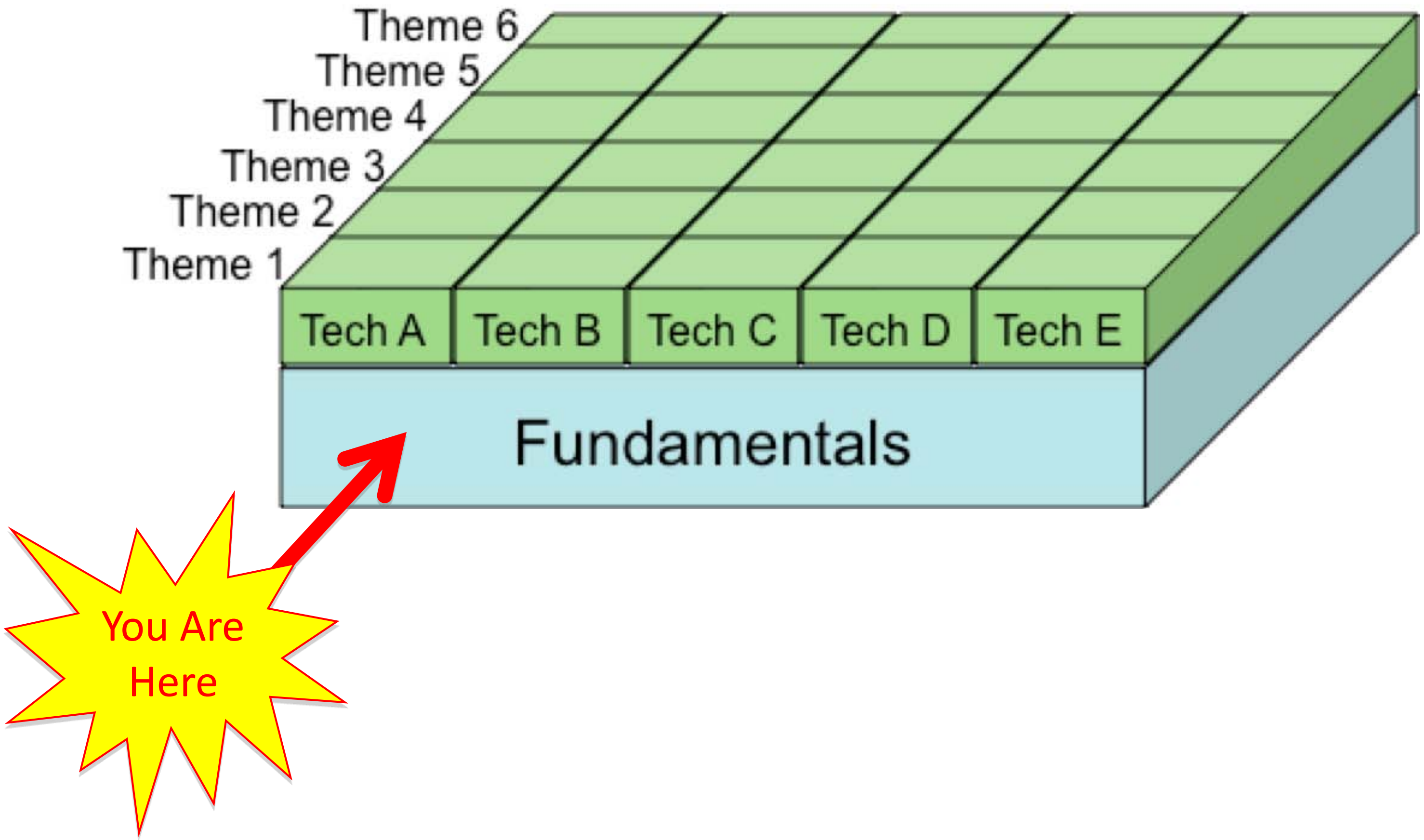
Charge Separation Part 1: Diode

Lecture 5 – 9/22/2011

MIT Fundamentals of Photovoltaics 2.626/2.627 – Fall 2011

Prof. Tonio Buonassisi

2.626/2.627 Roadmap



2.626/2.627: Fundamentals

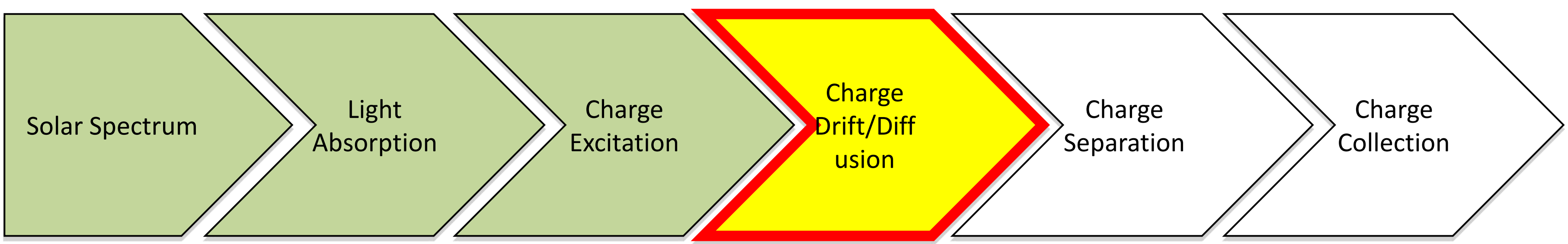
<u>Every</u> photovoltaic device must obey:

$$\text{Conversion Efficiency } (\eta) \equiv \frac{\text{Output Energy}}{\text{Input Energy}}$$

For most solar cells, this breaks down into:

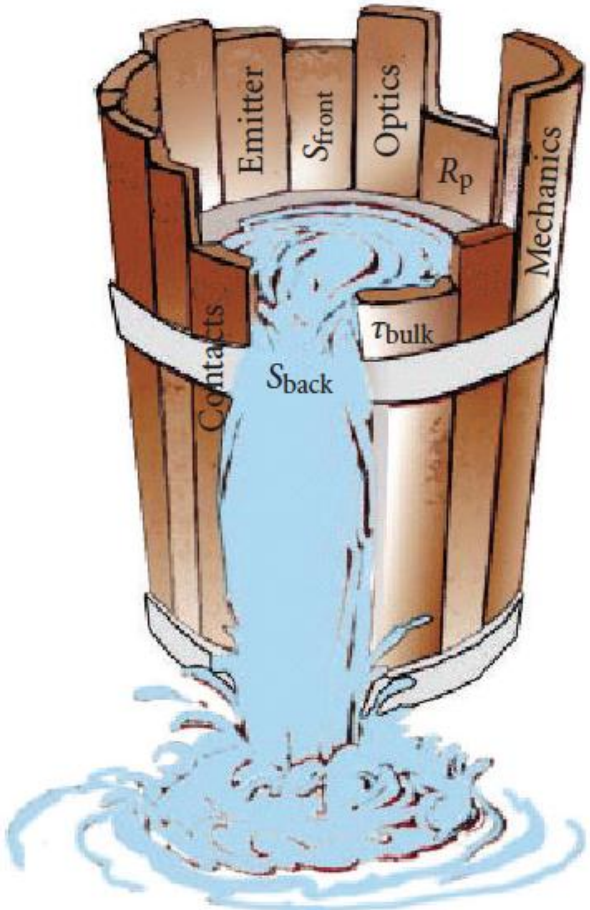
Inputs

Outputs



$$\eta_{\text{total}} = \eta_{\text{absorption}} \times \eta_{\text{excitation}} \times \eta_{\text{drift/diffusion}} \times \eta_{\text{separation}} \times \eta_{\text{collection}}$$

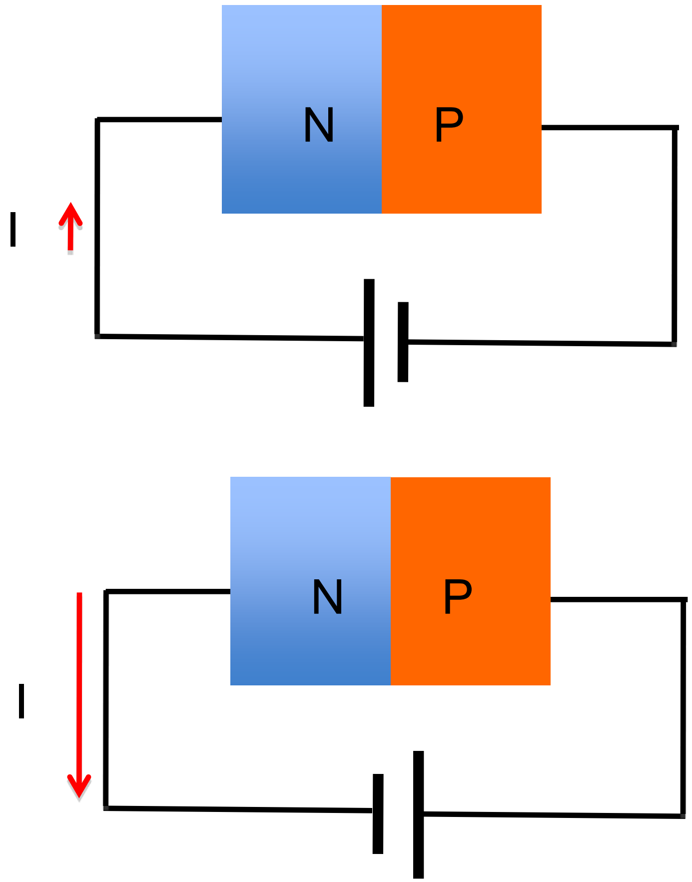
Liebig's Law of the Minimum



S. Glunz, *Advances in Optoelectronics* 97370 (2007)

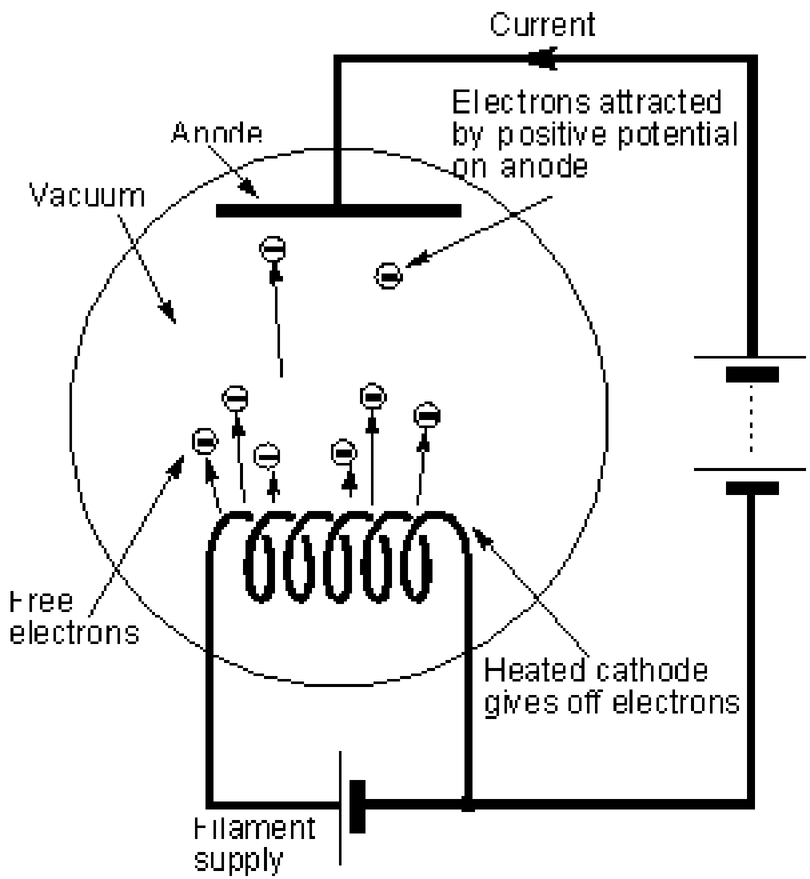
$$\eta_{total} = \eta_{absorption} \times \eta_{excitation} \times \eta_{drift/diffusion} \times \eta_{separation} \times \eta_{collection}$$

Diode: Essence of Charge Separation



- *What is a diode?*
- *How is it made?*
- *Why care about diodes?*

Diode: Essence of Charge Separation



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http://www.radio-electronics.com/info/data/thermionic-valves/vacuum-tube-theory/tube-tutorial-basics.php

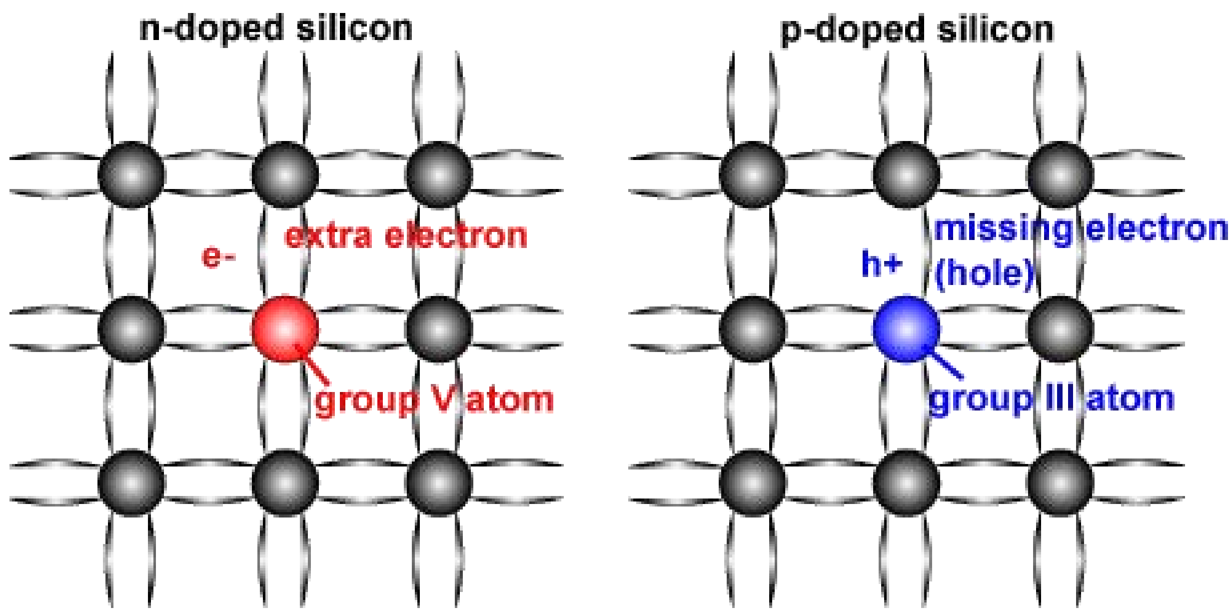
Learning Objectives: Diode

1. **Describe how conductivity of a semiconductor can be modified by the intentional introduction of dopants.**
2. Draw pictorially, with fixed and mobile charges, how built-in field of pn-junction is formed.
3. Current flow in a *pn*-junction: Describe the nature of drift, diffusion, and illumination currents in a diode. Show their direction and magnitude in the dark and under illumination.
4. Voltage across a *pn*-junction: Quantify the built-in voltage across a *pn*-junction. Quantify how the voltage across a *pn*-junction changes when an external bias voltage is applied.
5. Draw current-voltage (I-V) response, recognizing that minority carrier flux regulates current.

Dopant Atoms

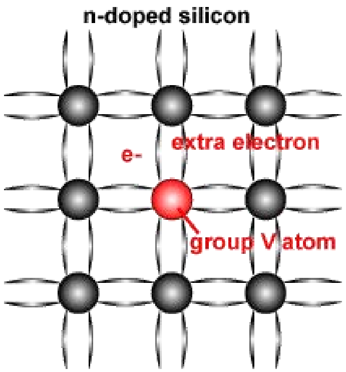
Periodic Table

5 B	6 C	7 N	8 O
13 Al	14 Si	15 P	16 S
31 Ga	32 Ge	33 As	34 Se
49 In	50 Sn	51 Sb	52 Te
81 Tl	82 Pb	83 Bi	84 Po



http://pvcdrom.pveducation.org/

Carrier Binding Energy to Shallow Dopant Atoms



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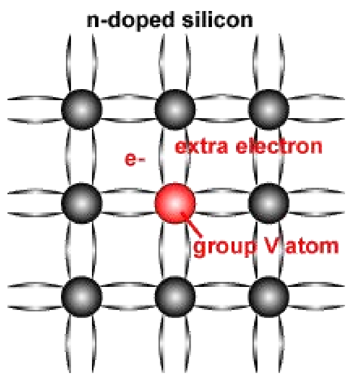
Carrier binding energy to a shallow (hydrogenic) dopant atom:

$$E = E_{\mathrm{H}} \frac{m^*}{m_{\mathrm{e}}} \frac{1}{\varepsilon^2} = (13.6 \text{ eV}) \cdot \frac{m^*}{m_{\mathrm{e}}} \frac{1}{\varepsilon^2}$$

Carrier Binding Energy to Shallow Dopant Atoms



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Carrier binding energy to a shallow (hydrogenic) dopant atom:

Effective mass correction

$$E = E_{\mathrm{H}} \frac{m^*}{m_{\mathrm{e}}} \frac{1}{\varepsilon^2} = (13.6\ \mathrm{eV}) \cdot \frac{m^*}{m_{\mathrm{e}}} \frac{1}{\varepsilon^2}$$

Electron screening

Learning Objectives: Diode

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Gauss' Law: Review

Spatially variant fixed charge creates an electric field:

$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

ξ = electric field
ρ = charge density
ε = material permittivity

Example: Capacitor

$$\nabla \cdot \xi = \frac{\rho}{\varepsilon}$$

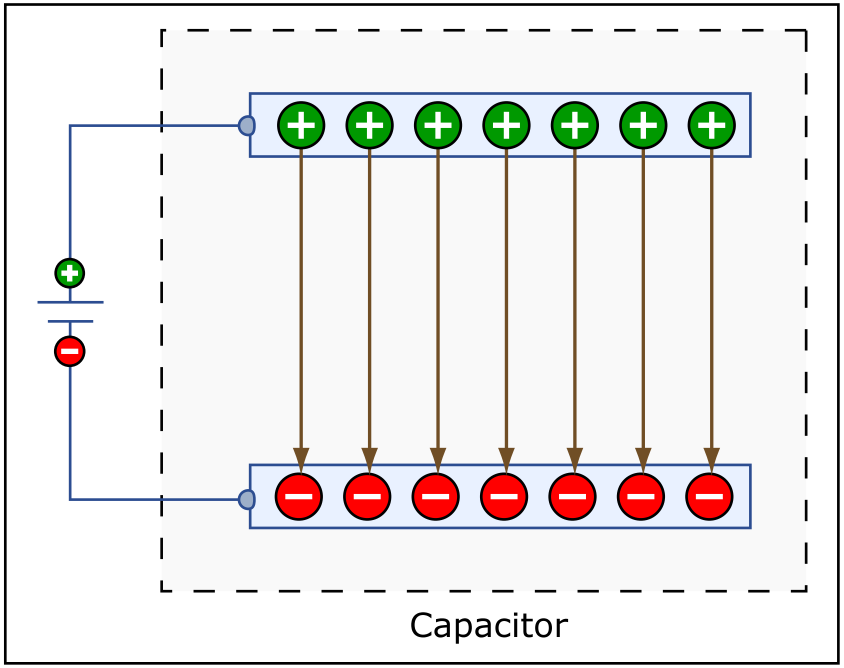

Image by MIT OpenCourseWare.

