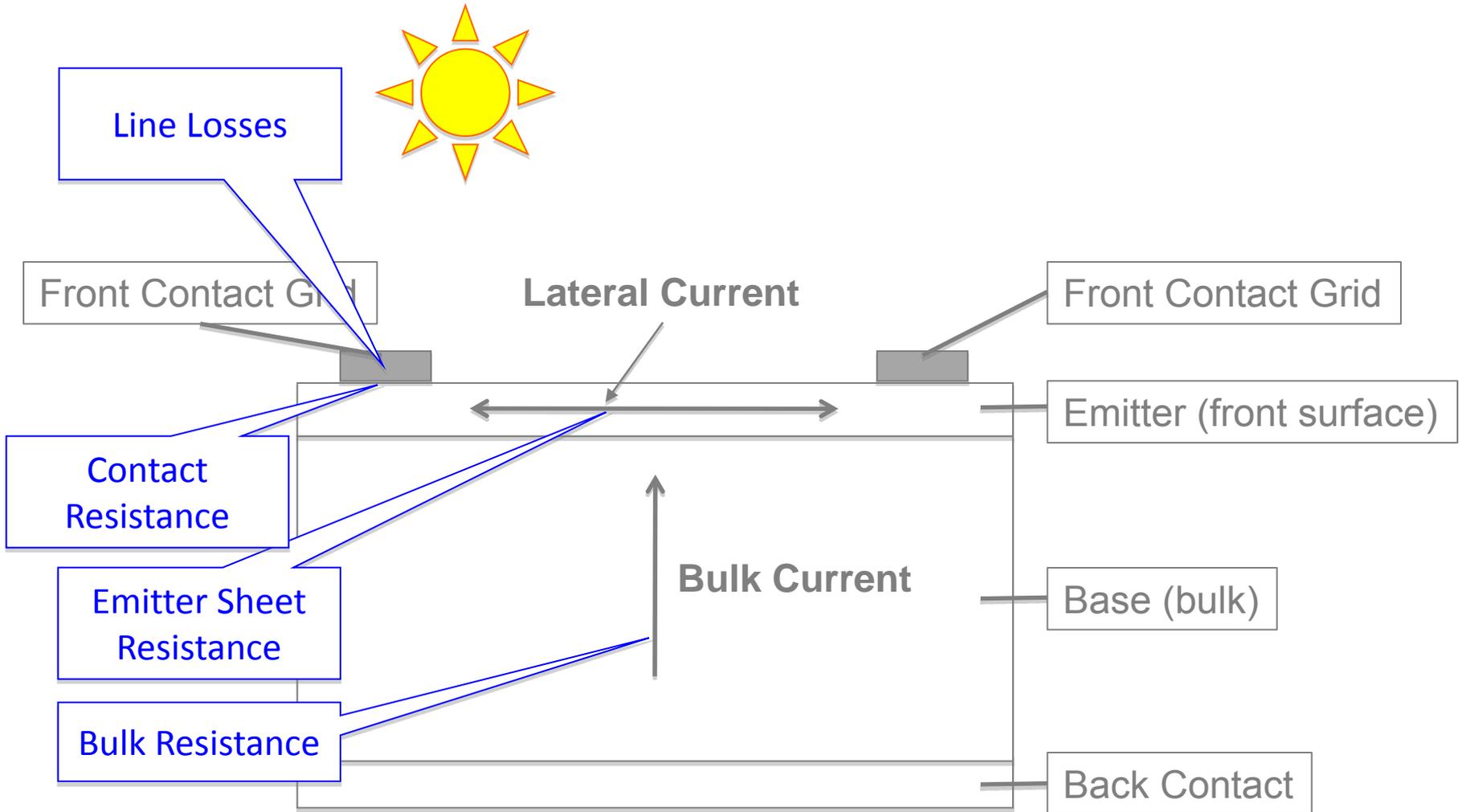
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# Components of Series Resistance



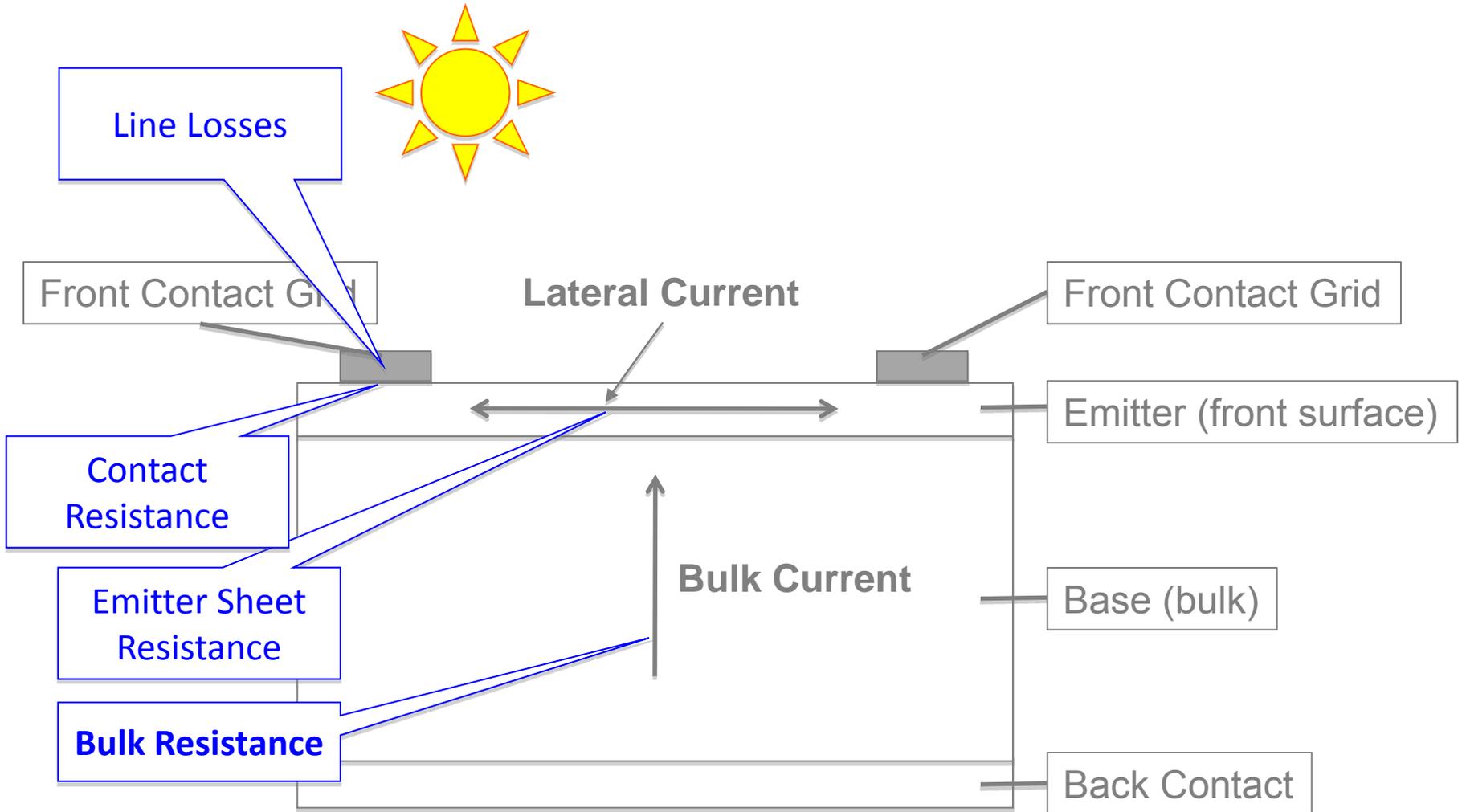
# Components of Series Resistance



# Components of $R_s$ : Bulk Resistance

(i.e., How to choose absorber thickness)

# Components of Series Resistance



# Bulk (Base) Resistance

Resistivity:  $\rho = \frac{1}{qn\mu}$   $\left\{ \begin{array}{l} q = \text{charge} \\ n = \text{carrier density} \\ \mu = \text{carrier mobility} \end{array} \right.$

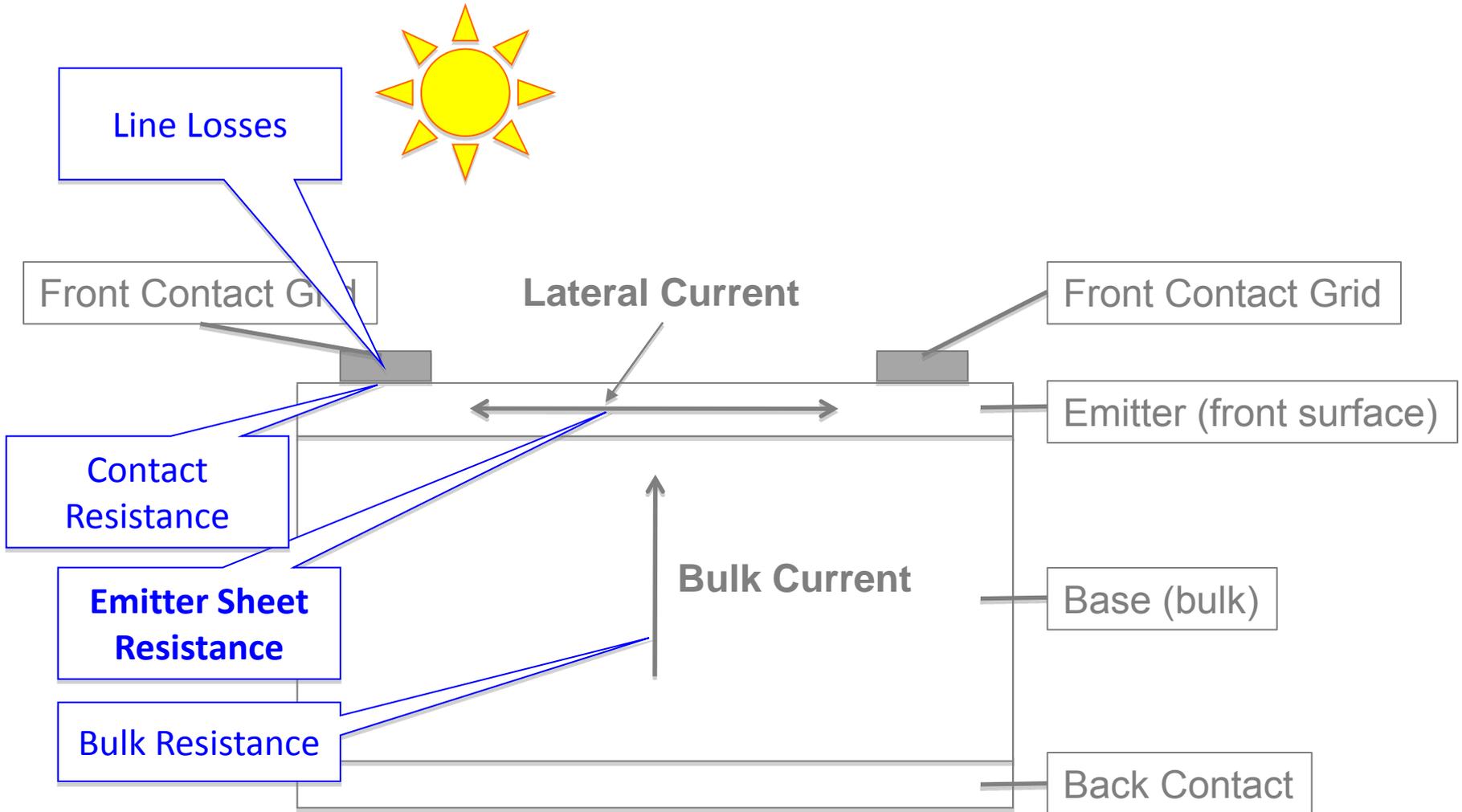
Base Resistance:  $R_b = \rho \frac{l}{A}$   $\left\{ \begin{array}{l} l = \text{length of conductive path} \\ A = \text{Area of current flow} \\ \rho = \text{base resistivity} \end{array} \right.$