Gauss' Law: Review

Spatially variant fixed charge creates an electric field:

$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

ξ = electric field
ρ = charge density
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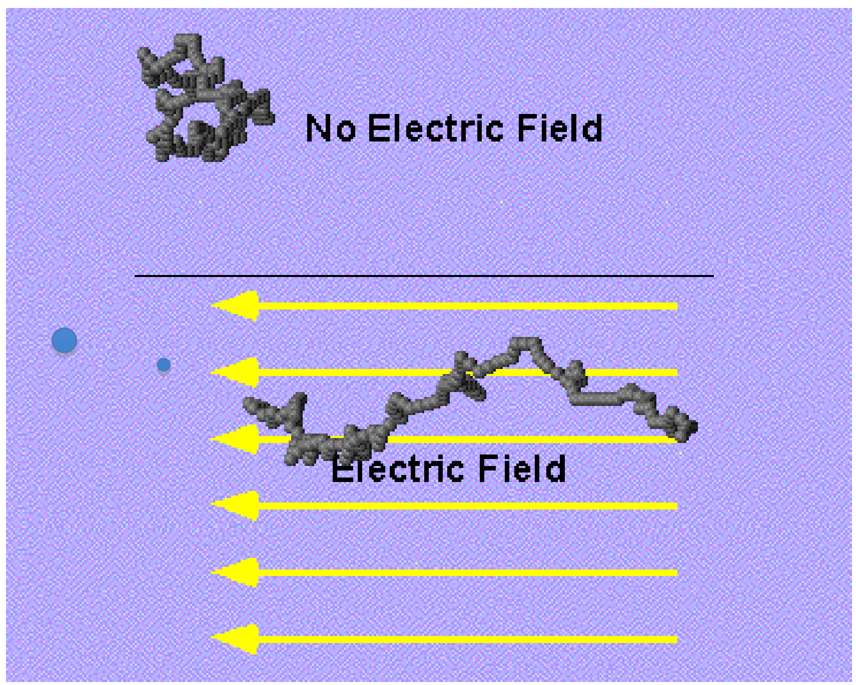
Drift Current: Net charge moves parallel to electric field

Described by Drift Equation

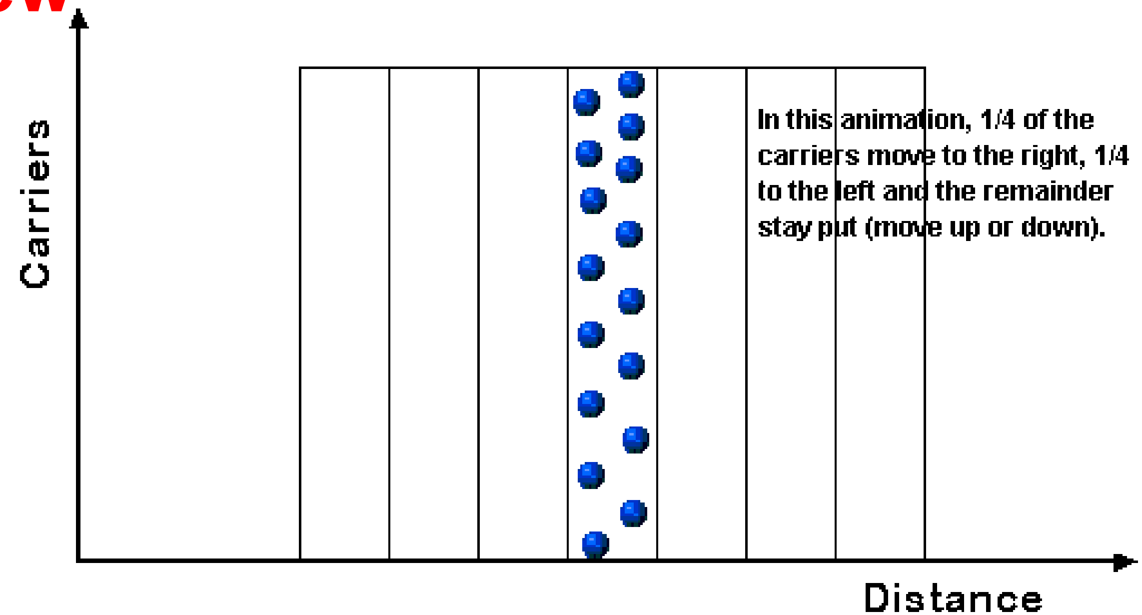
$$J_h = q\mu_h p\xi$$

$$J_e = q\mu_e n\xi$$



From: PVCDROM

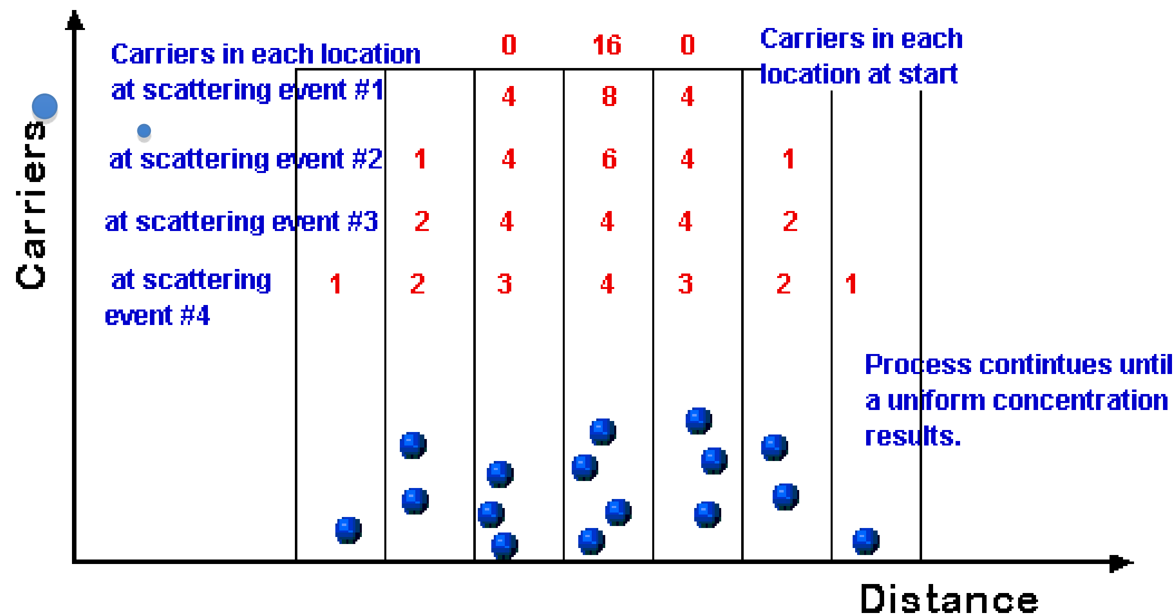
Diffusion: Review



Described by Fick's Law

$$J_{\mathrm{h}} = -qD_{\mathrm{h}} \frac{dp}{dx}$$

$$J_{\mathrm{e}} = qD_{\mathrm{e}} \frac{dn}{dx}$$



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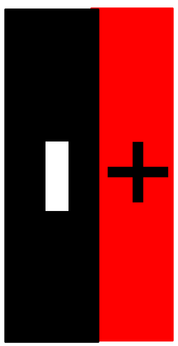
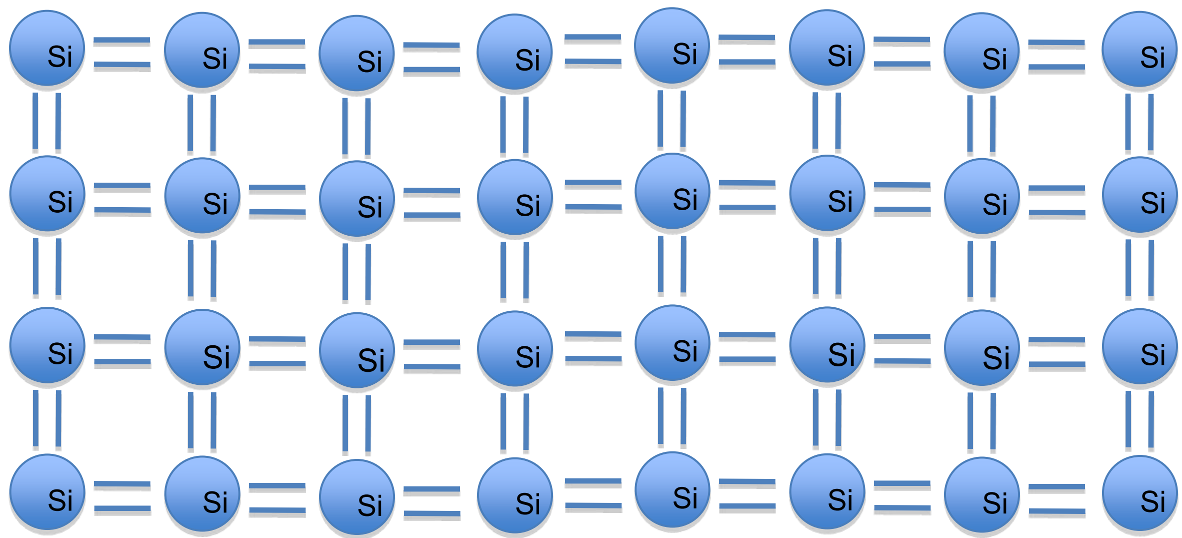
Buonassisi (MIT) 2011

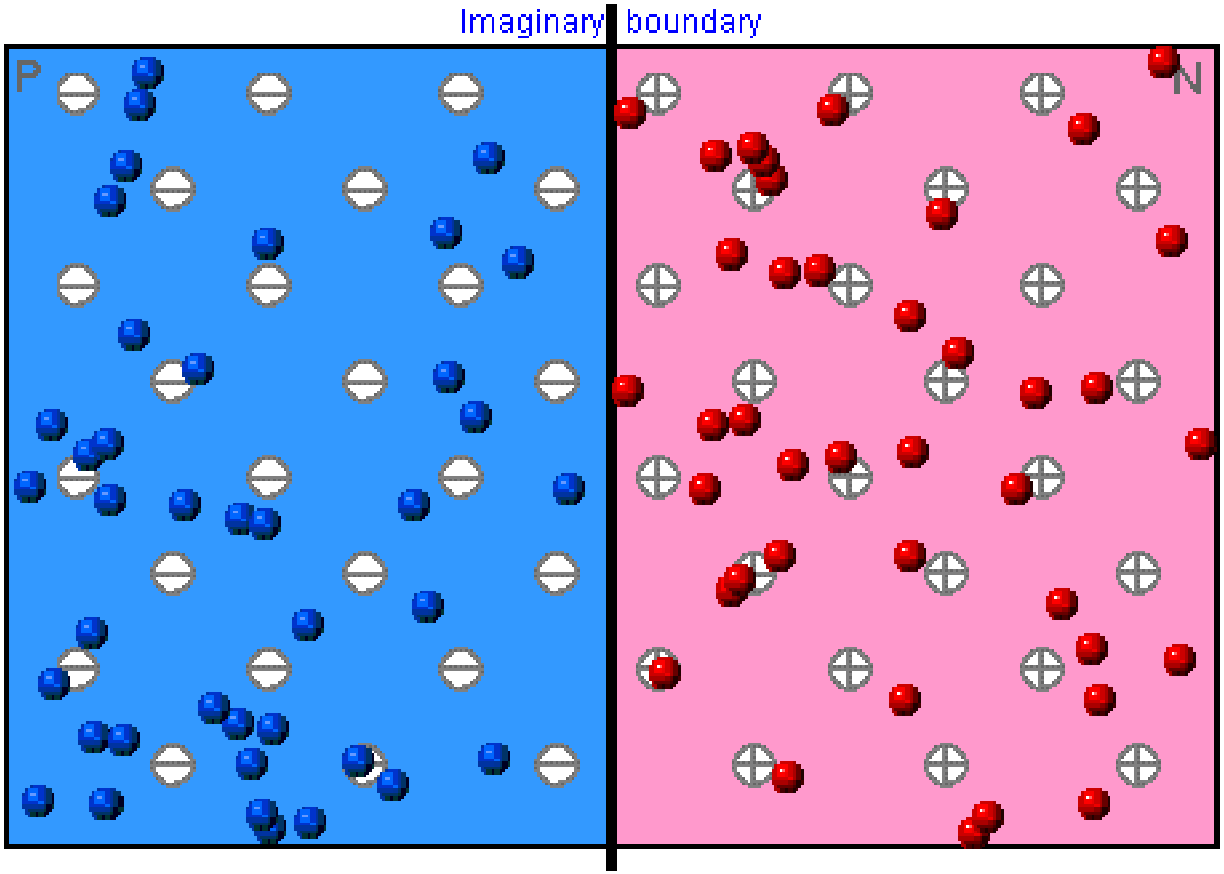
Recall the Checker Board Example





Let’s imagine the n- and p-type materials in contact, but with an imaginary barrier in between them.

How a pn-junction comes into being



With the P and N type materials separated the carriers diffuse around randomly.

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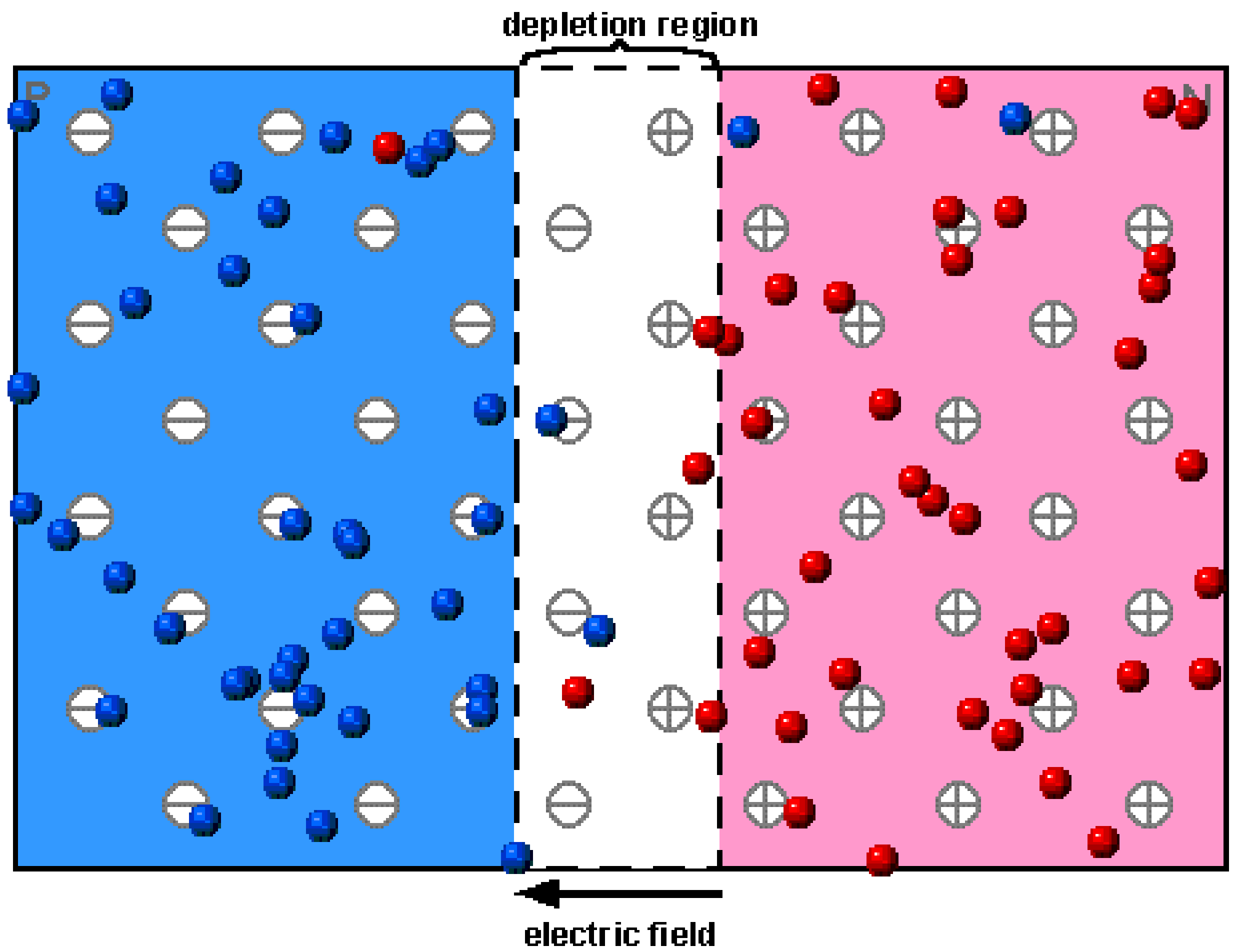
When that imaginary boundary is removed, electrons and holes diffuse into the other side.

How a pn-junction comes into being



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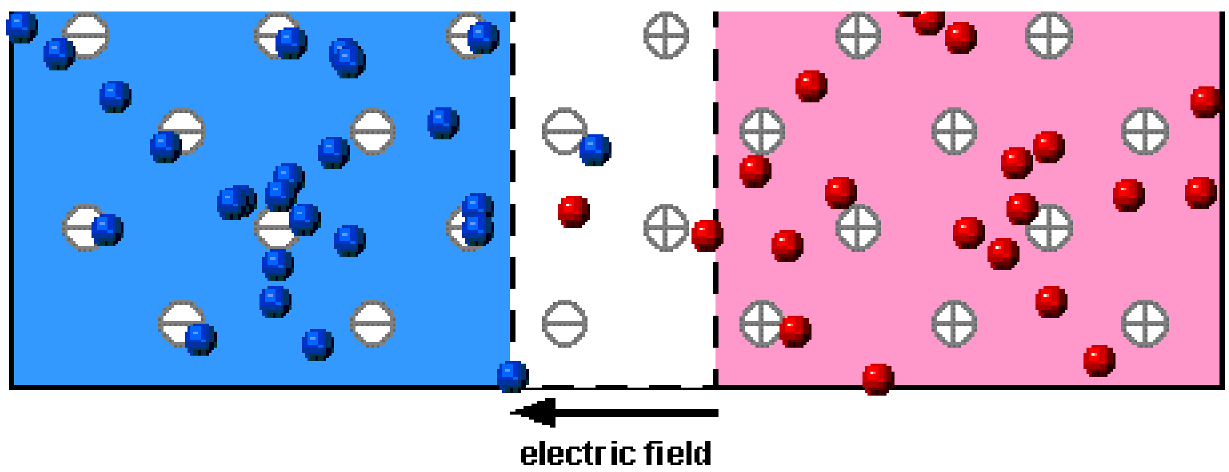
Eventually, the accumulation of like charges [(h^+ + P^+) or (e^- + B^-)] balances out the diffusion, and steady state condition is reached.

How a pn-junction comes into being



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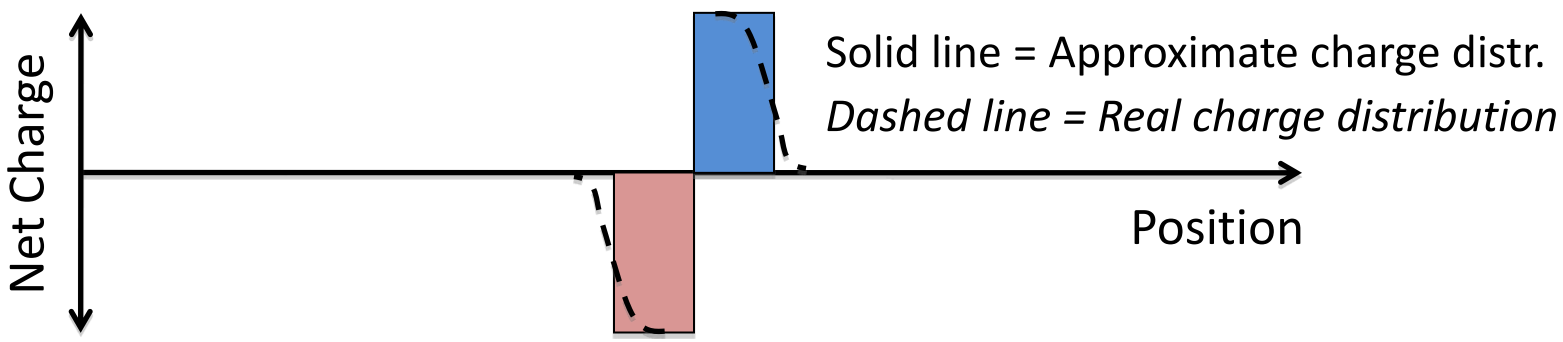
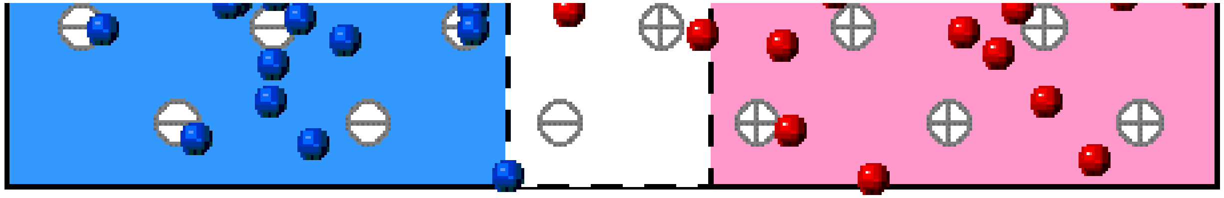


The net charge can be approximated as shown above.

How a pn-junction comes into being



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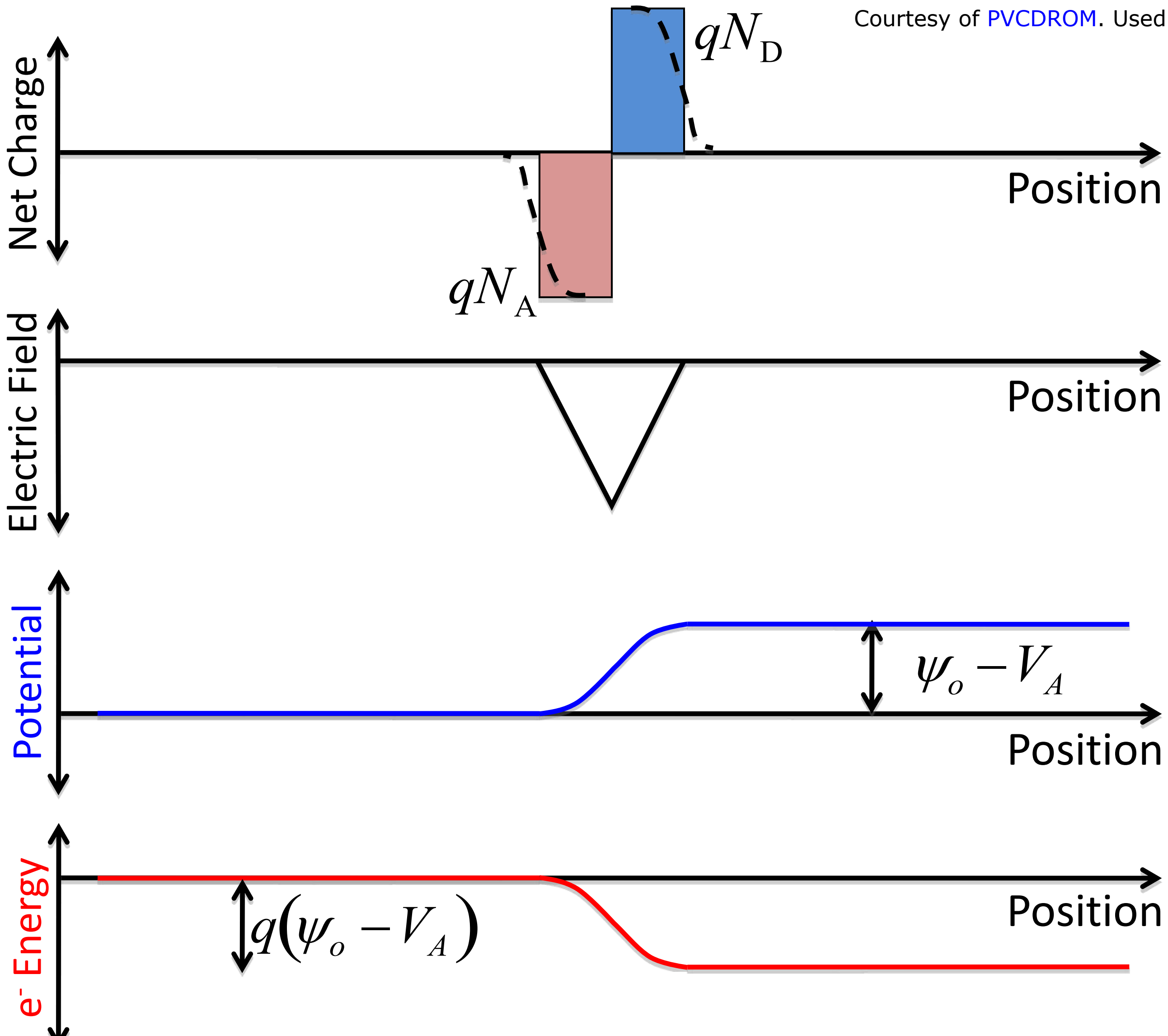
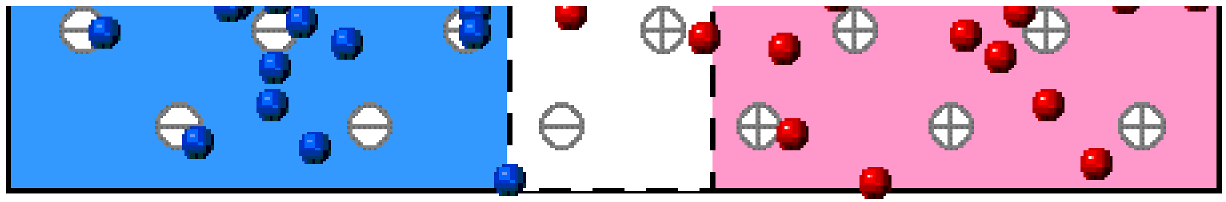


$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

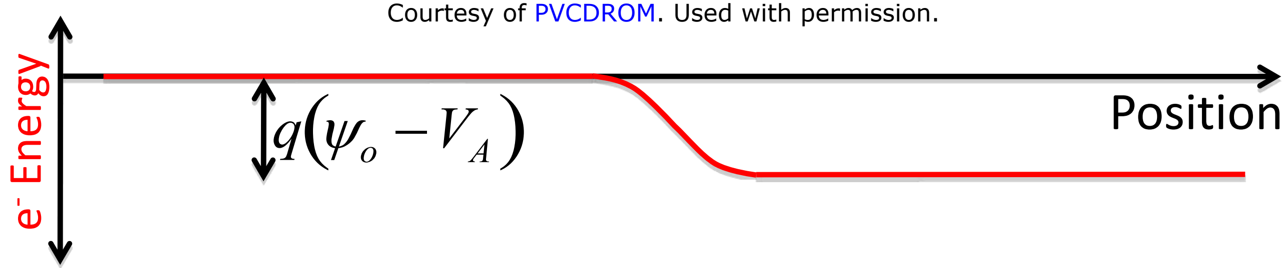
$$\frac{d\psi}{dx} = -\xi$$

$$E = q\psi$$

Summary of Current Understanding



$$E = q\psi$$



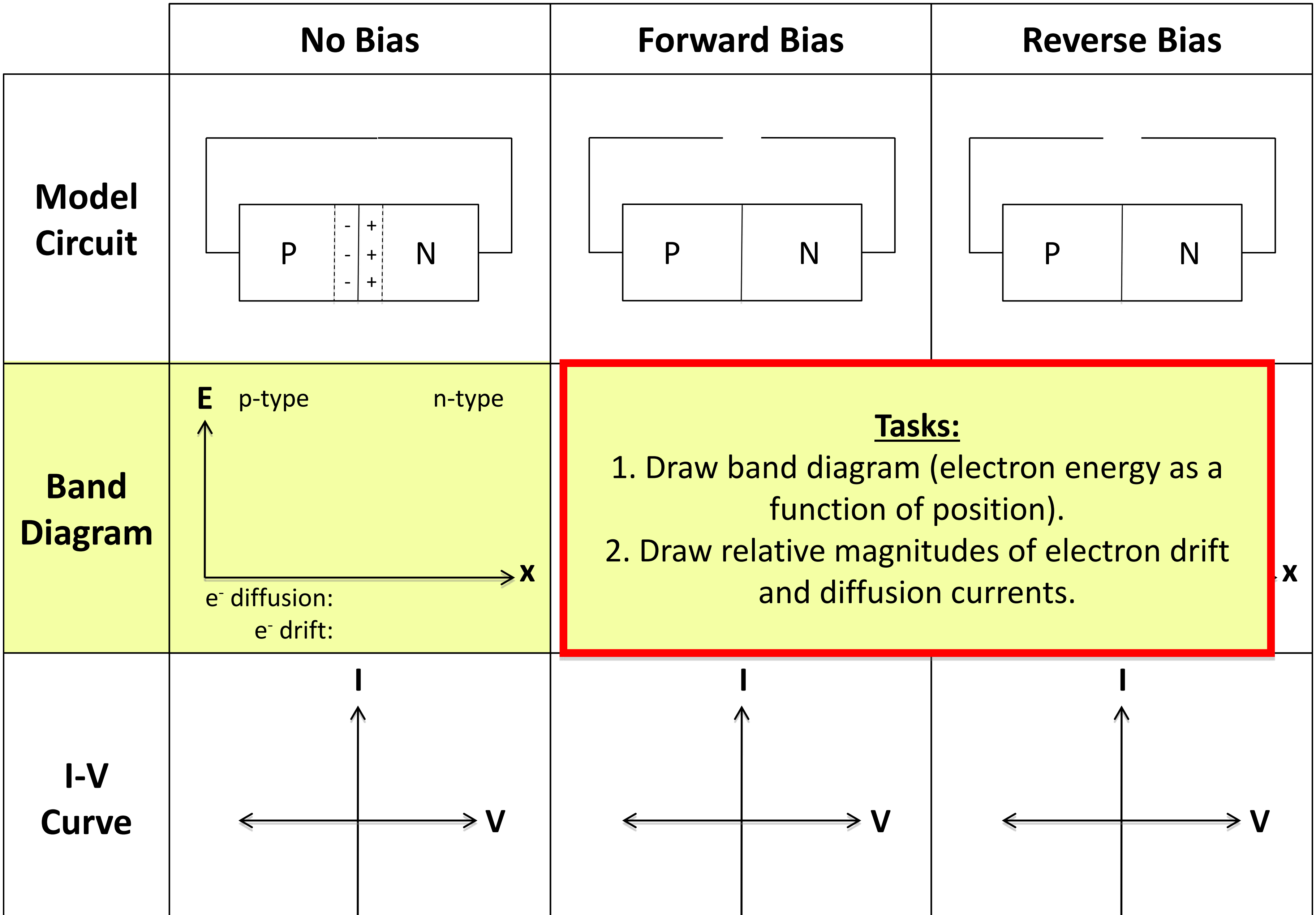
1. When light creates an electron-hole pair, a *pn*-junction can separate the positive and negative charges because of the built-in electric field.
2. This built-in electric field is established at a pn-junction because of the balance of electron & hole drift and diffusion currents.

In-Class Exercise

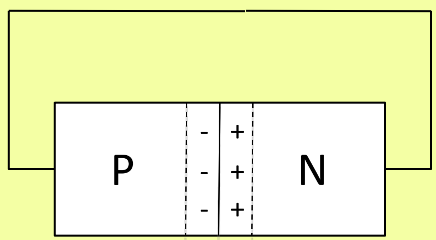
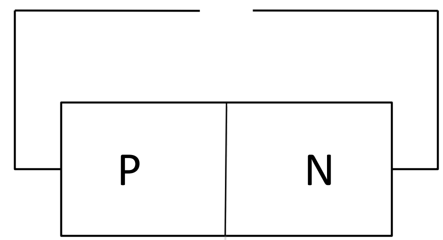
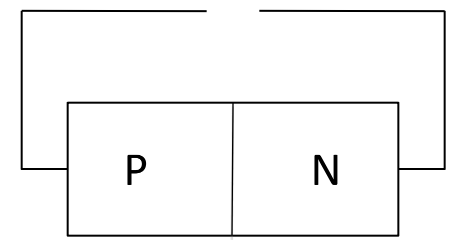
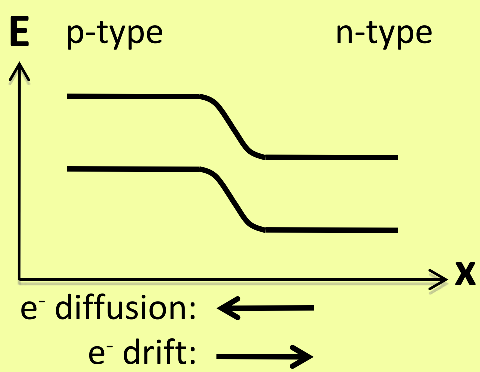
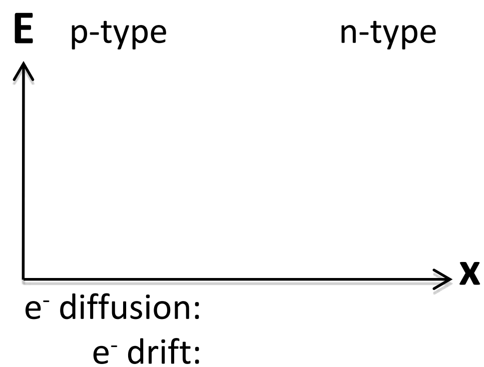
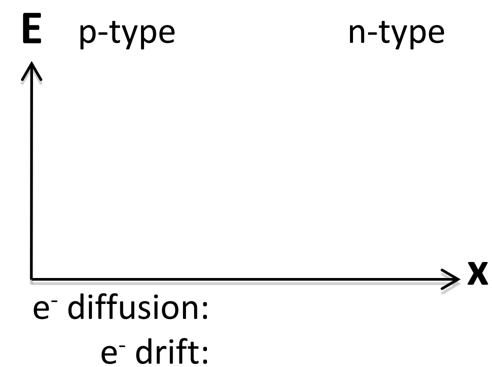
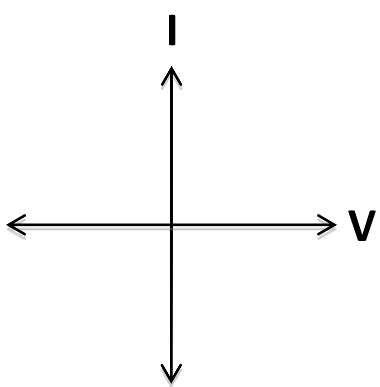
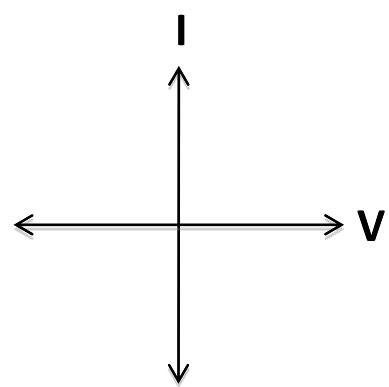
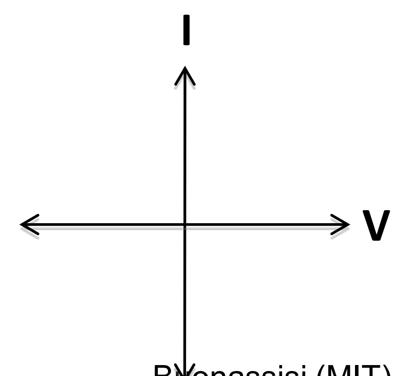
pn-junction, under dark conditions