*NB: Beware of non-linearities! (e.g., dependence of  $\mu$  on  $n$ ).*

# Components of $R_s$ : Emitter Sheet Resistance

(i.e., How to design front contact metallization)

# Components of Series Resistance



# Emitter Sheet Resistance

*Bulk Resistivity is defined according to the following expression:*

$$\rho = \frac{1}{q \cdot \mu \cdot N}$$

$\rho$  = resistivity  
 $\mu$  = carrier mobility  
 $N$  = Carrier concentration (dopant concentration)

Units of  $\rho$ :  $\Omega\text{-cm}$

*For a thin layer, a “sheet resistance” can be described:*

$$\rho_s = \frac{1}{q \int_0^t \mu_e(x) \cdot N_D(x) dx}$$

$x$  = layer thickness

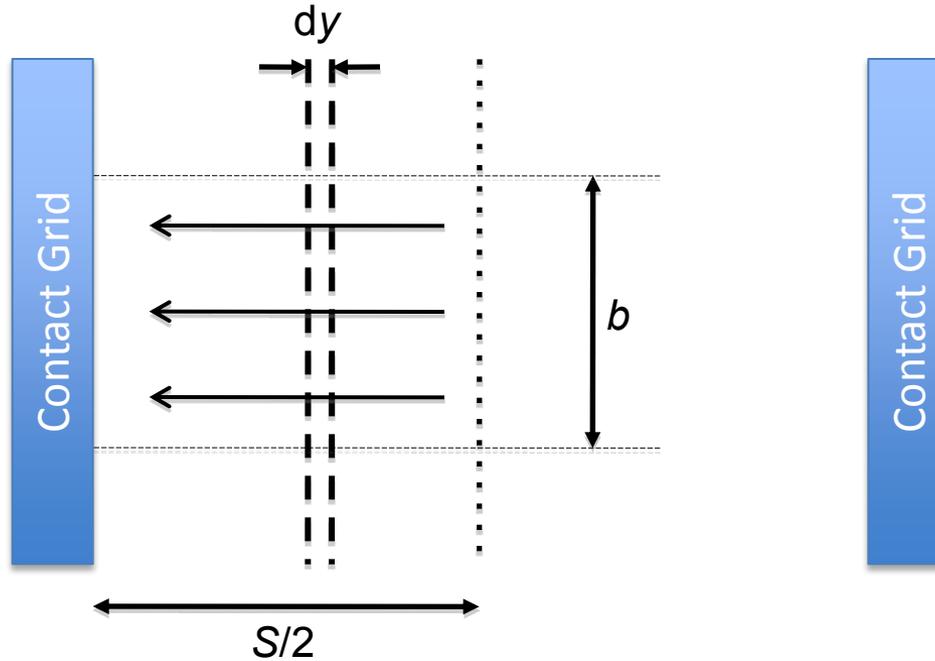
Units of  $\rho_s$ :  $\Omega$  , or  $\Omega/\square$

 *For uniform layers,  
a simplification:*

$$\rho_s = \frac{1}{q \cdot \mu \cdot N \cdot t}$$

# Sheet Resistance Losses

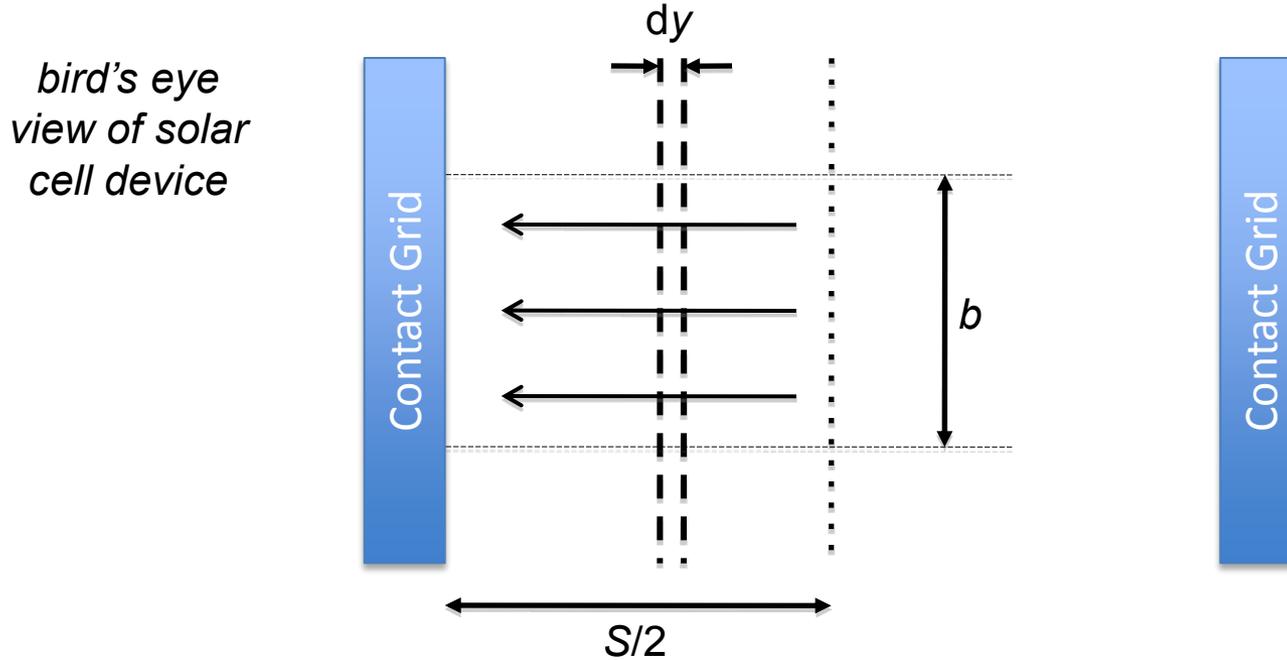
*bird's eye  
view of solar  
cell device*



*The total power loss is thus:*

$$P_{\text{loss}} = \int I^2 dR = \int_0^{S/2} \frac{J^2 b^2 y^2 \rho_s}{b} dy = \frac{J^2 b \rho_s S^3}{24}$$

# Sheet Resistance Losses

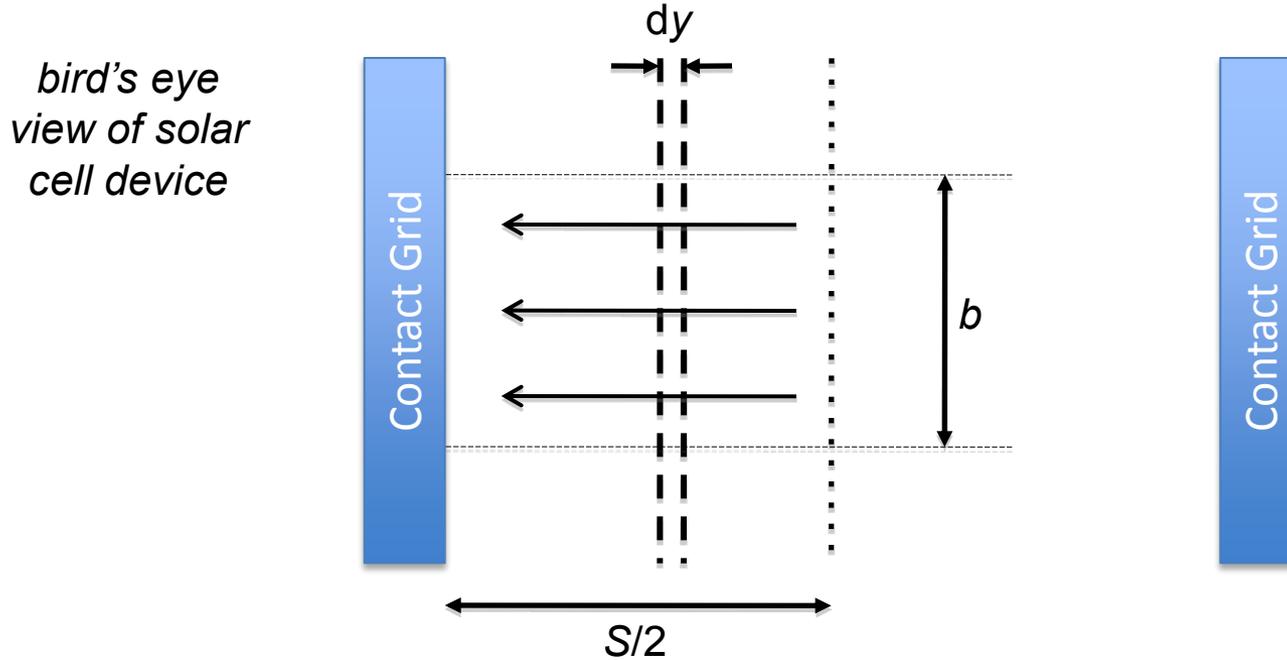


At the maximum power point, the generated power in the emitter ROI is:  $V_{mp} J_{mp} b \frac{S}{2}$

Hence, the fractional power loss at the maximum power point (MPP) is:

$$p = \frac{P_{\text{loss}}}{P_{\text{mpp}}} = \frac{\rho_s S^2 J_{\text{mp}}}{12 V_{\text{mp}}}$$