	No Bias	Forward Bias	Reverse Bias
Model Circuit	P - - - + + + N	P N	P N
Band Diagram	E p-type n-type x e^- diffusion: e^- drift:	E p-type n-type x e^- diffusion: e^- drift:	E p-type n-type x e^- diffusion: e^- drift:
I-V Curve	I V	I V	I V

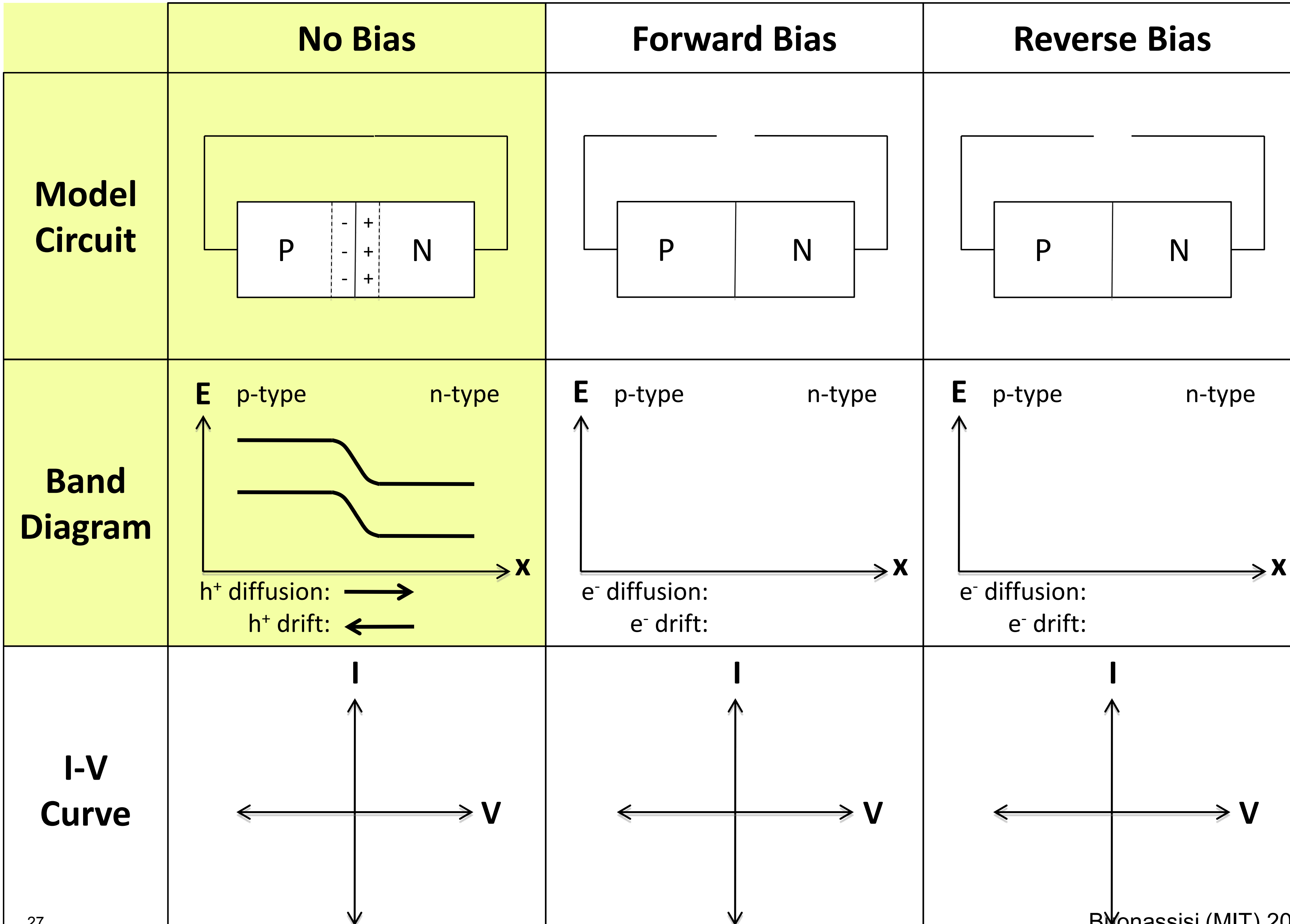
pn-junction, under dark conditions



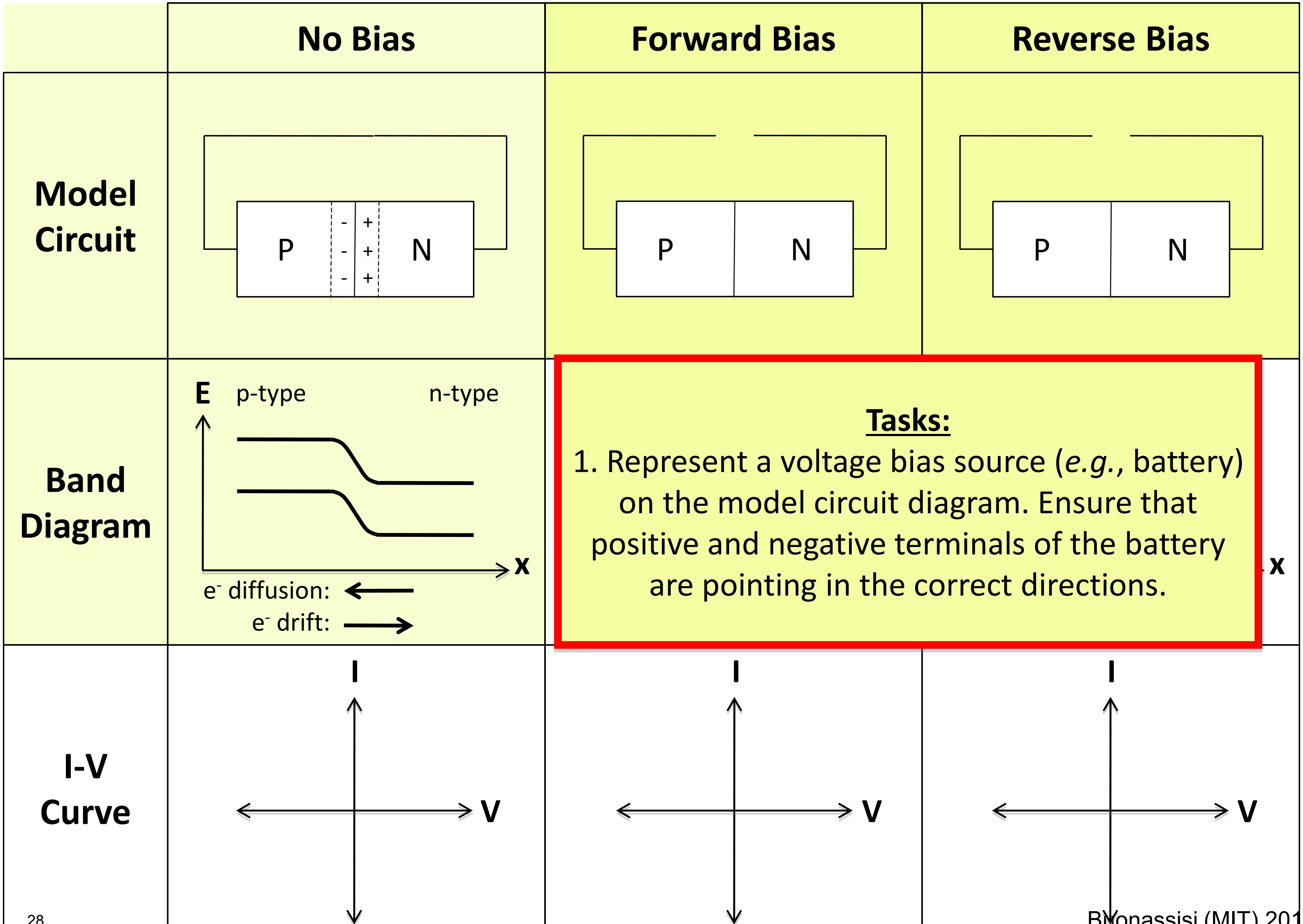
pn-junction, under dark conditions

	No Bias	Forward Bias	Reverse Bias
Model Circuit			
Band Diagram			
I-V Curve			

pn-junction, under dark conditions



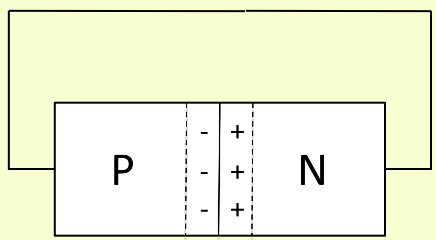
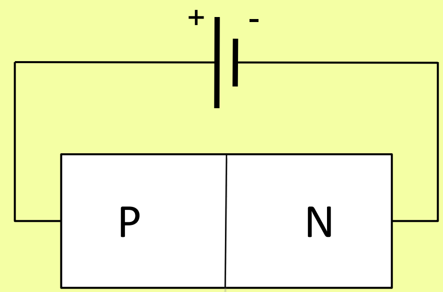
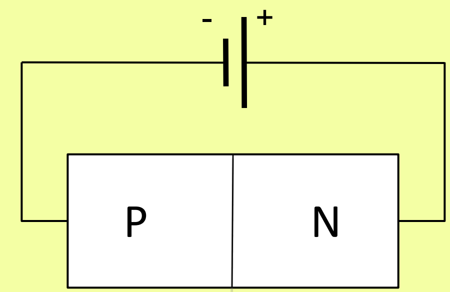
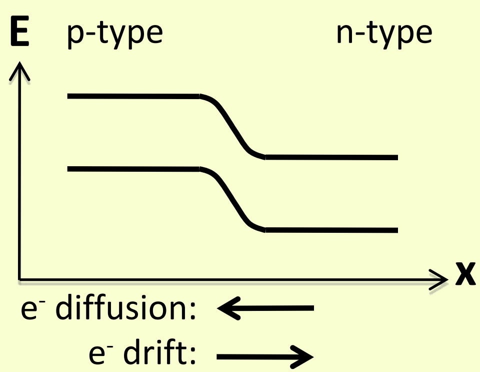
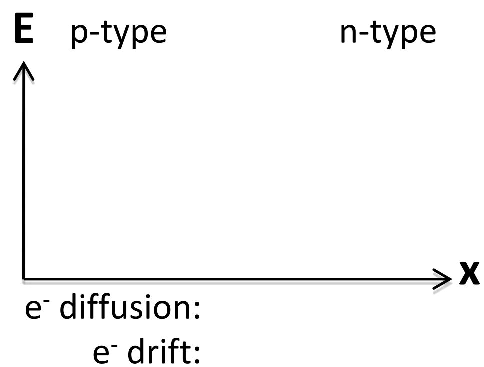
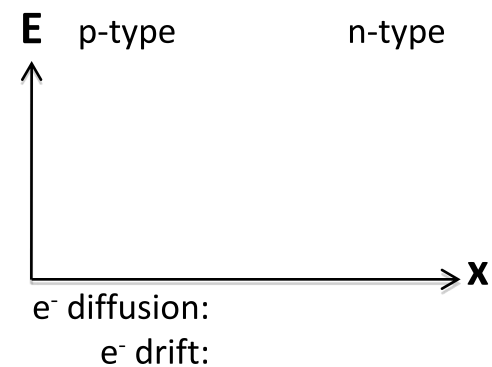
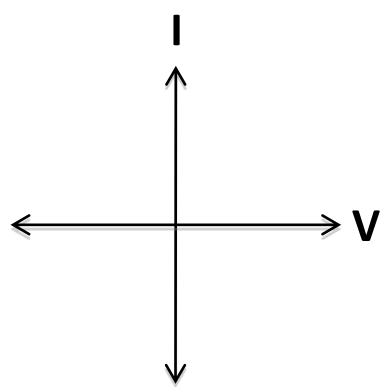
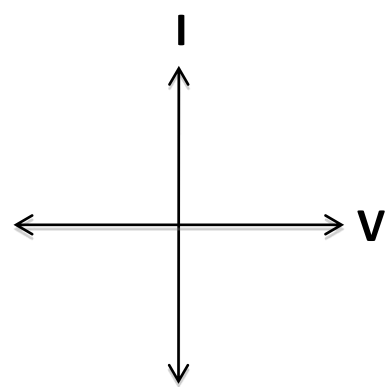
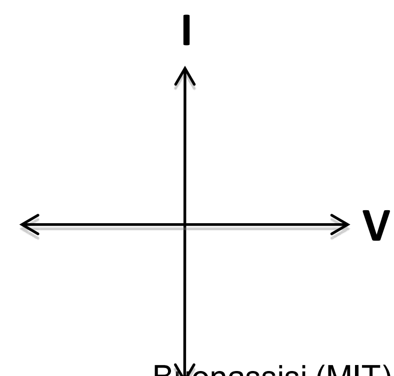
pn-junction, under dark conditions



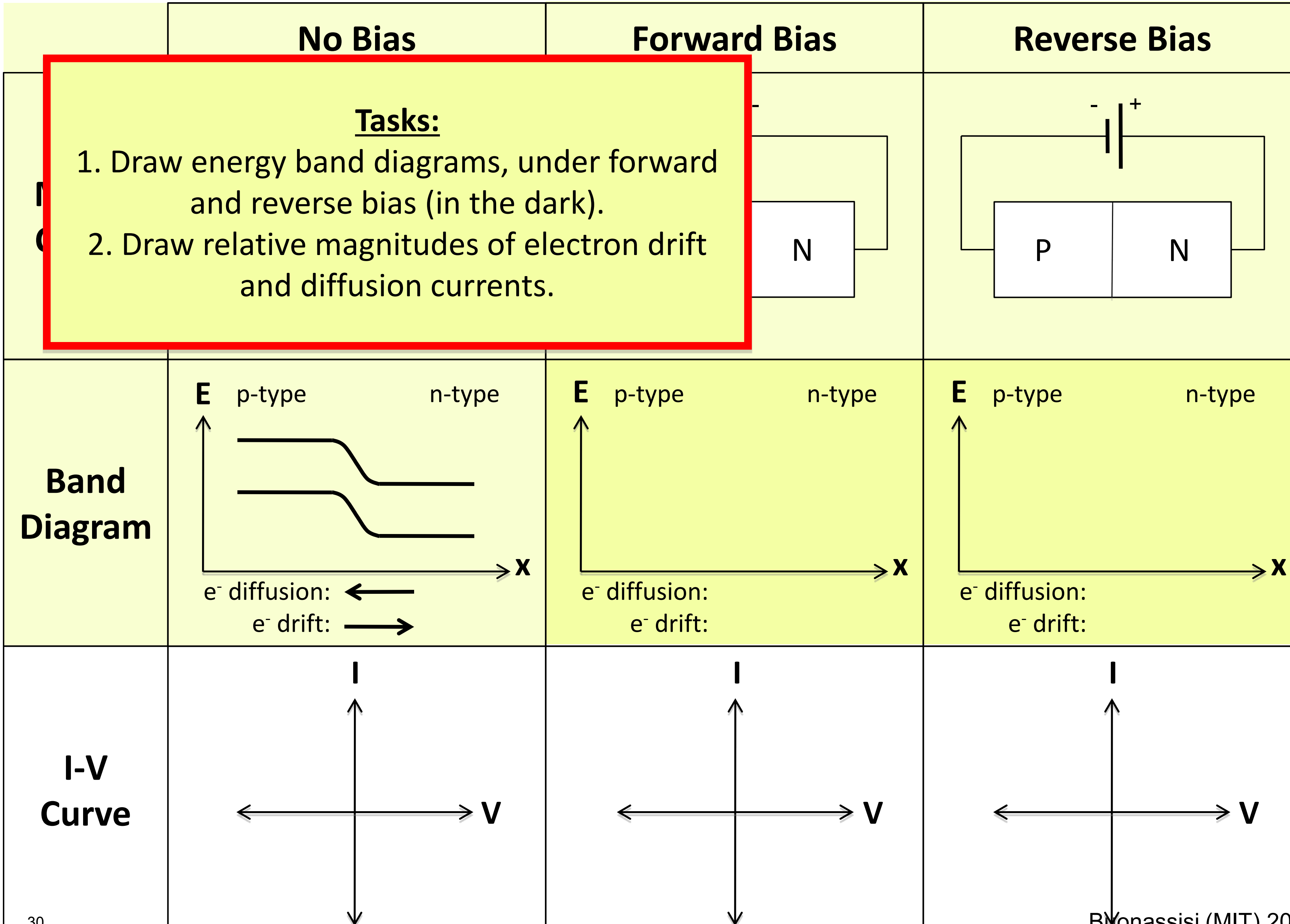
Tasks:

1. Represent a voltage bias source (*e.g.*, battery) on the model circuit diagram. Ensure that positive and negative terminals of the battery are pointing in the correct directions.

pn-junction, under dark conditions

	No Bias	Forward Bias	Reverse Bias
Model Circuit			
Band Diagram			
I-V Curve			

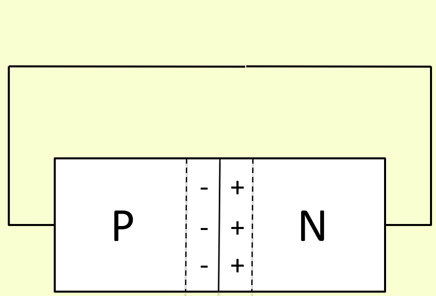
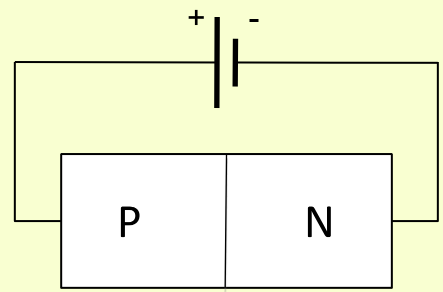
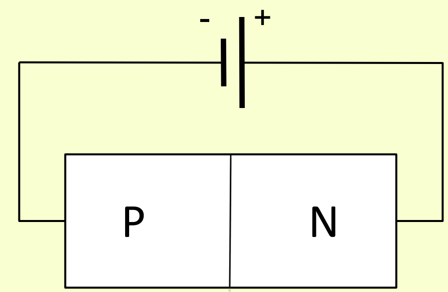
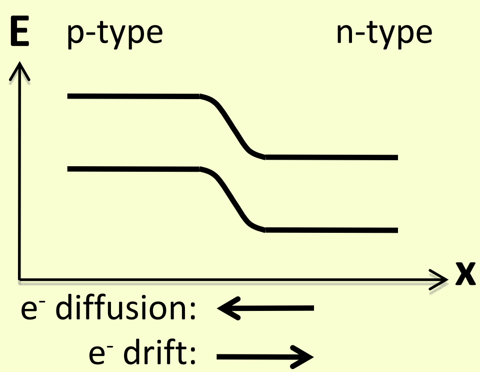
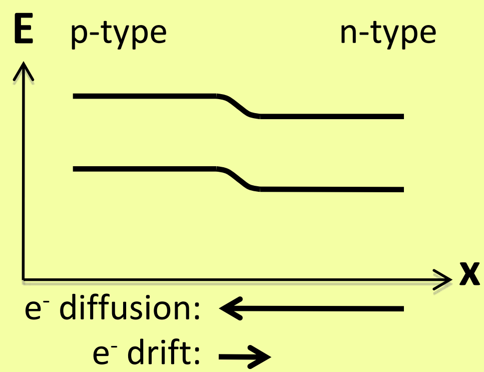
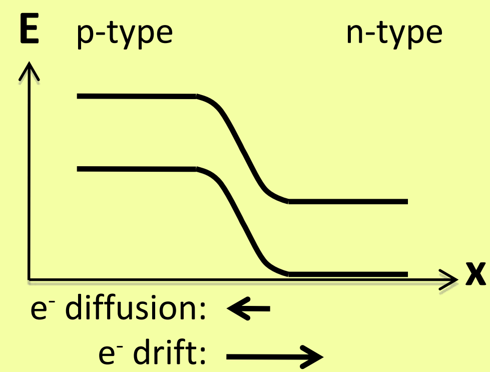
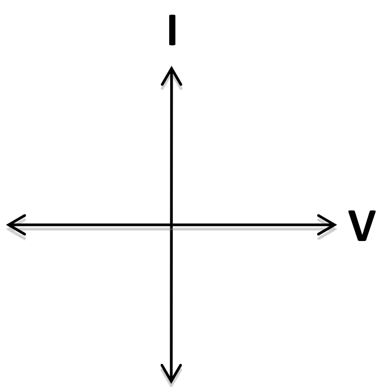
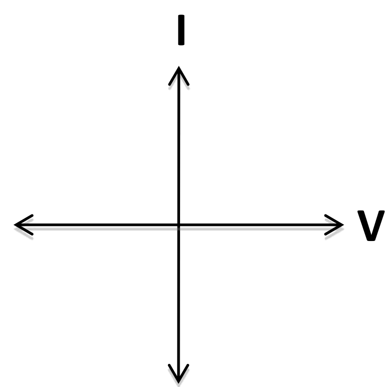
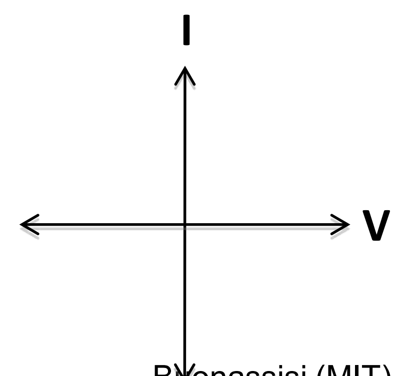
pn-junction, under dark conditions



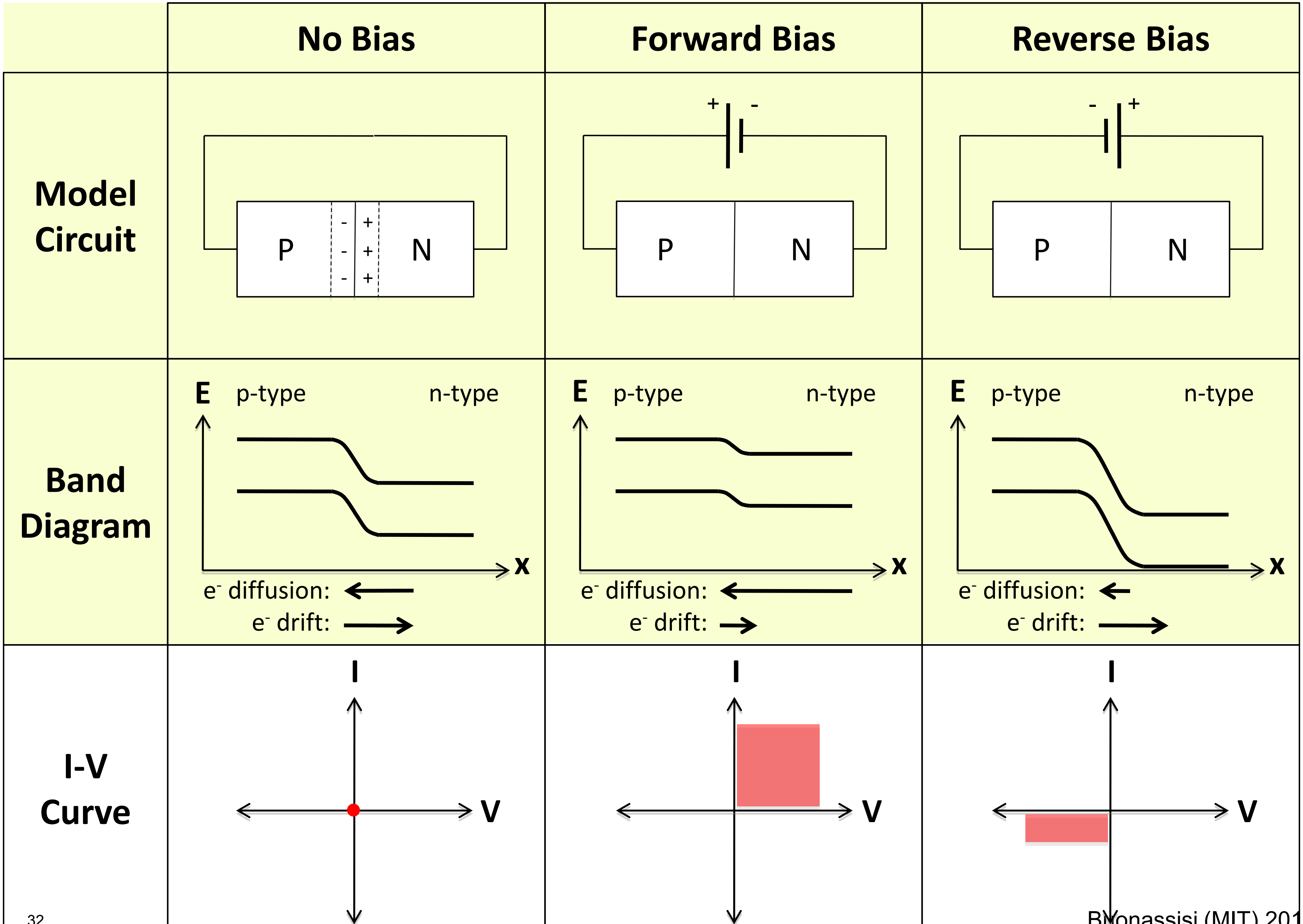
Tasks:

1. Draw energy band diagrams, under forward and reverse bias (in the dark).
2. Draw relative magnitudes of electron drift and diffusion currents.

pn-junction, under dark conditions

	No Bias	Forward Bias	Reverse Bias
Model Circuit			
Band Diagram			
I-V Curve			

pn-junction, under dark conditions



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4. Voltage across a *pn*-junction: Quantify the built-in voltage across a *pn*-junction. Quantify how the voltage across a *pn*-junction changes when an external bias voltage is applied.
5. Draw current-voltage (I-V) response, recognizing that minority carrier flux regulates current.

Carrier Motion

Under equilibrium conditions in a homogeneous material: Individual carriers constantly experience Brownian motion, but the net charge flow is zero.

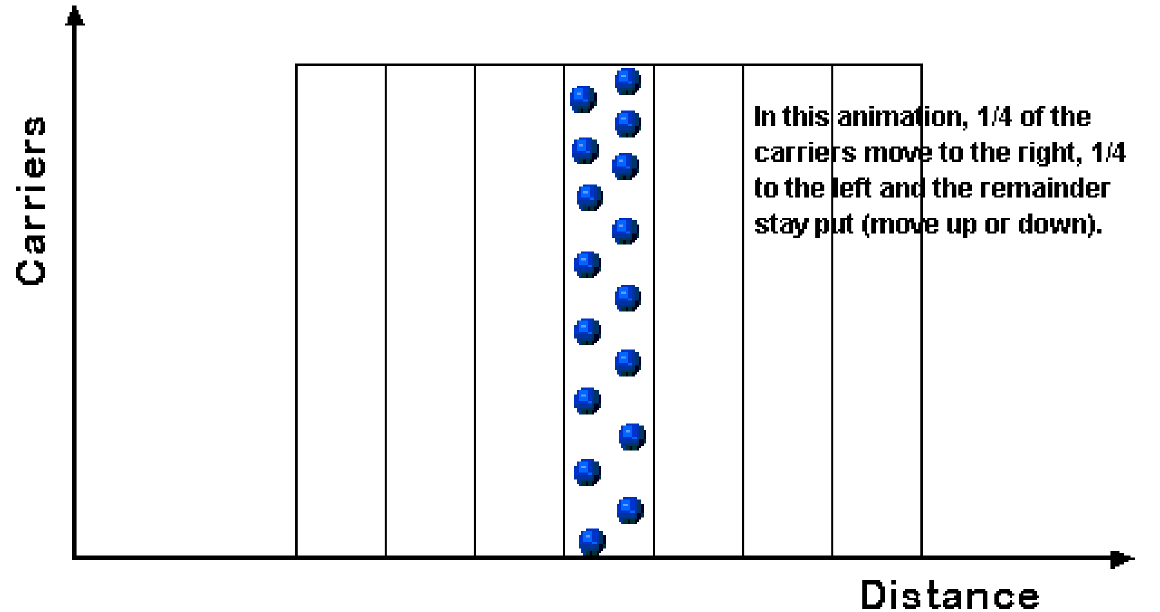
To achieve net charge flow (current), carriers must move via diffusion or drift.

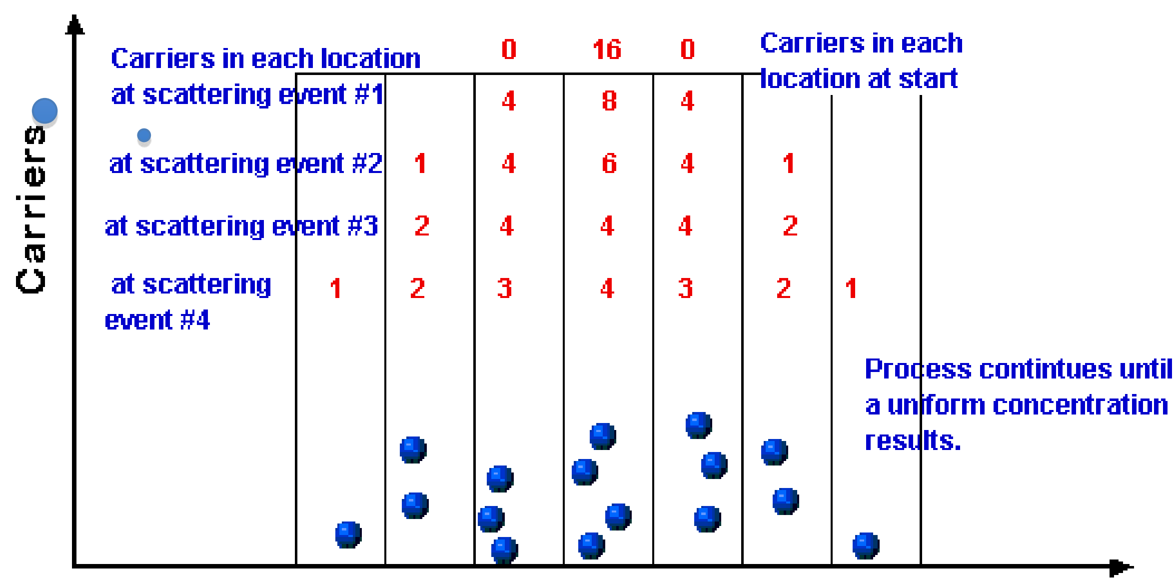
Diffusion

Described by Fick's Law

$$J_{\mathrm{h}} = -qD_{\mathrm{h}}\frac{dp}{dx}$$

$$J_{\mathrm{e}} = qD_{\mathrm{e}}\frac{dn}{dx}$$





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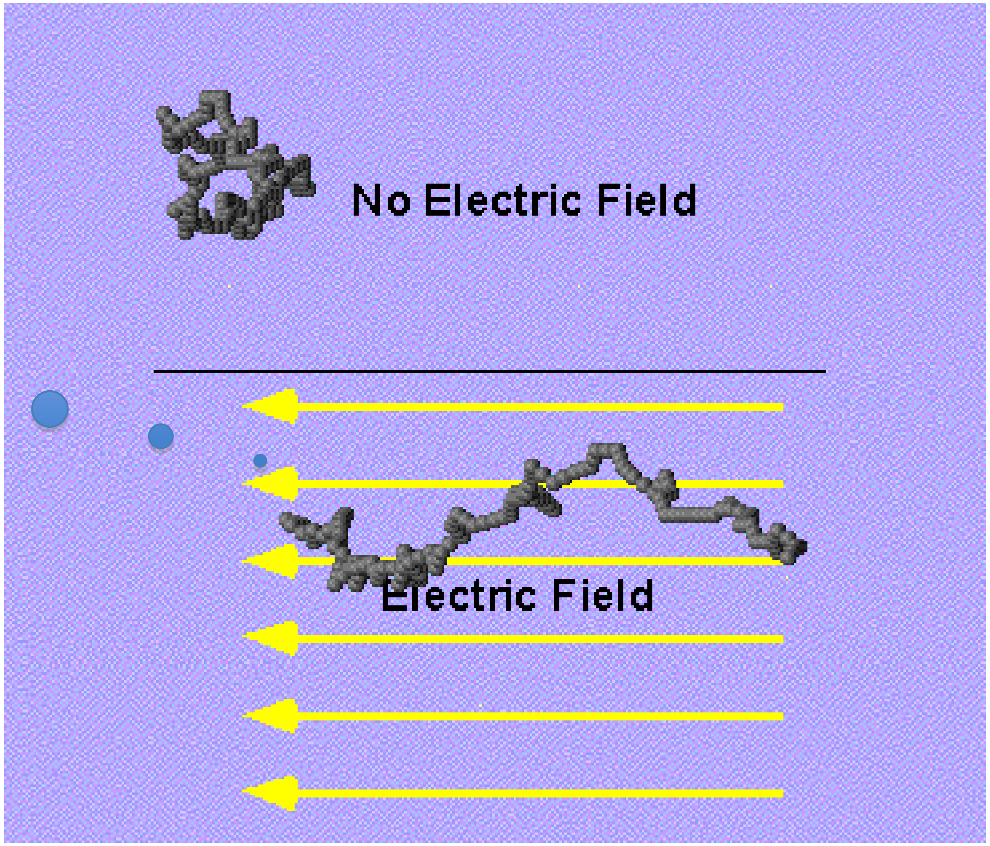
Drift Current

$$J_{\mathrm{h}} = q\mu_{\mathrm{h}} p\xi$$

$$J_{\mathrm{e}} = q\mu_{\mathrm{e}} n\xi$$



From PVCDROM

Current Density Equations

$$J_e = \underbrace{q\mu_n n\xi}_{\text{Dominates when } \xi \text{ is large}} + \underbrace{qD_e \frac{dn}{dx}}_{\text{Dominates when } \xi \text{ is small}}$$

$$J_h = \underbrace{q\mu_h p\xi}_{\text{Dominates when } \xi \text{ is large}} - \underbrace{qD_h \frac{dp}{dx}}_{\text{Dominates when } \xi \text{ is small}}$$

Einstein Relationships:
Relation between drift and diffusion:

$$D_e = \left(\frac{kT}{q}\right)\mu_n$$

$$D_h = \left(\frac{kT}{q}\right)\mu_p$$

What's ξ?