# Sheet Resistance Losses



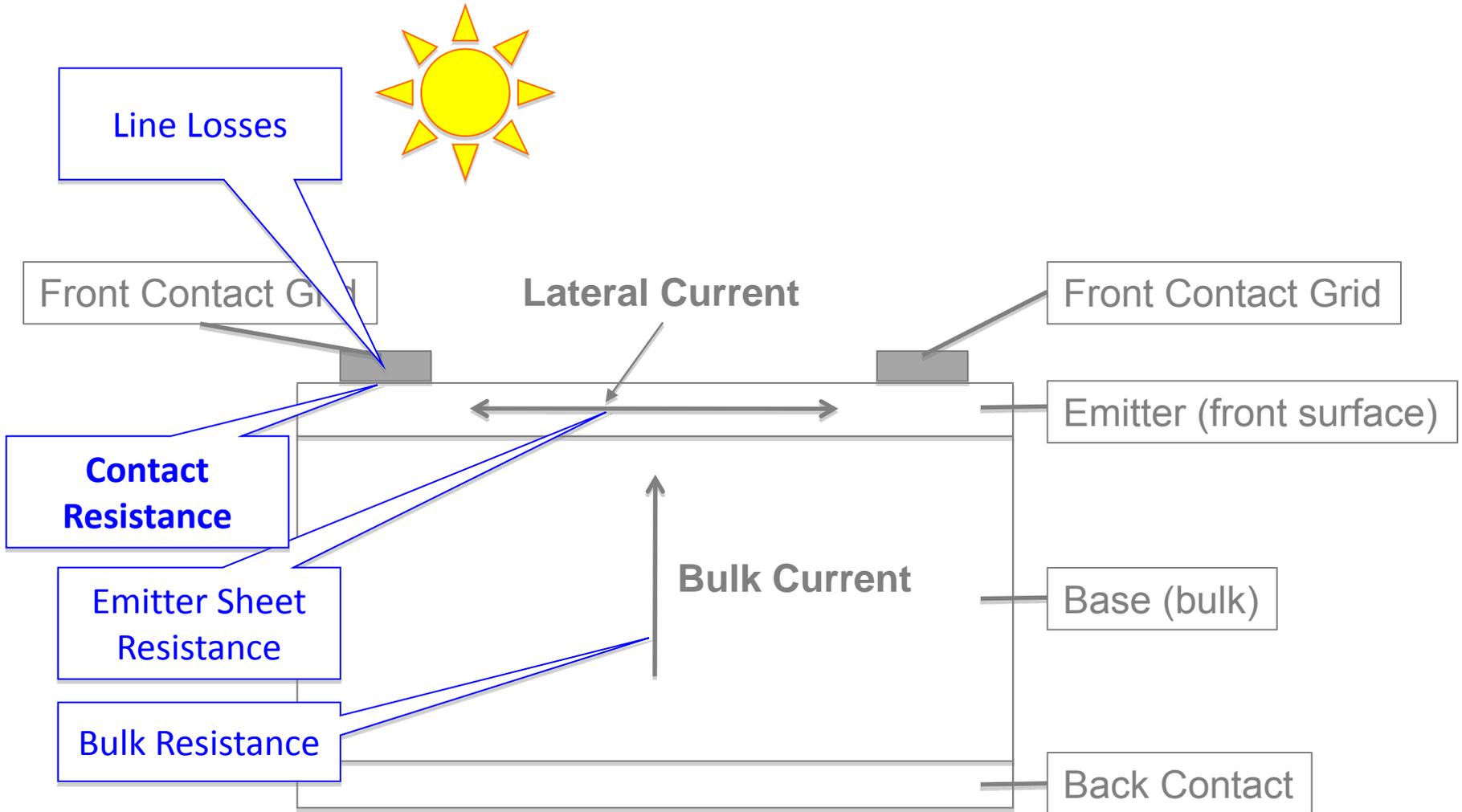
Consider a solar cell with  $\rho_s = 40 \Omega/\square$ ,  $J_{mp} = 30 \text{ mA/cm}^2$ , and  $V_{mp} = 450 \text{ mV}$ .  
 If we want less than a 4% power loss (i.e.,  $p < 0.04$ ) through the emitter, then

$$S < \sqrt{\frac{12pV_{mp}}{\rho_s J_{mp}}} = 4 \text{ mm}$$

Agrees with finger spacing in commercial solar cells!

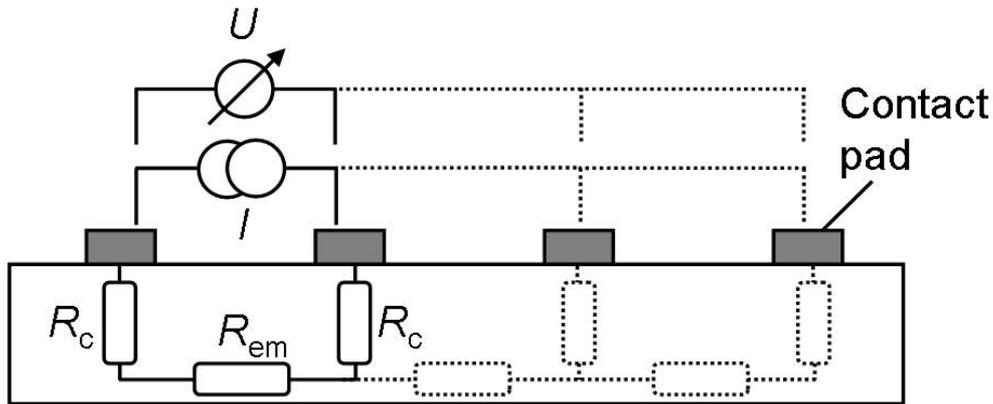
# Components of $R_s$ : Contact Resistance

# Components of Series Resistance

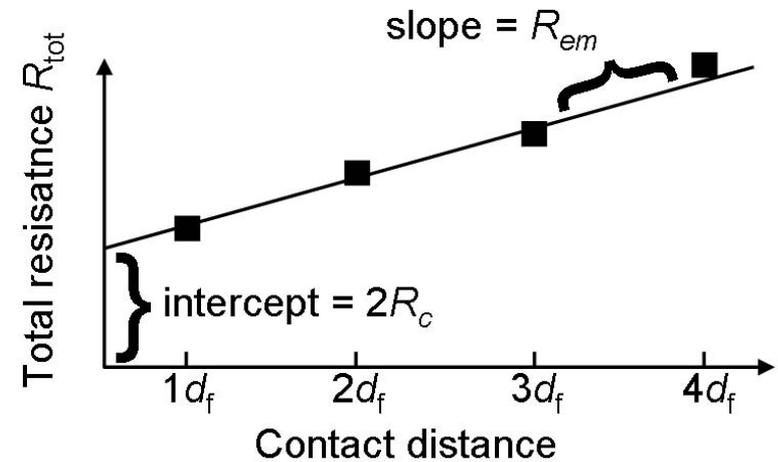


# Method to Measure Contact Resistance (TLM Method)

*Sequential measurements of resistance between reference finger and measured finger.*



$R_c$  = contact resistance

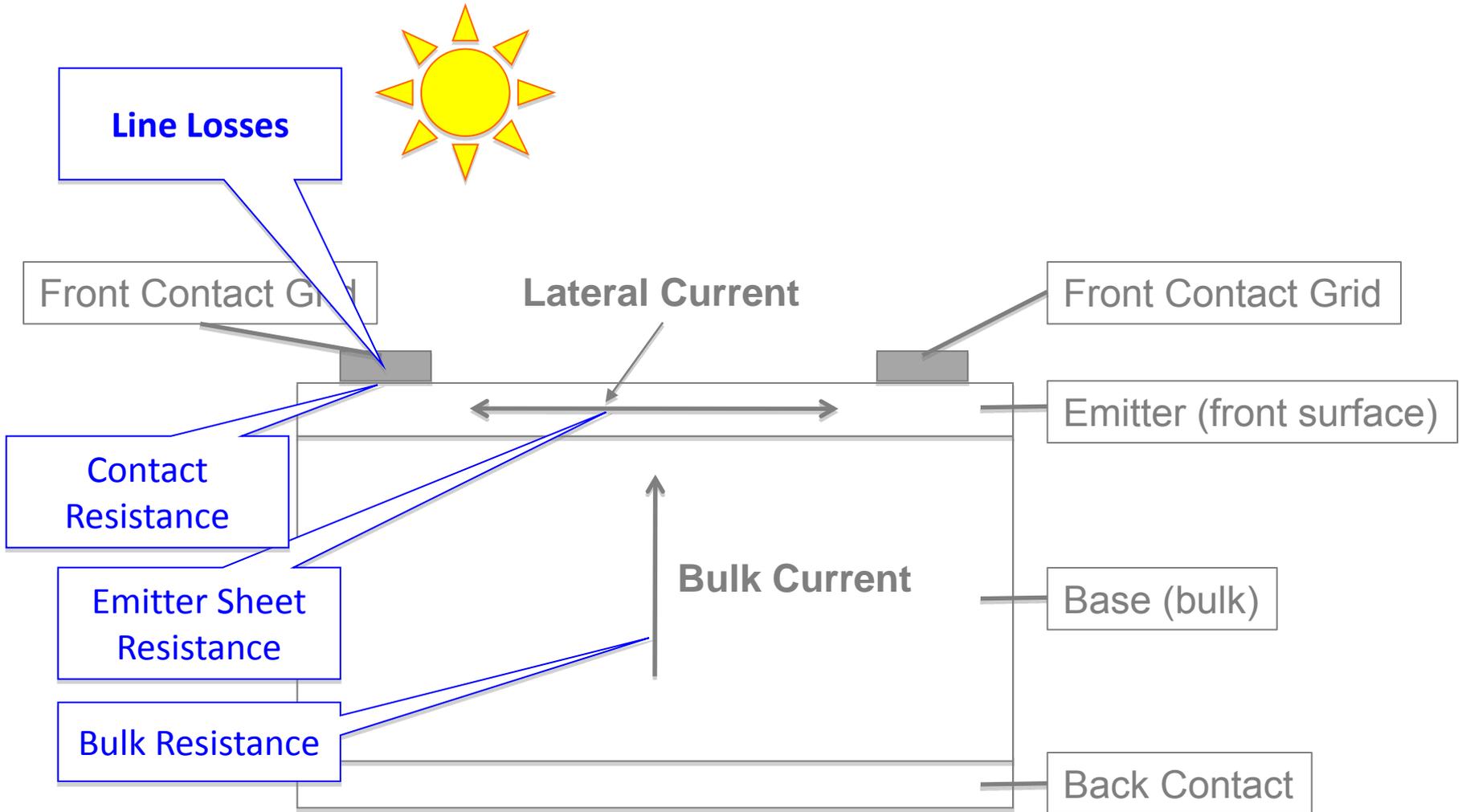


Figures courtesy of Stefan Kontermann

Courtesy of Stefan Kontermann. Used with permission.

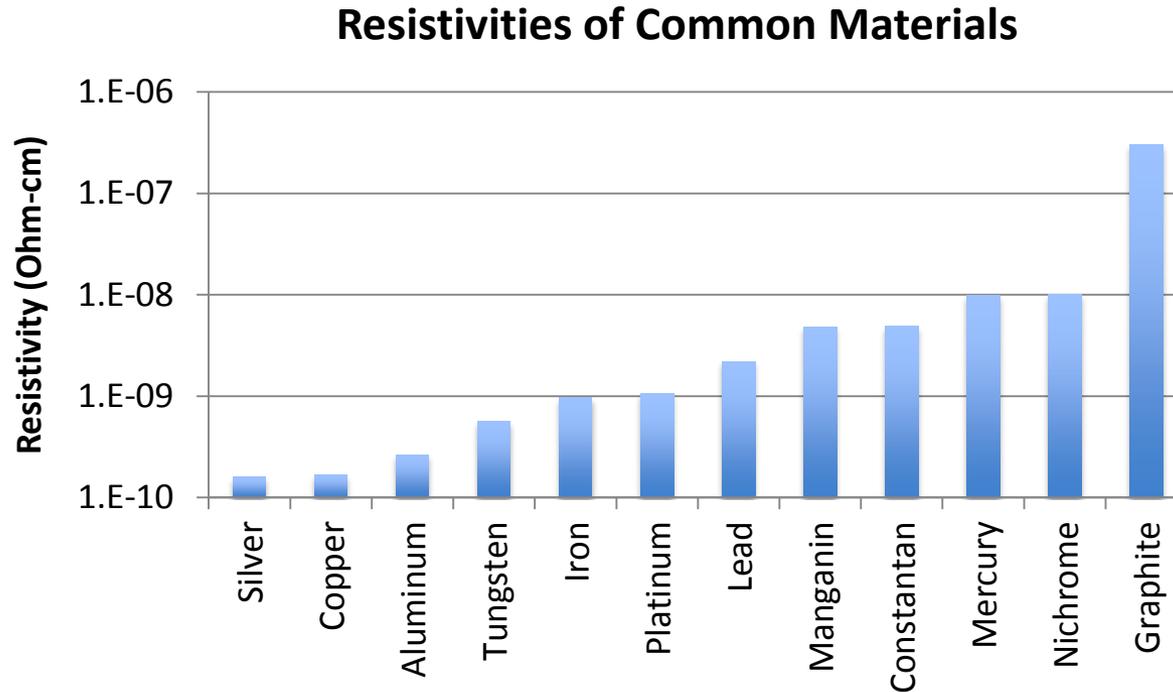
# Components of $R_s$ : Line Losses

# Components of Series Resistance



# Line Losses

Line Resistance:  $R_b = \rho \frac{l}{A}$   $\left\{ \begin{array}{l} l = \text{length of conductive path} \\ A = \text{Area of current flow} \\ \rho = \text{metal resistivity} \end{array} \right.$



# Learning Objectives: Toward a 1D Device Model

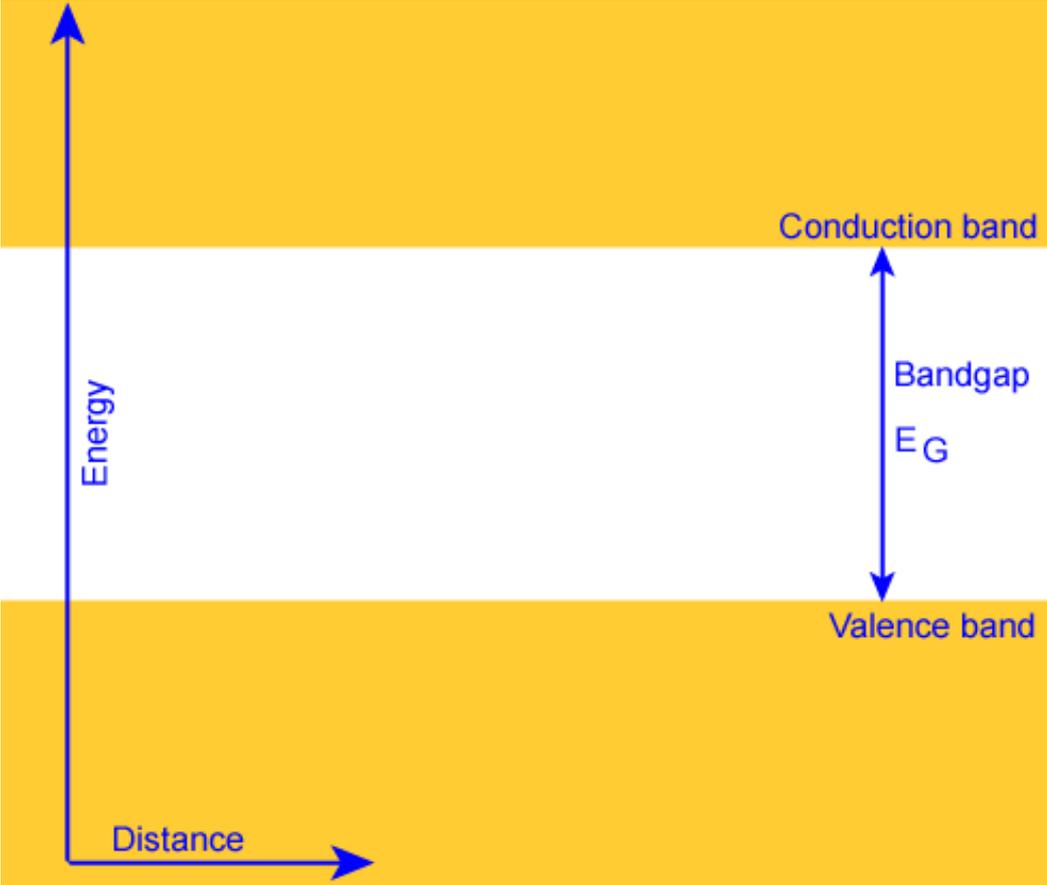
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Question: When I illuminate my device, do I perturb the band structure or Fermi energy?

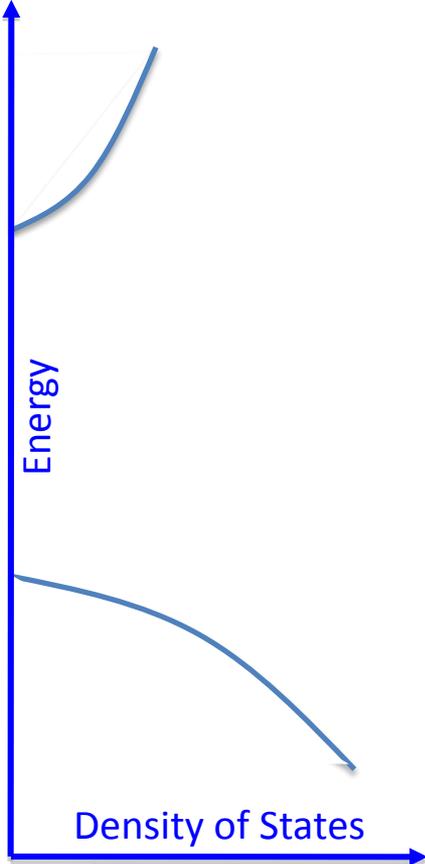
Calculate Fermi Energy (function of  
dopant + illumination +  
temperature)

# Conductivity

### Band Diagram (E vs. x)



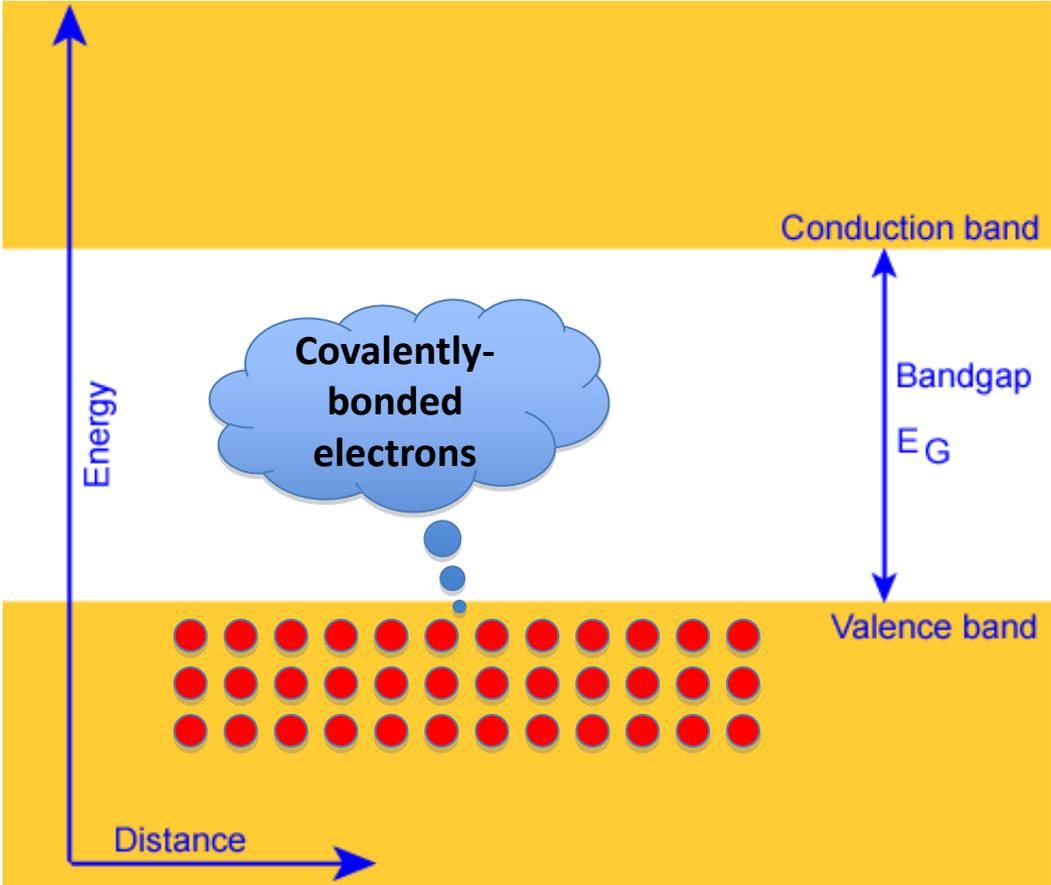
### Density of States



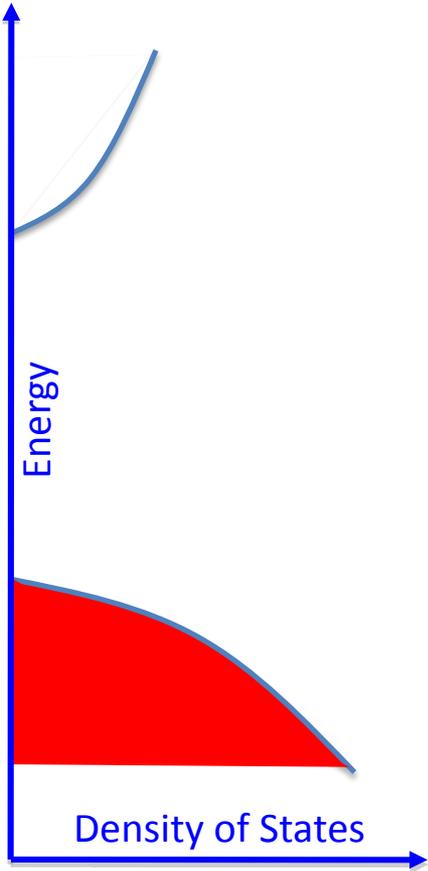
# Conductivity: Dependence on Temperature

At absolute zero, no conductivity (perfect insulator).