From differential form of Gauss' Law (a.k.a. Poisson's Equation):

$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

ρ = charge density
ε = material permittivity

We know the charge density is:

$$\rho = q\left(p - n + N_\mathrm{D}^+ - N_\mathrm{A}^-\right)$$

$$\rho \approx q\left(p - n + N_\mathrm{D} - N_\mathrm{A}\right)$$

N_D^+ = ionized donor concentration
N_A^- = ionized acceptor concentration

Assuming all dopants are ionized at room temperature

In summa:

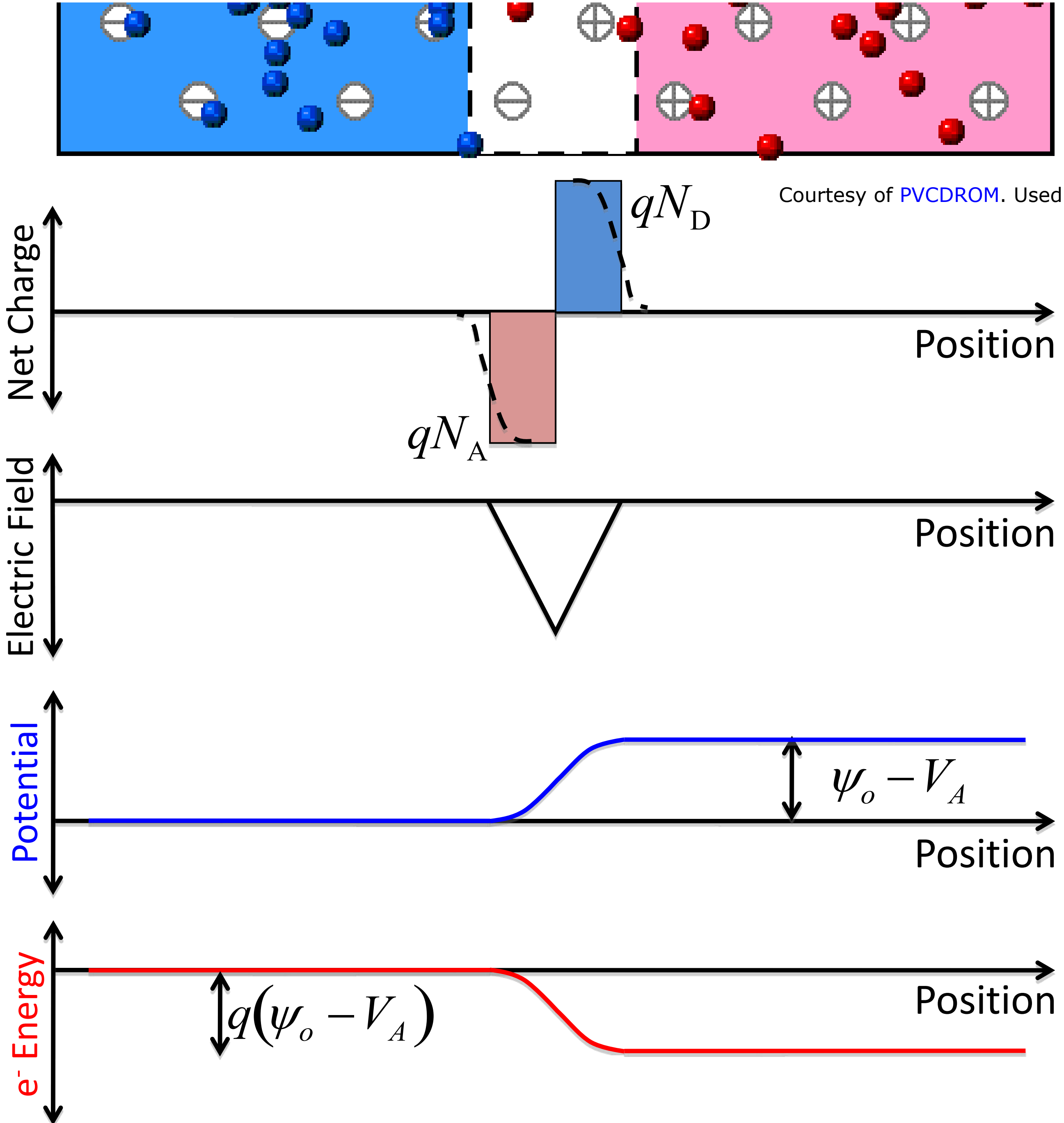
$$\frac{d\xi}{dx} = \frac{q}{\varepsilon}\left(p - n + N_\mathrm{D} - N_\mathrm{A}\right)$$

What's ξ?

$$\frac{d\xi}{dx} = \frac{\rho}{\varepsilon}$$

$$= -\xi$$

$$E = q\psi$$



Continuity Equations

$$\text{rate entering} - \text{rate exiting} = \frac{A}{q}\left\{J_e(x) - \left[J_e(x+\delta x)\right]\right\}$$

$$= \frac{A}{q}\frac{dJ_e}{dx}\delta x$$

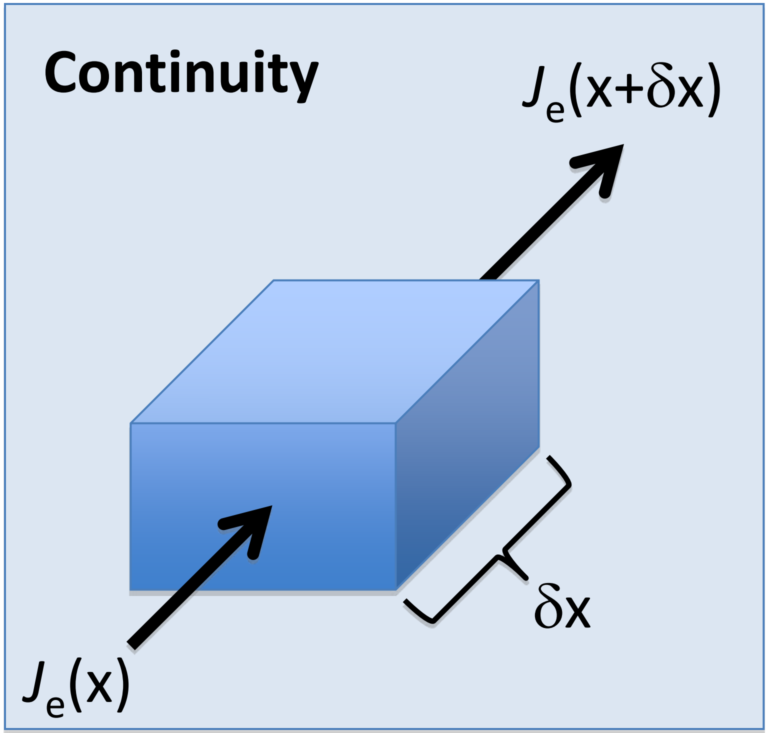
$$\text{rate of generation} - \text{rate of recombination} = A\delta x(G-U)$$

For electrons:

$$\frac{1}{q}\frac{dJ_e}{dx} = U - G$$

For holes:

$$\frac{1}{q}\frac{dJ_h}{dx} = -(U-G)$$



System of Equations Describing Transport in Semiconductors

$$J_e = q\mu_n n\xi + qD_e \frac{dn}{dx}$$

$$J_h = q\mu_h p\xi - qD_h \frac{dp}{dx}$$

Drift and Diffusion

$$\frac{d\xi}{dx} = \frac{q}{\varepsilon}\left(p - n + N_D - N_A\right)$$

Electric Field

$$\frac{1}{q}\frac{dJ_e}{dx} = U - G$$

$$\frac{1}{q}\frac{dJ_h}{dx} = -(U - G)$$

Continuity Equations

Possible to Solve Analytically?

No! Coupled set of non-linear differential equations.

Must solve numerically (e.g., using computer simulations)…

…or make series of approximations to solve analytically.

Learning Objectives: Diode

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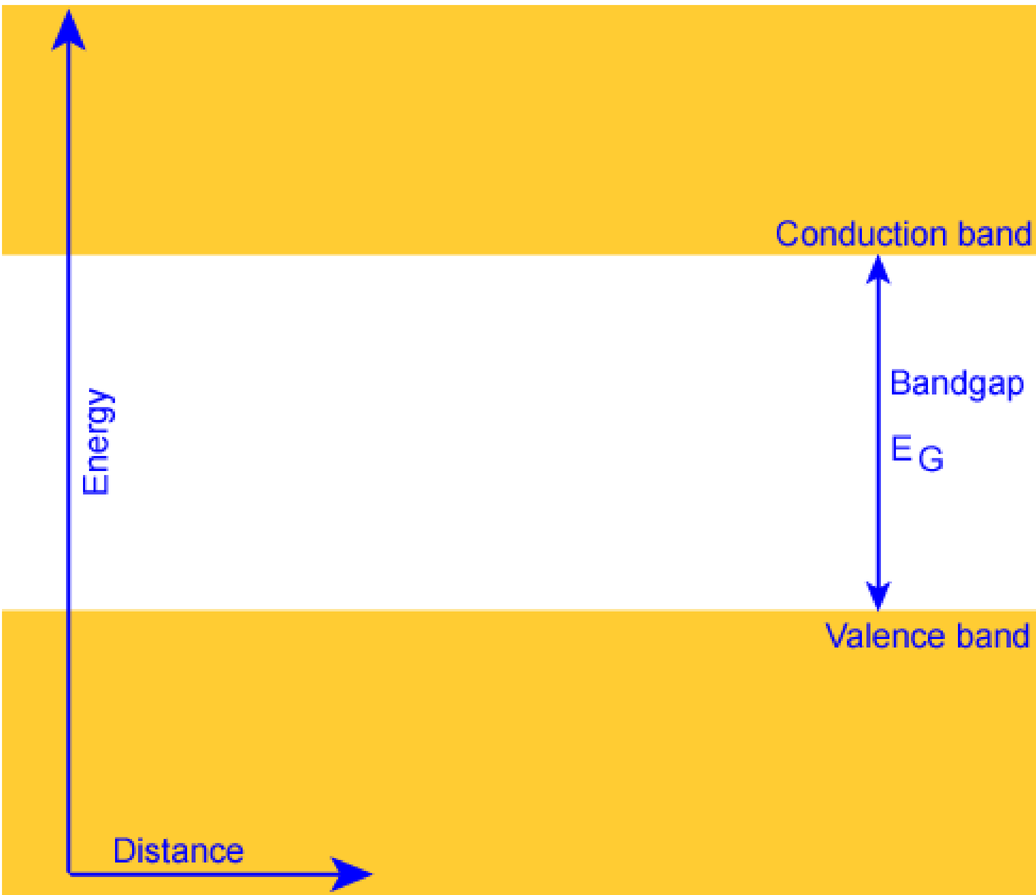
New Concept: ***Chemical Potential***

Band Diagram (E vs. x)



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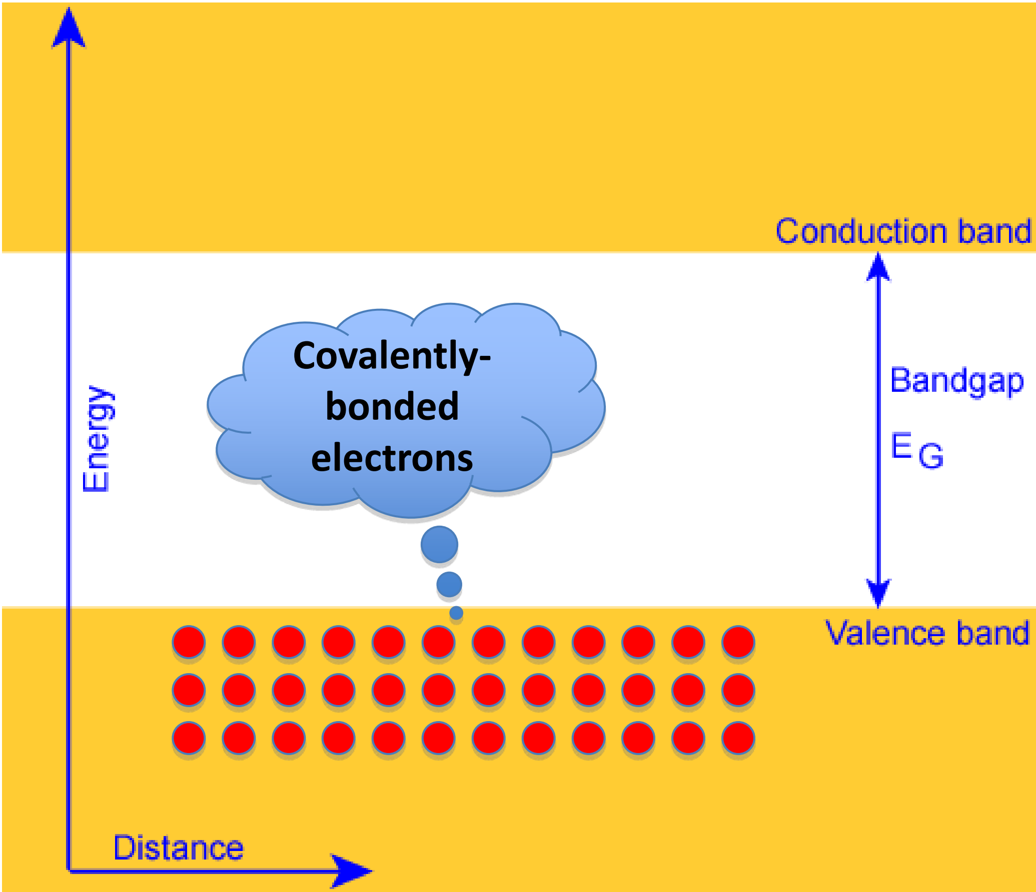
http://pvcdrom.pveducation.org/

New Concept: ***Chemical Potential***

At absolute zero, no conductivity (perfect insulator).

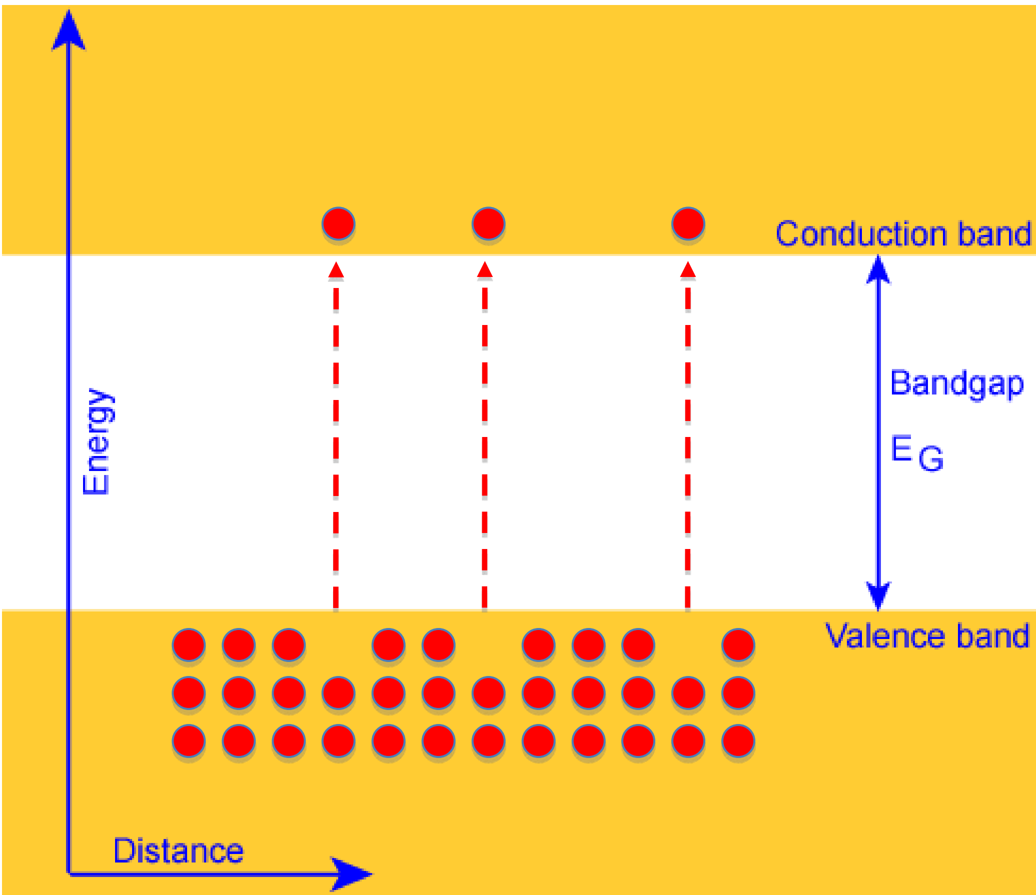
Band Diagram (E vs. x)



New Concept: ***Chemical Potential***

At T > 0 K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. x)



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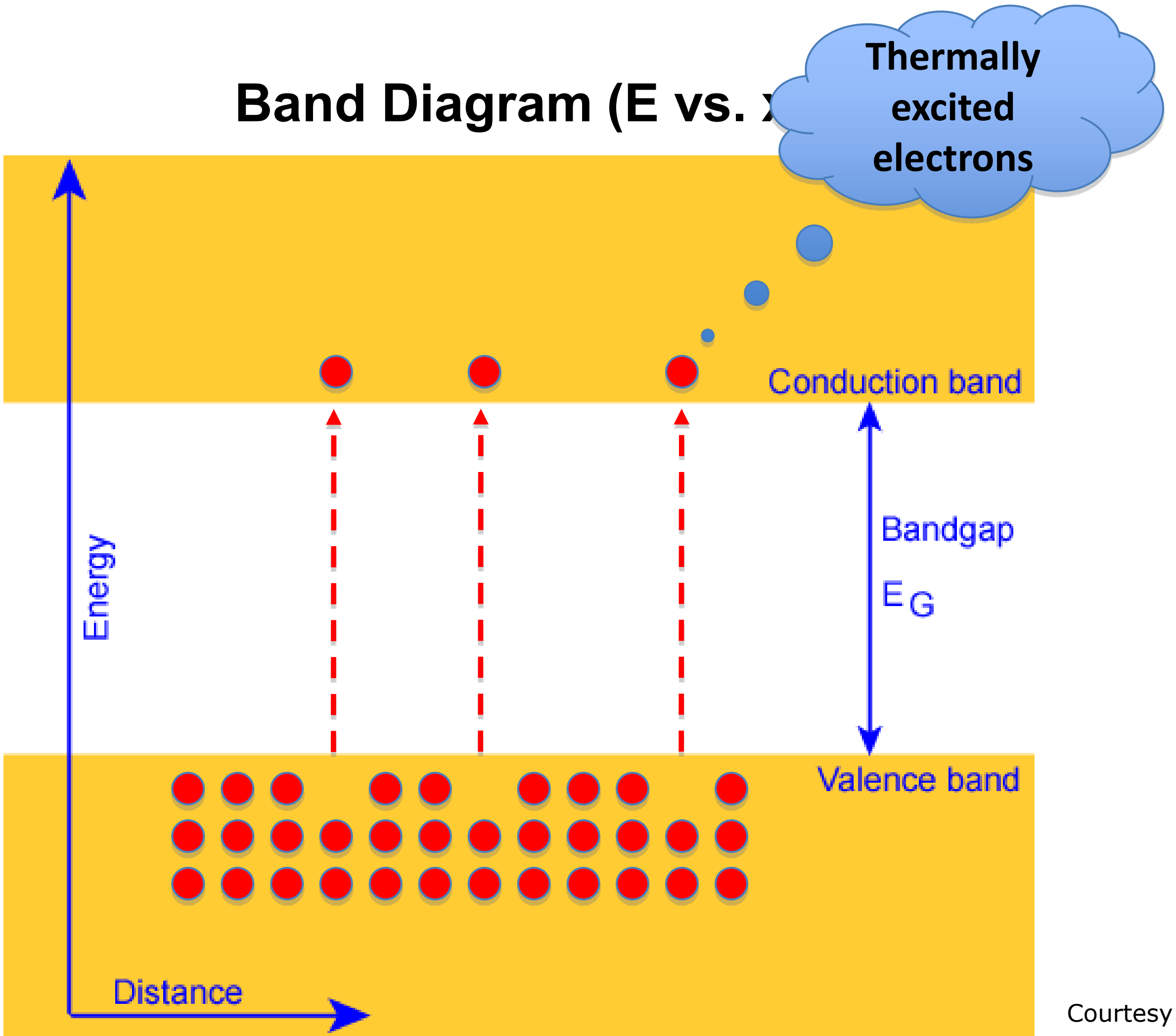

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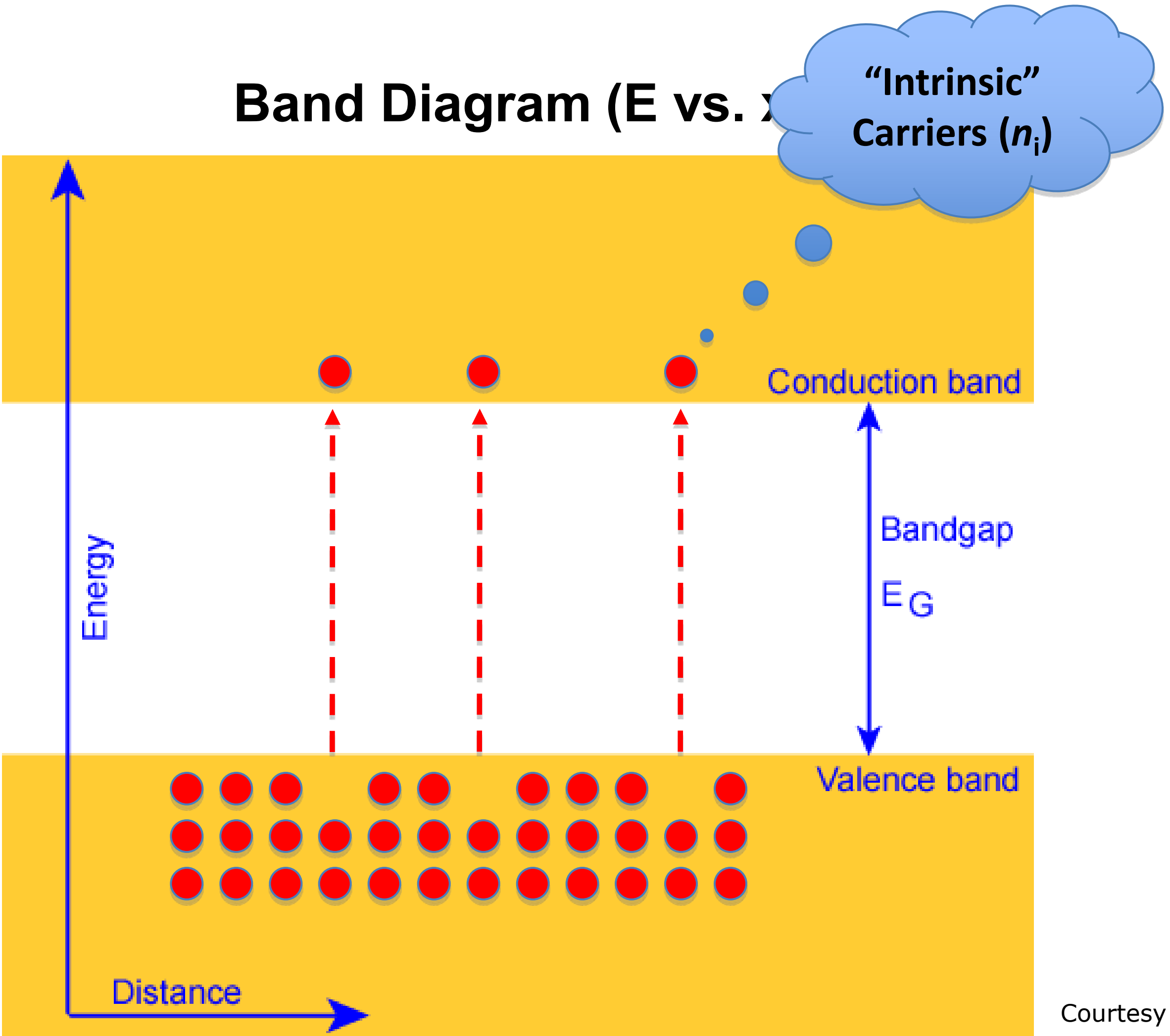

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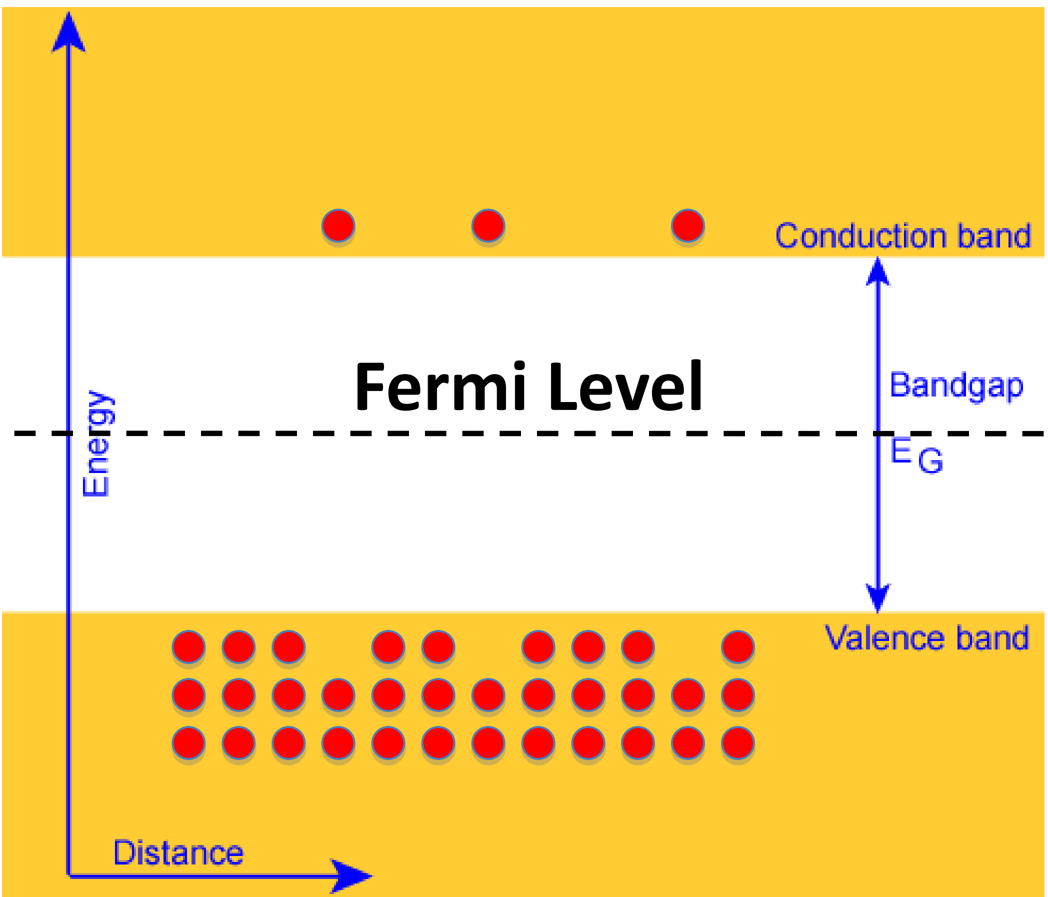
New Concept: ***Chemical Potential***

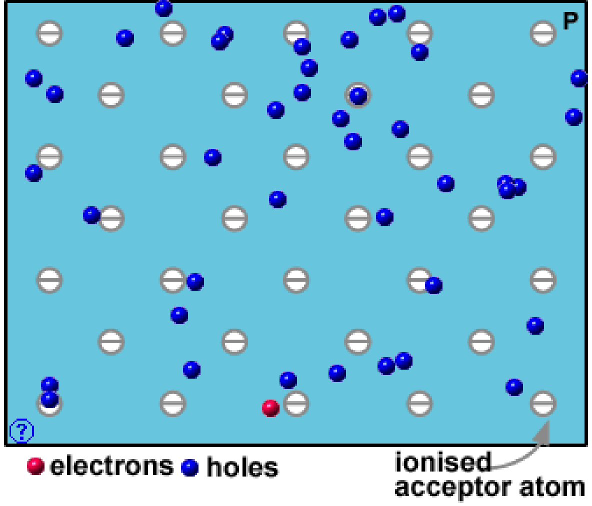
At T > 0 K, some carriers are thermally excited across the bandgap.

Band Diagram (E vs. x)

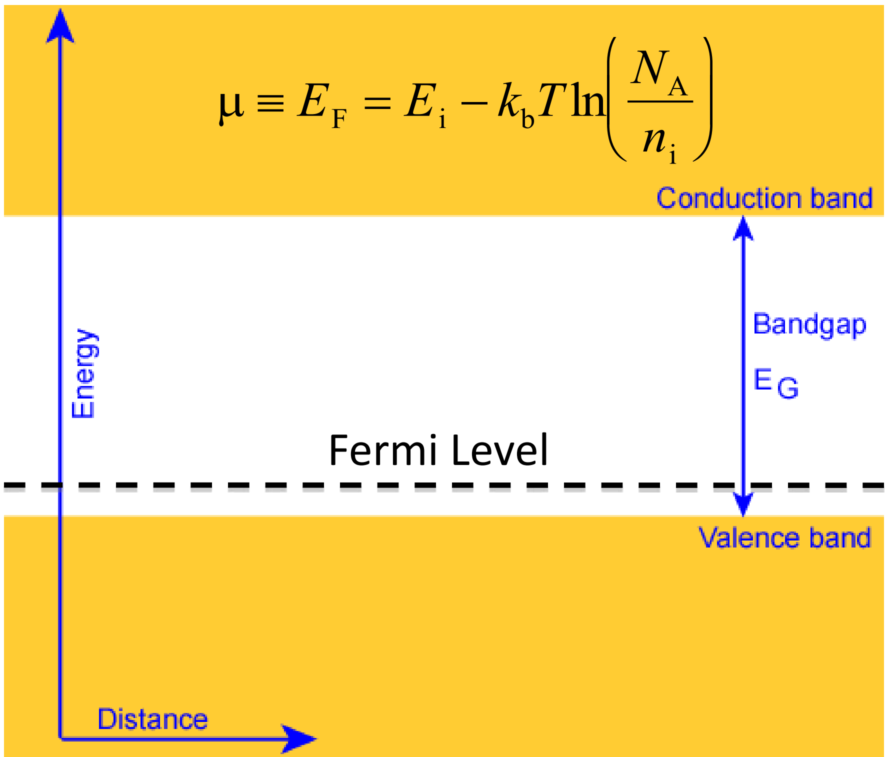


- The *chemical potential* describes the average energy necessary to add or remove an infinitesimally small quantity of electrons to the system.
- In a semiconductor, the chemical potential is referred to as the "Fermi level."

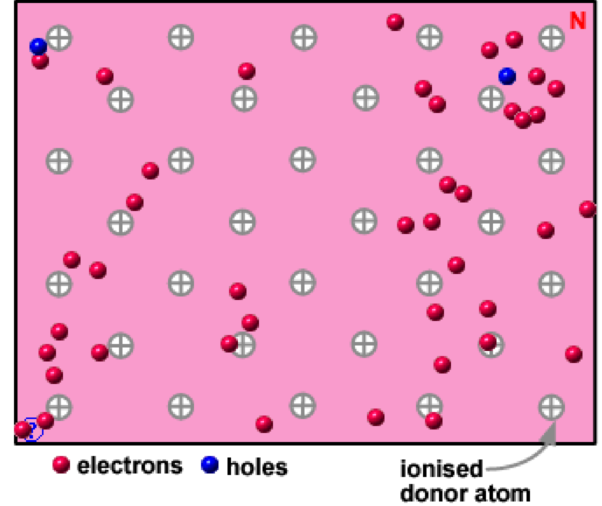
p-type



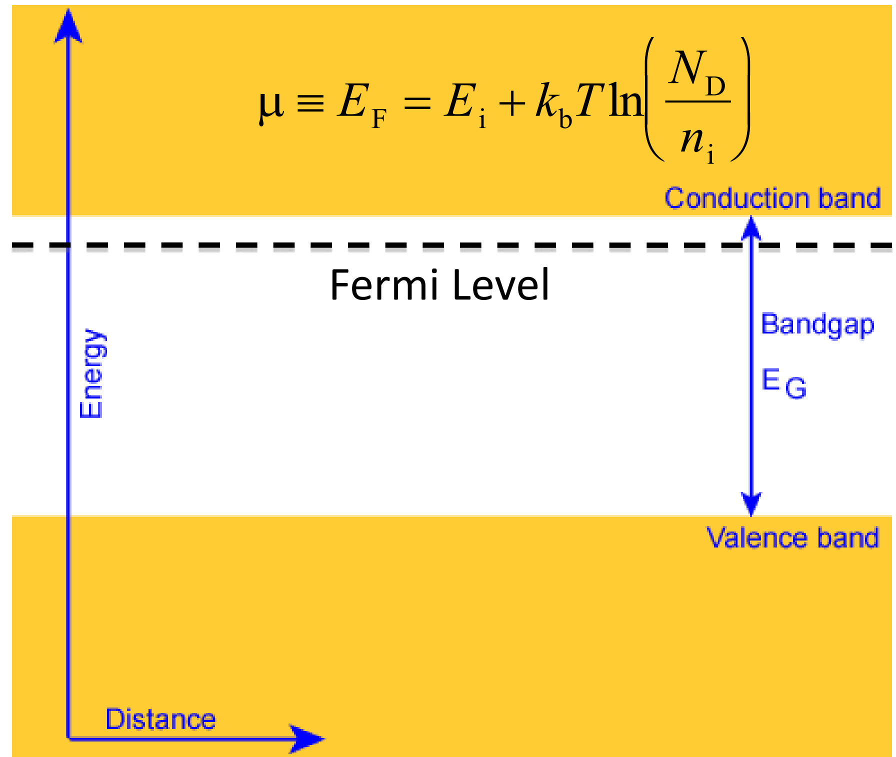
$$\mu \equiv E_F = E_i - k_b T \ln\left(\frac{N_A}{n_i}\right)$$