Band Diagram (E vs. x)



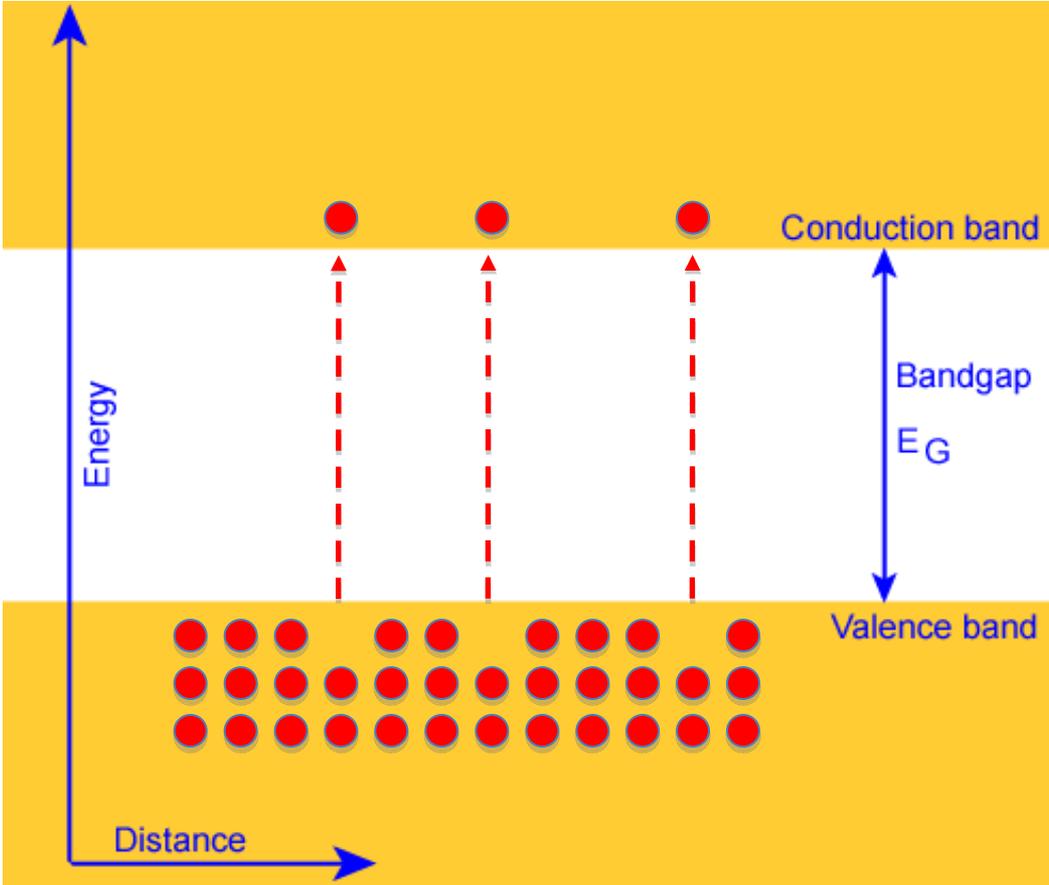
Density of States



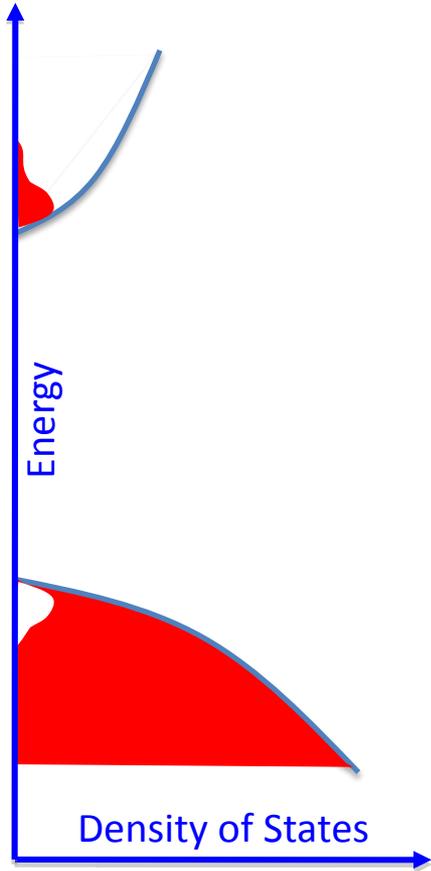
# Conductivity: Dependence on Temperature

At  $T > 0$  K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. x)

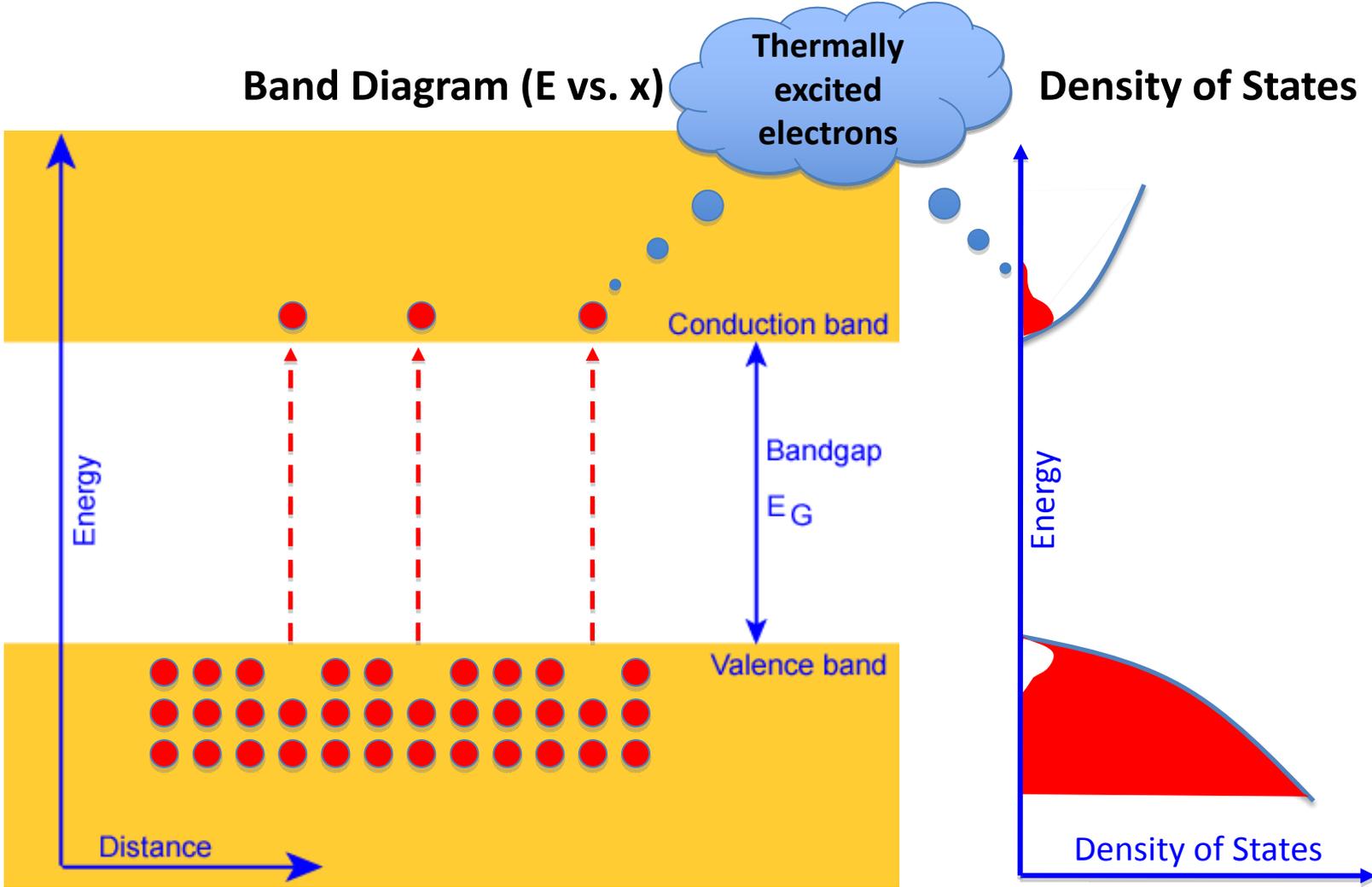


Density of States



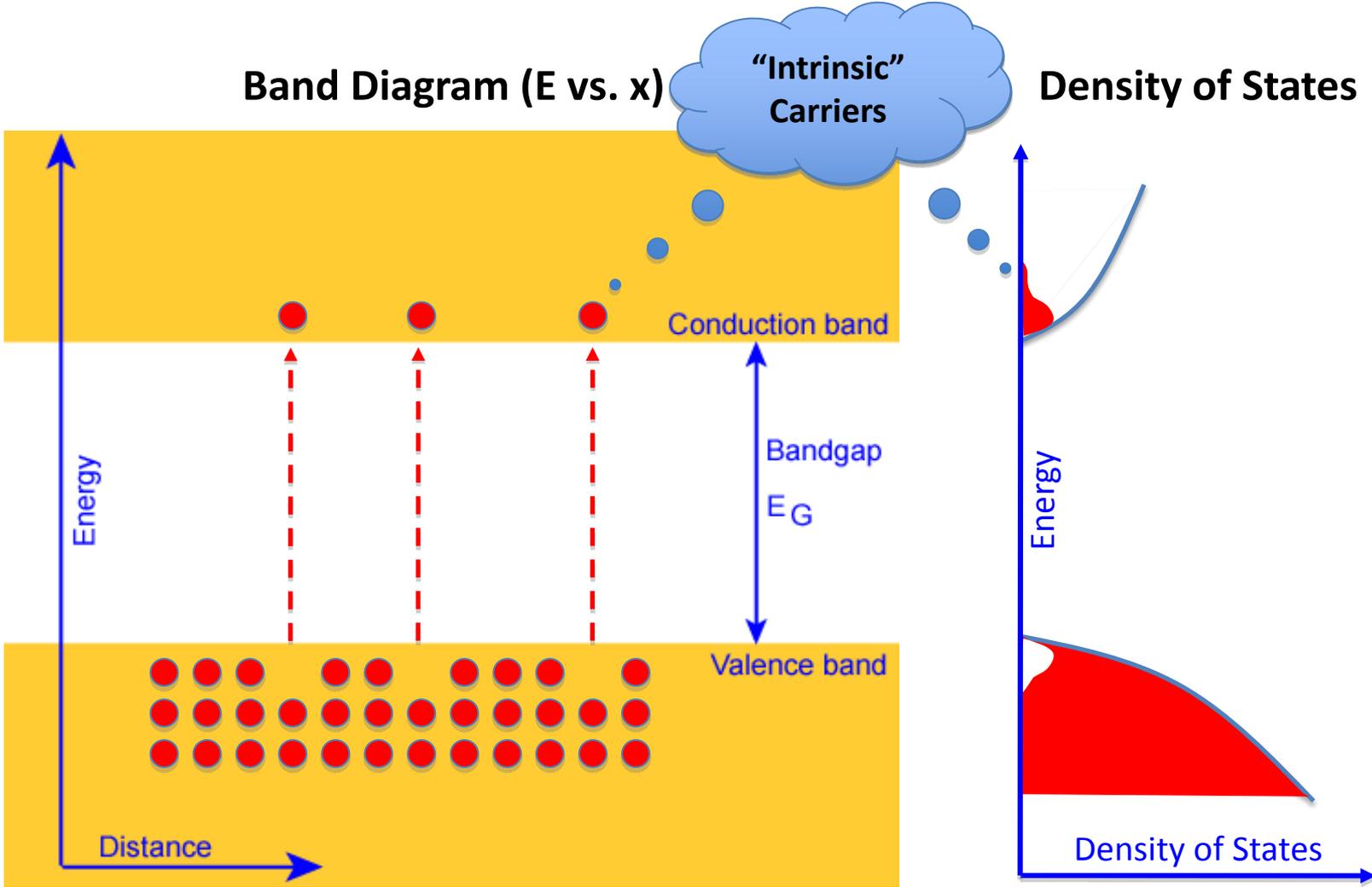
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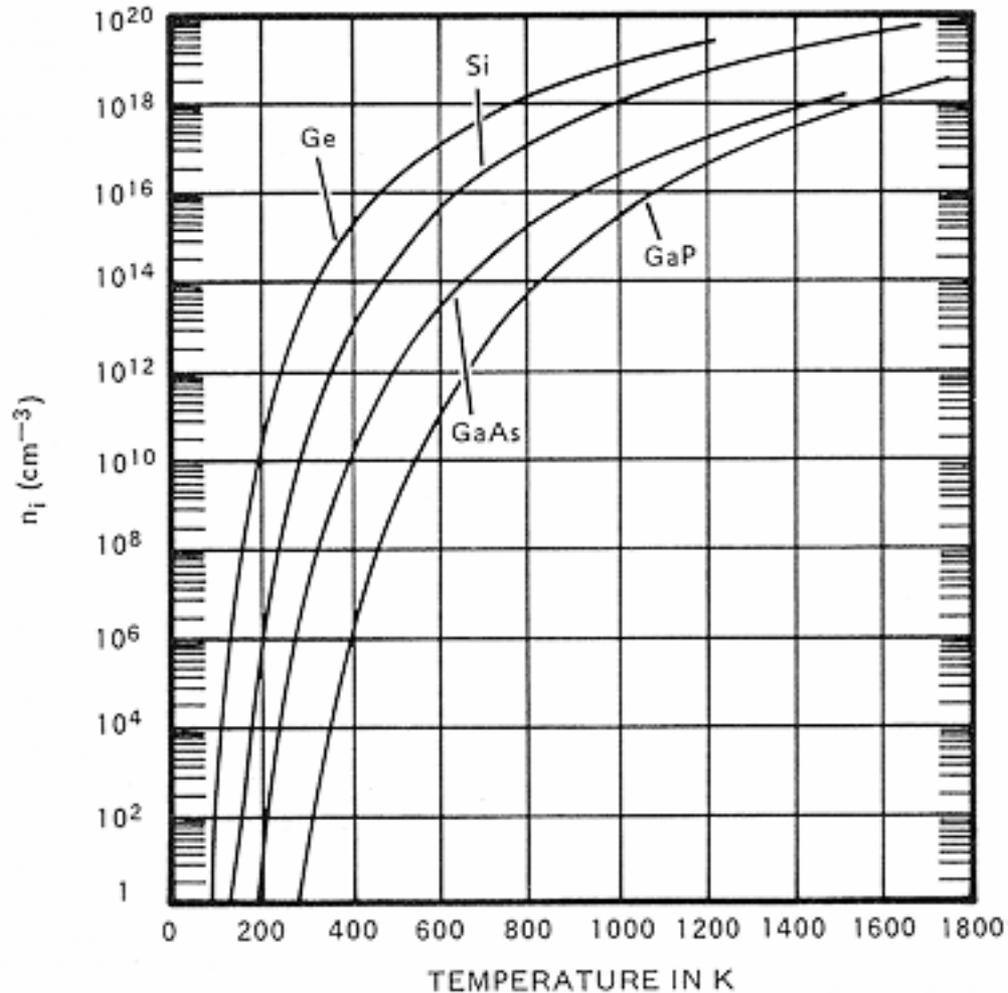


# Conductivity: Dependence on Temperature

At  $T > 0$  K, some carriers are thermally excited across the bandgap.



# Temperature Dependence of Intrinsic Carrier Concentration



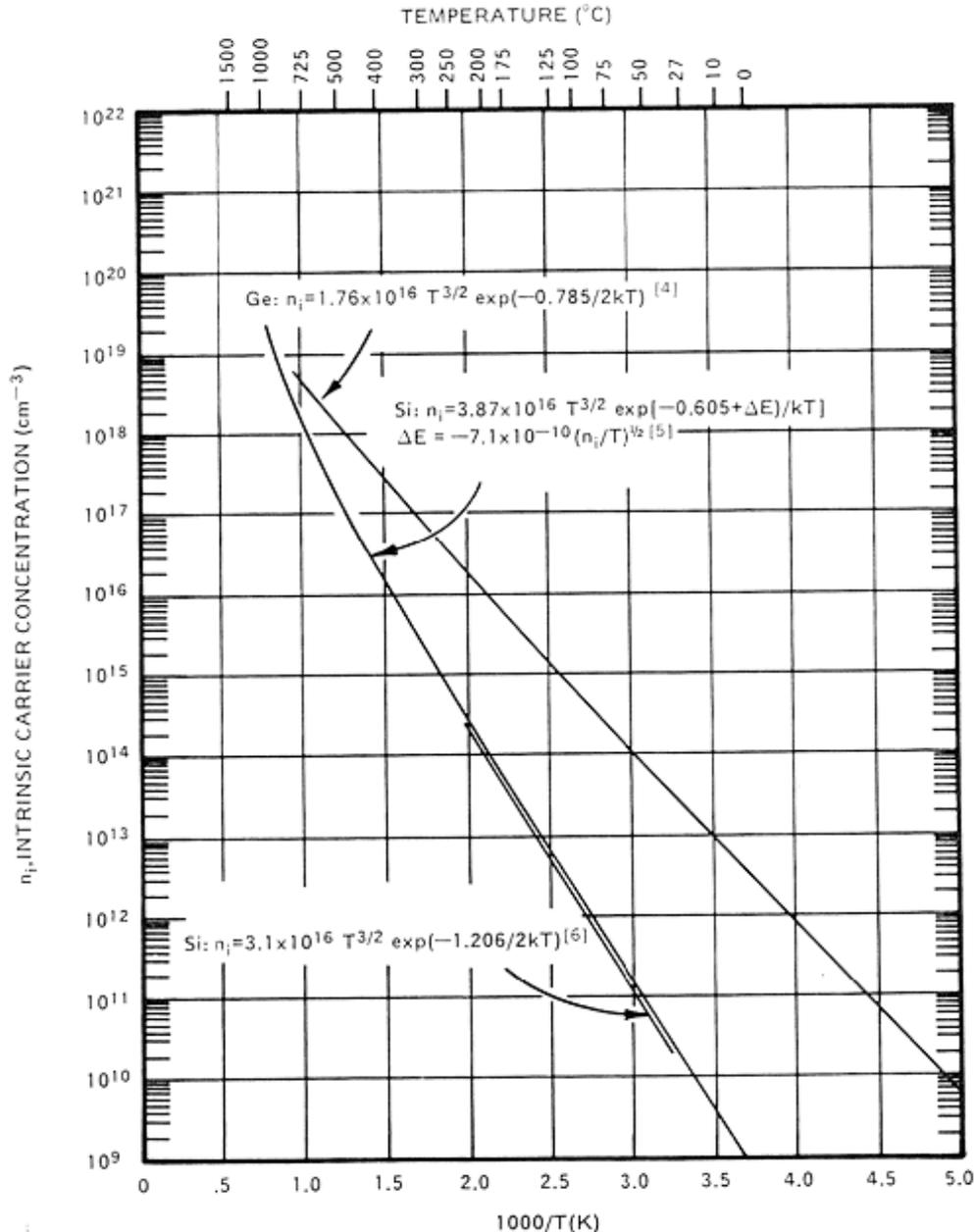
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Arrhenius Equation,  
generic form:

$$N = N_0 \cdot \exp \left[ -E_A / k_b T \right]$$

Buonassisi (MIT) 2011

# Temperature Dependence of Intrinsic Carrier Concentration



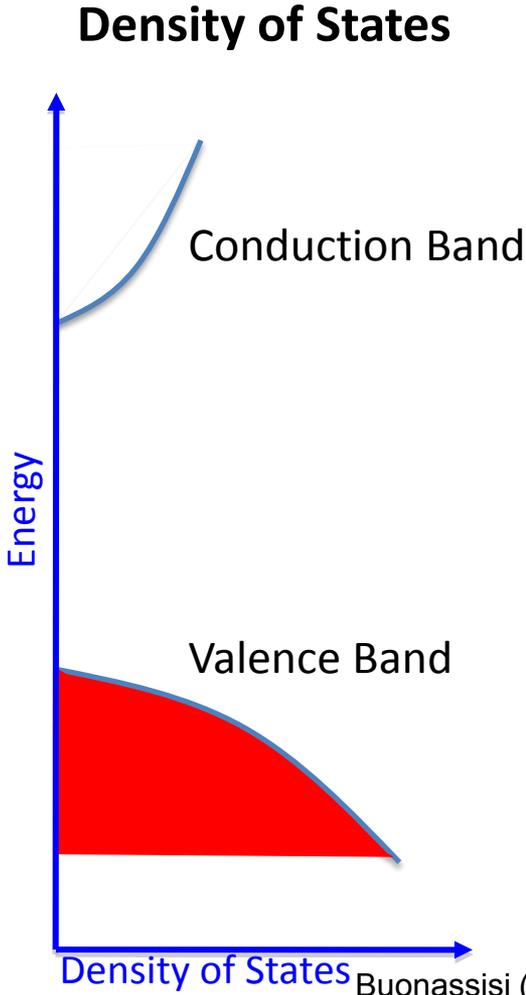
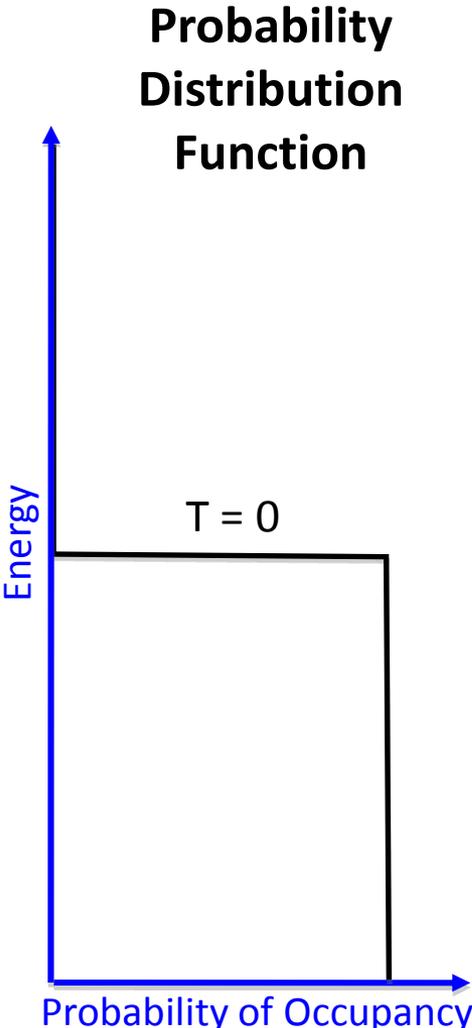
Arrhenius Equation,  
generic form:

$$N = N_0 \cdot \exp \left[ -E_A / k_b T \right]$$

Buonassisi (MIT) 2011

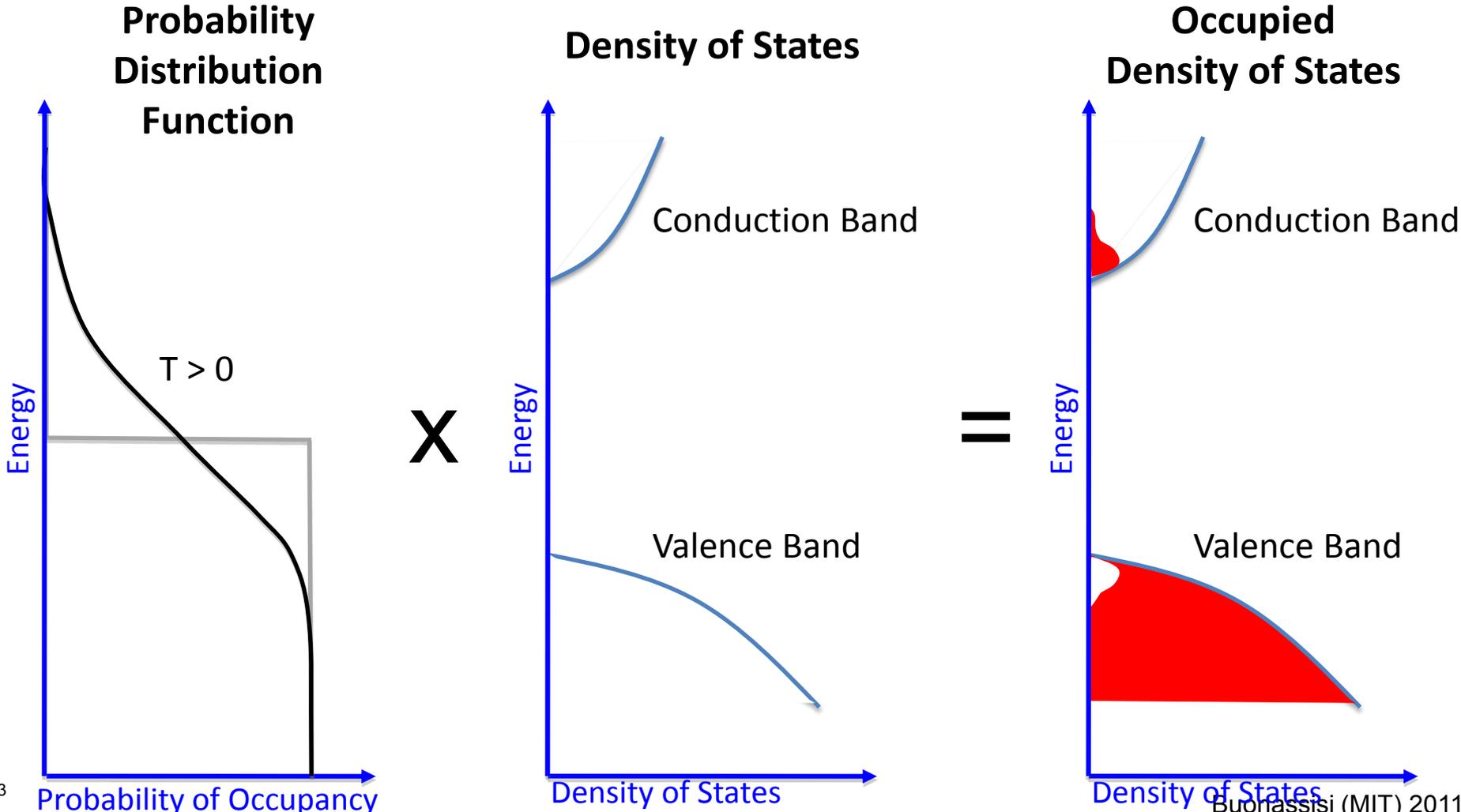
# Conductivity: Dependence on Temperature

At absolute zero, no conductivity (perfect insulator).



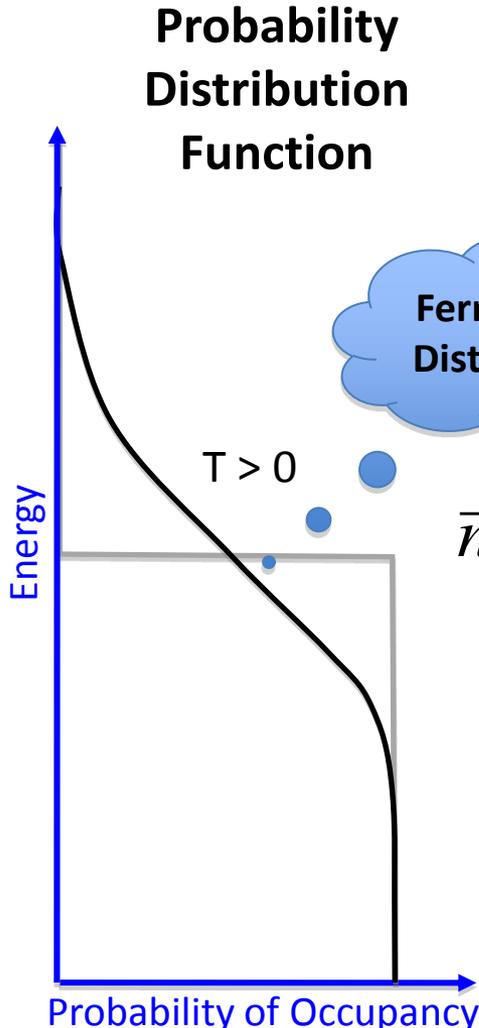
# Conductivity: Dependence on Temperature

At a finite temperature, finite conductivity (current can flow).

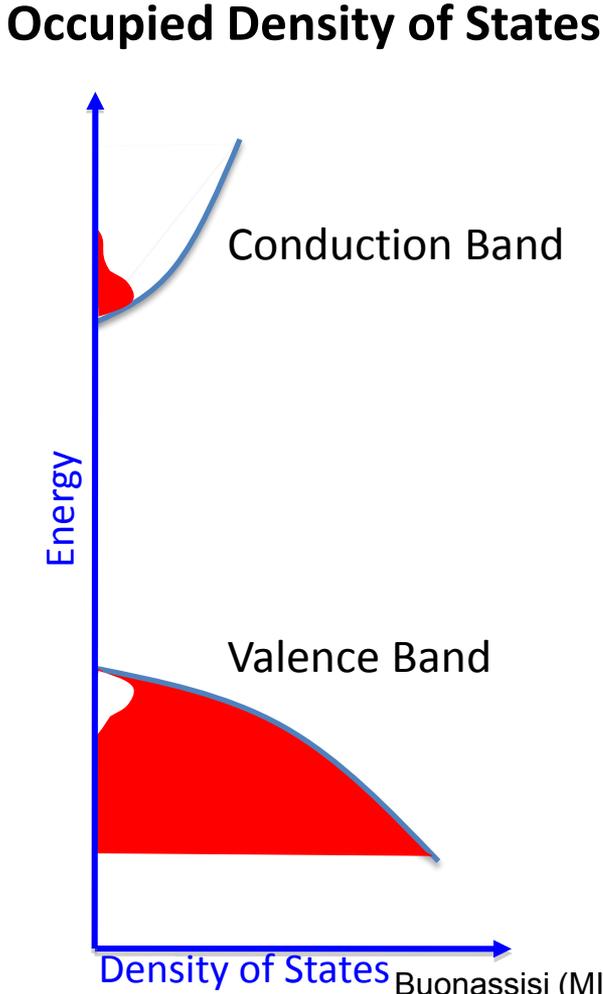


# Conductivity: Dependence on Temperature

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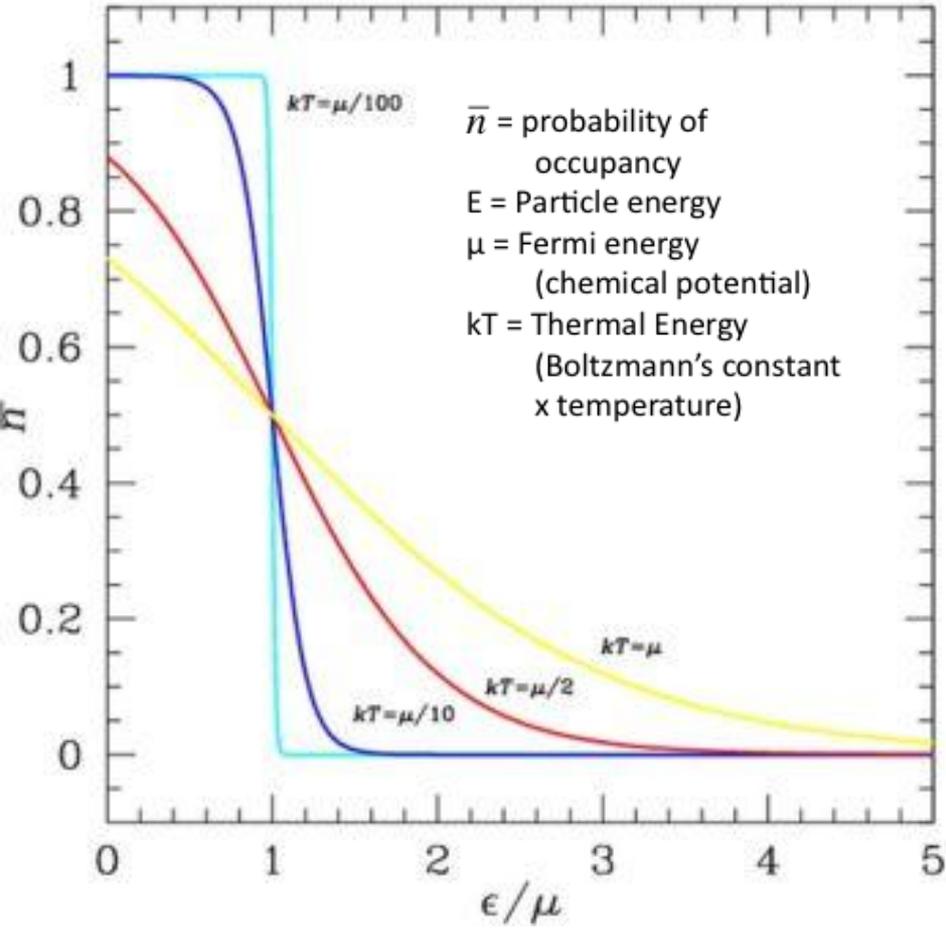


$$\bar{n} = \frac{1}{e^{(E - E_F)/k_b T} + 1}$$



To reduce noise in a Si CCD camera, should you increase or decrease temperature?

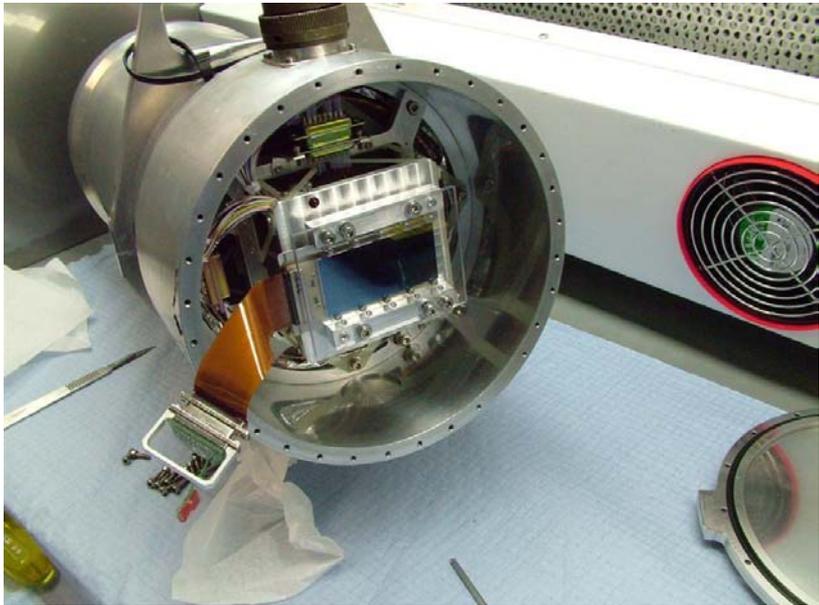
# Lower Temperature = Lower Intrinsic Carrier Concentration



Public domain image (Source: Wikimedia Commons).

<http://www.answers.com/topic/semiconductor>

## CCD inside a LN dewar



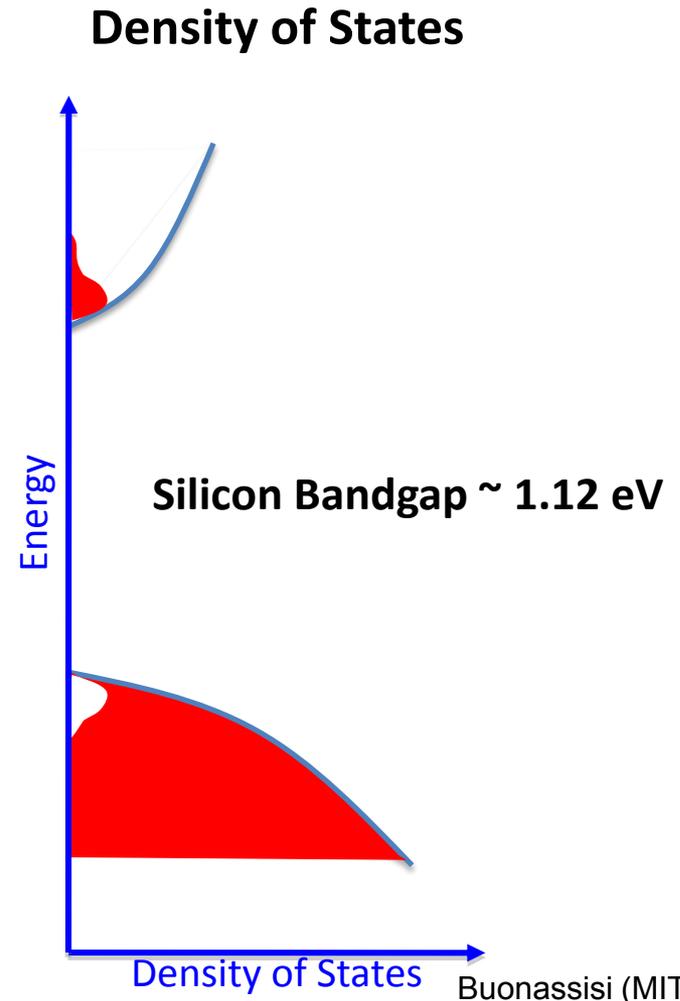
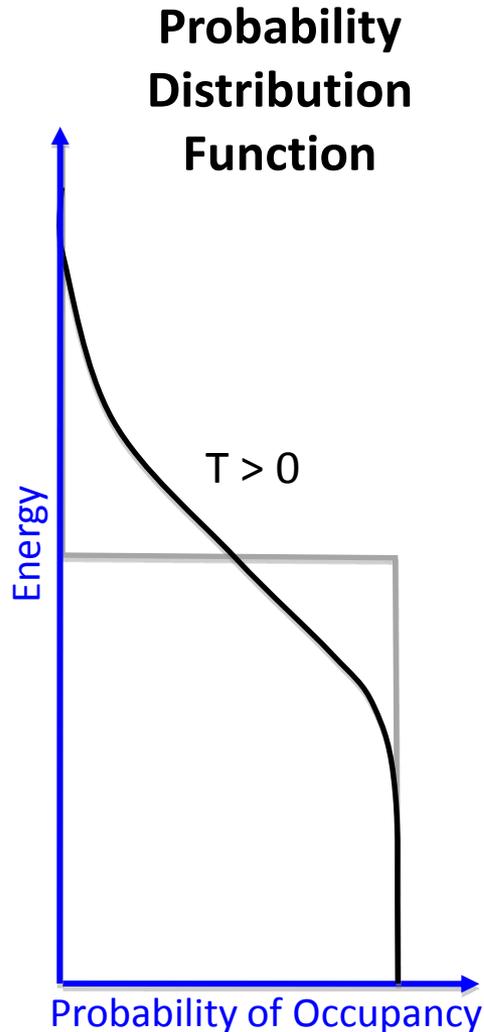
<http://msowww.anu.edu.au/observing/detectors/wfi.php>

Courtesy of The Australian National University. Used with permission.

Question: Transistors made from which semiconductor material experience greater electronic noise at room temperature:  
Germanium or Silicon?

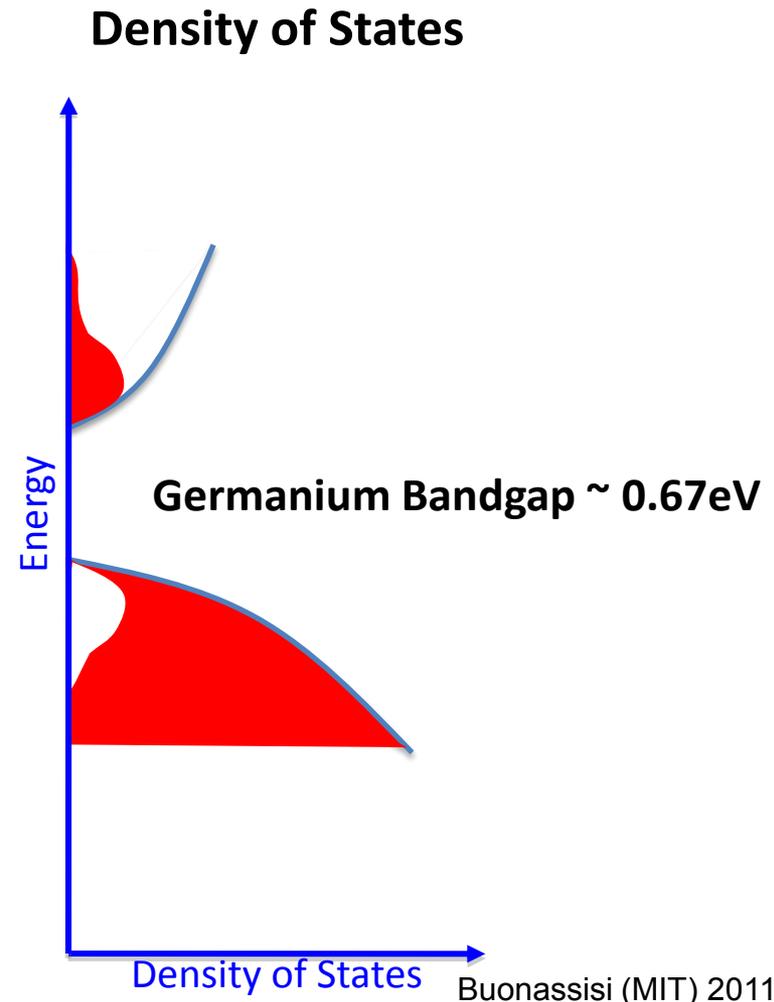
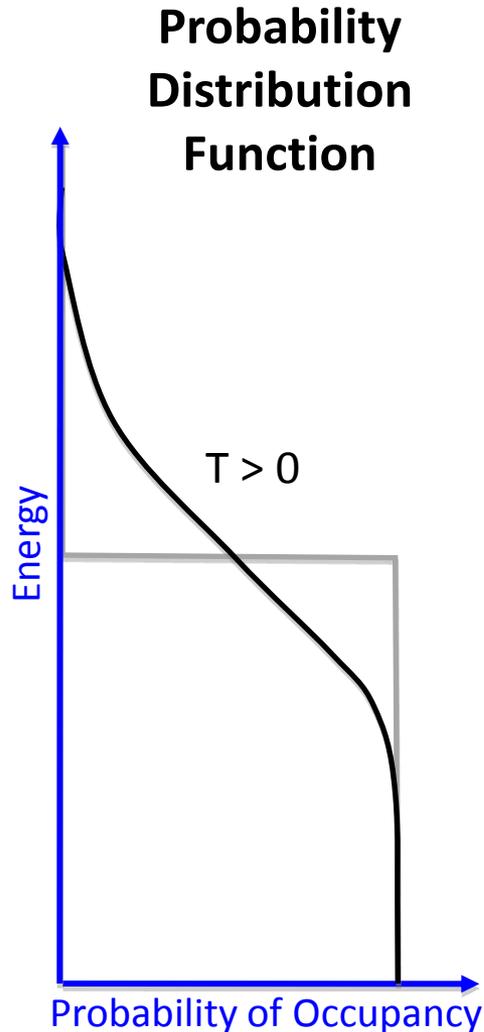
# Intrinsic Conductivity: Dependence on Bandgap

At a finite temperature, finite conductivity (current can flow).



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# Intrinsic Conductivity: Dependence on Bandgap

Please see table at <https://web.archive.org/web/20130818190346/http://www.siliconfareast.com/sigegaas.htm>

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