n-type



$$\mu \equiv E_F = E_i + k_b T \ln\left(\frac{N_D}{n_i}\right)$$



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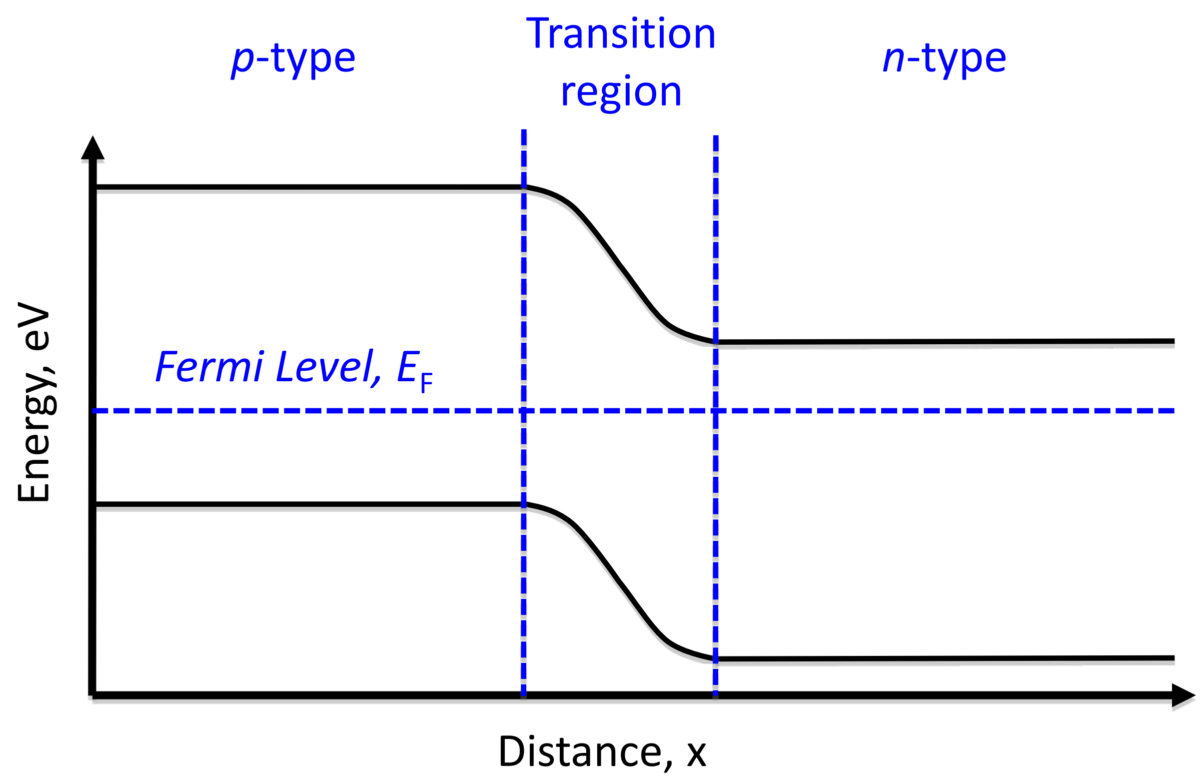
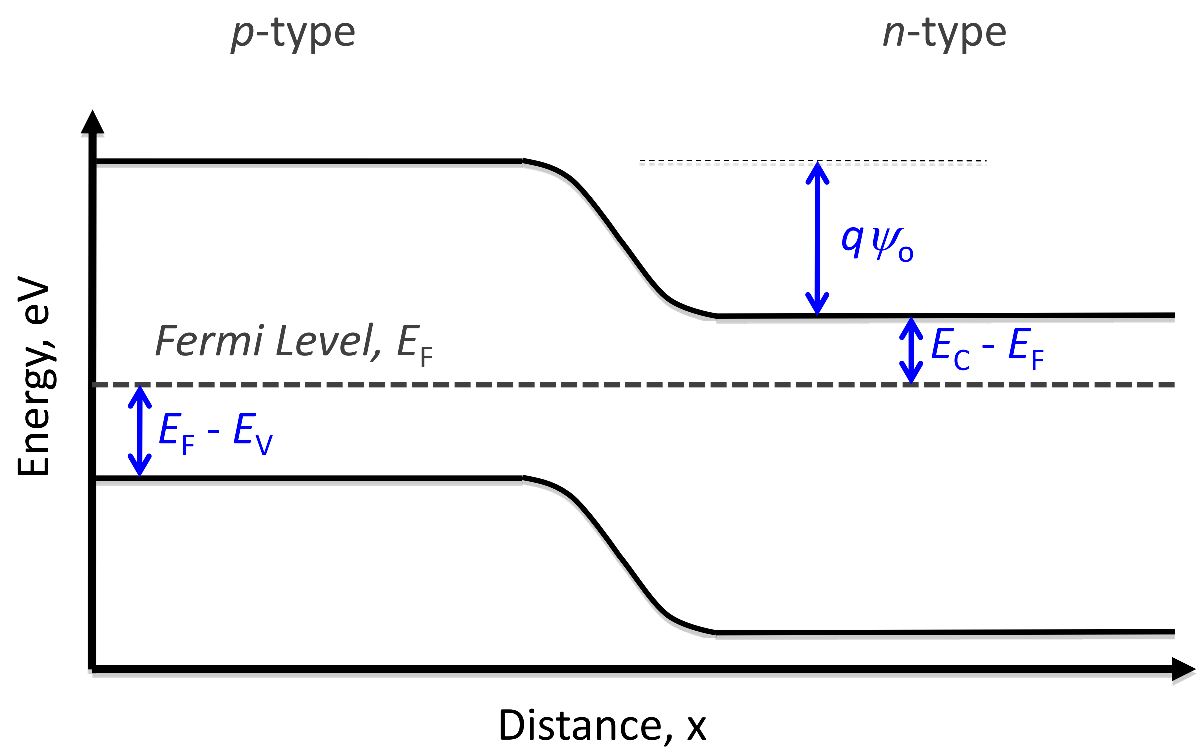
We assume: All dopants are ionized!

Voltage Across a *pn*-Junction



Voltage Across a *pn*-Junction



$$q\psi_o = E_g - (E_F - E_V) - (E_C - E_F)$$

$$= \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Built-in pn-junction potential a function of dopant concentrations.

Derivation

$$q\psi_{\mathrm{o}} = E_{\mathrm{g}} - \left(E_{\mathrm{F}} - E_{\mathrm{V}}\right) - \left(E_{\mathrm{C}} - E_{\mathrm{F}}\right)$$

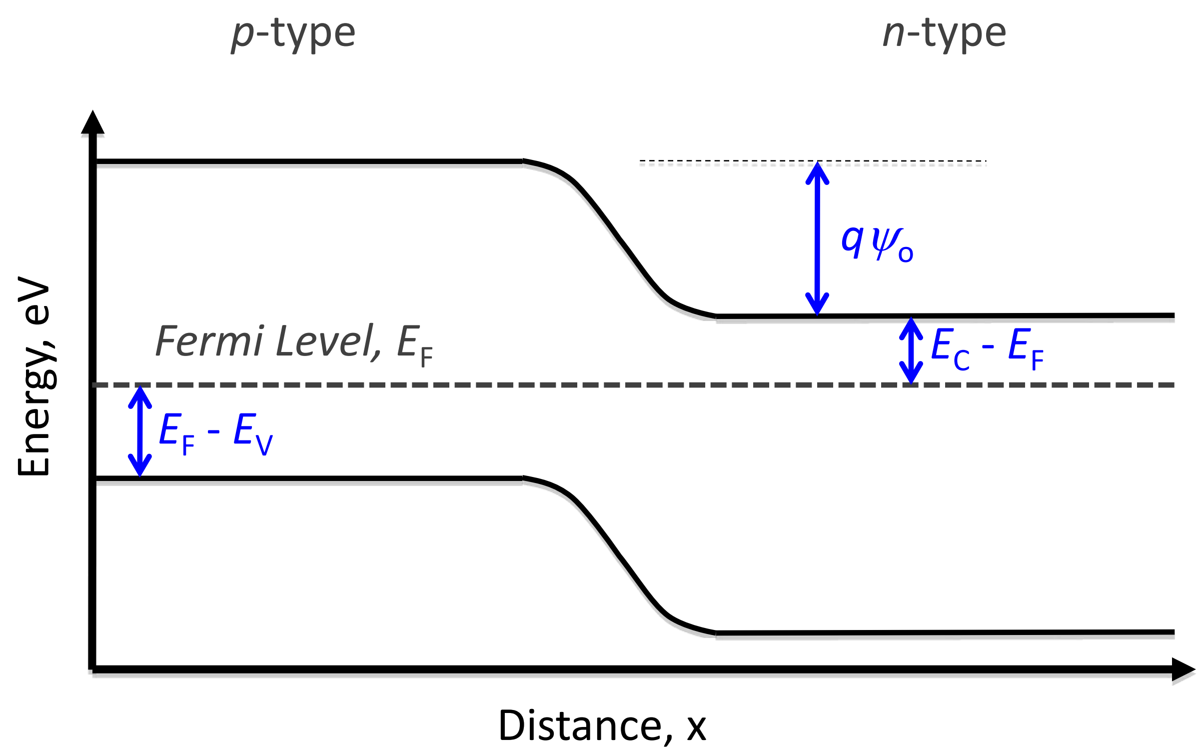
$$= E_{\mathrm{g}} - kT\ln\left(\frac{N_{\mathrm{V}}}{N_{\mathrm{A}}}\right) - kT\ln\left(\frac{N_{\mathrm{C}}}{N_{\mathrm{D}}}\right)$$

$$= E_{\mathrm{g}} - kT\ln\left(\frac{N_{\mathrm{C}}N_{\mathrm{V}}}{N_{\mathrm{A}}N_{\mathrm{D}}}\right)$$

$$\psi_{\mathrm{o}} = \frac{kT}{q}\ln\left(\frac{N_{\mathrm{A}}N_{\mathrm{D}}}{n_i^2}\right)$$

Built-in pn-junction potential a function of dopant concentrations.

Voltage Across a *pn*-Junction

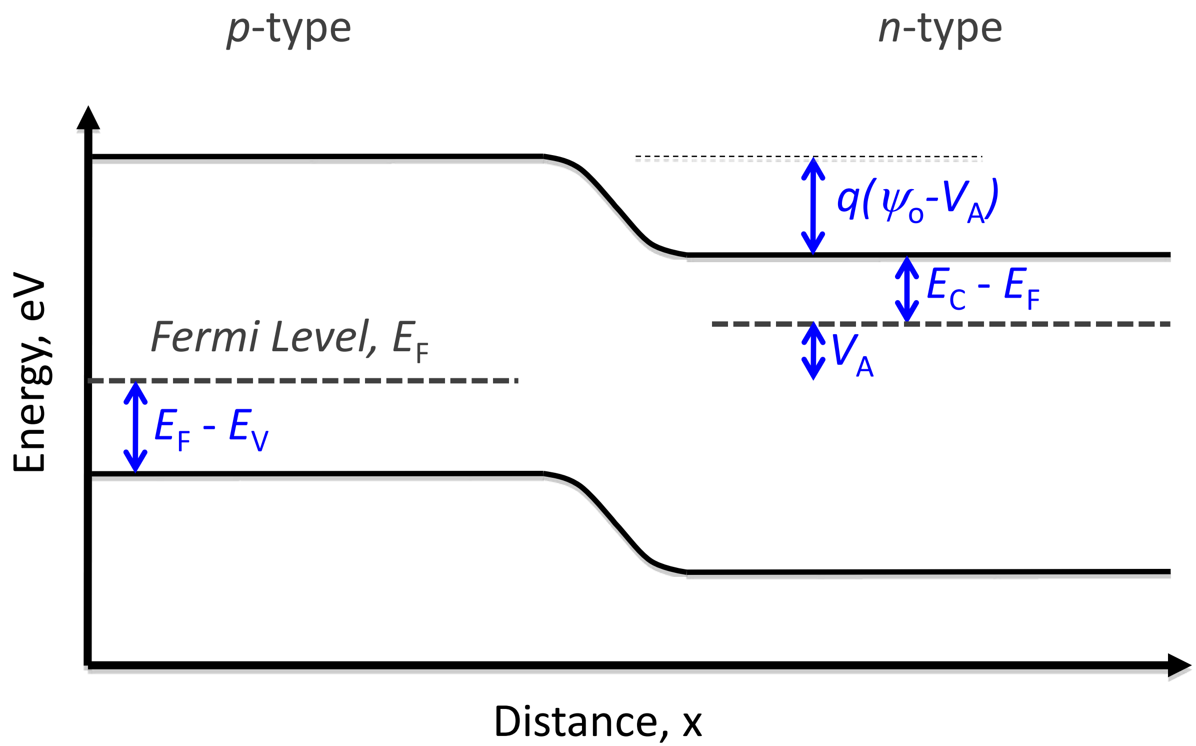


$$q\psi_o = E_g - (E_F - E_V) - (E_C - E_F)$$

$$= \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

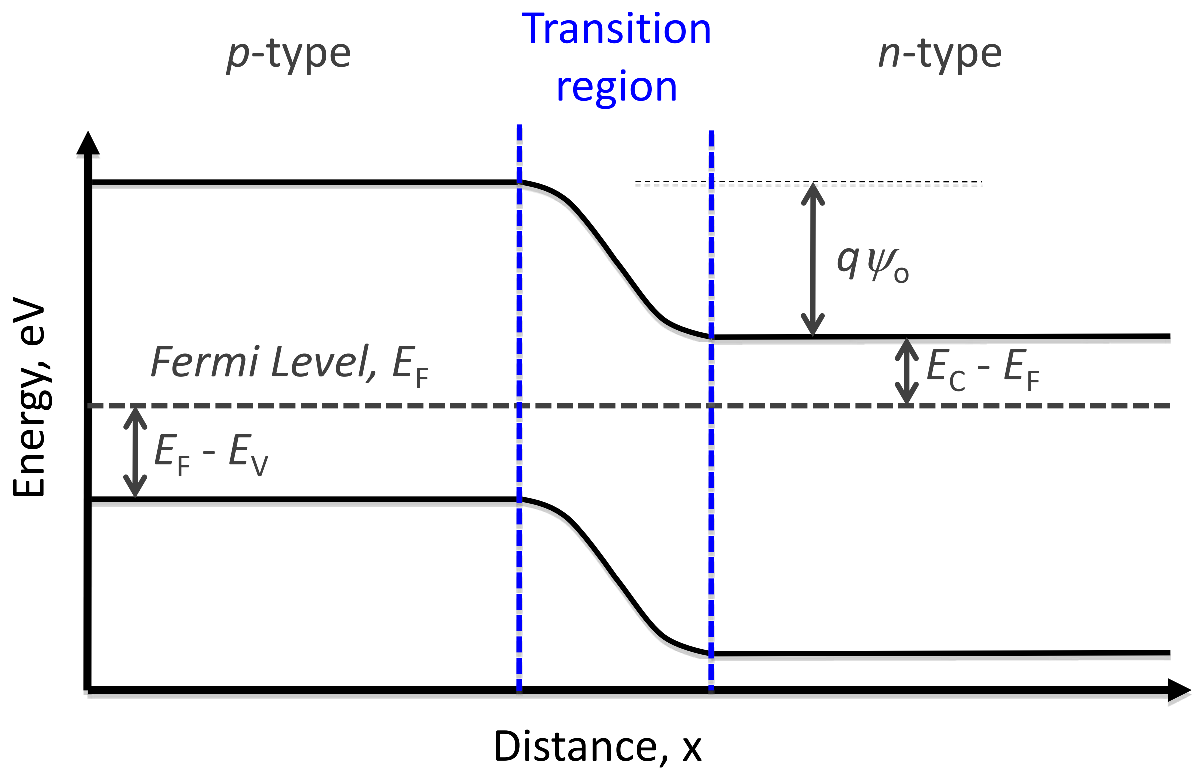
Built-in pn-junction potential a function of dopant concentrations.

Voltage Across a **Biased** *pn*-Junction



$$q\left(\psi_{\mathrm{o}}-V_{\mathrm{A}}\right)=k_{\mathrm{b}}T\ln\left(\frac{N_{\mathrm{A}}N_{\mathrm{D}}}{n_i^2}\right)-V_{\mathrm{A}}$$

Effect of Bias on Width of Space-Charge Region



$$q\psi_o = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

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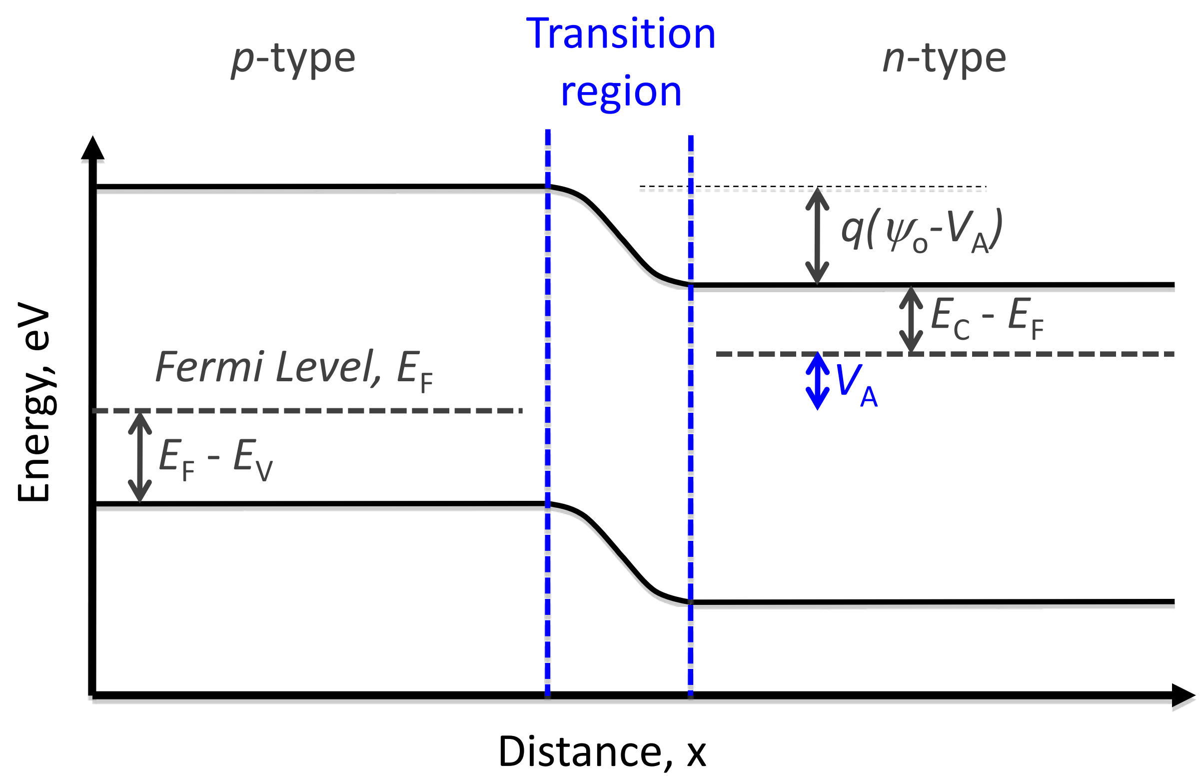
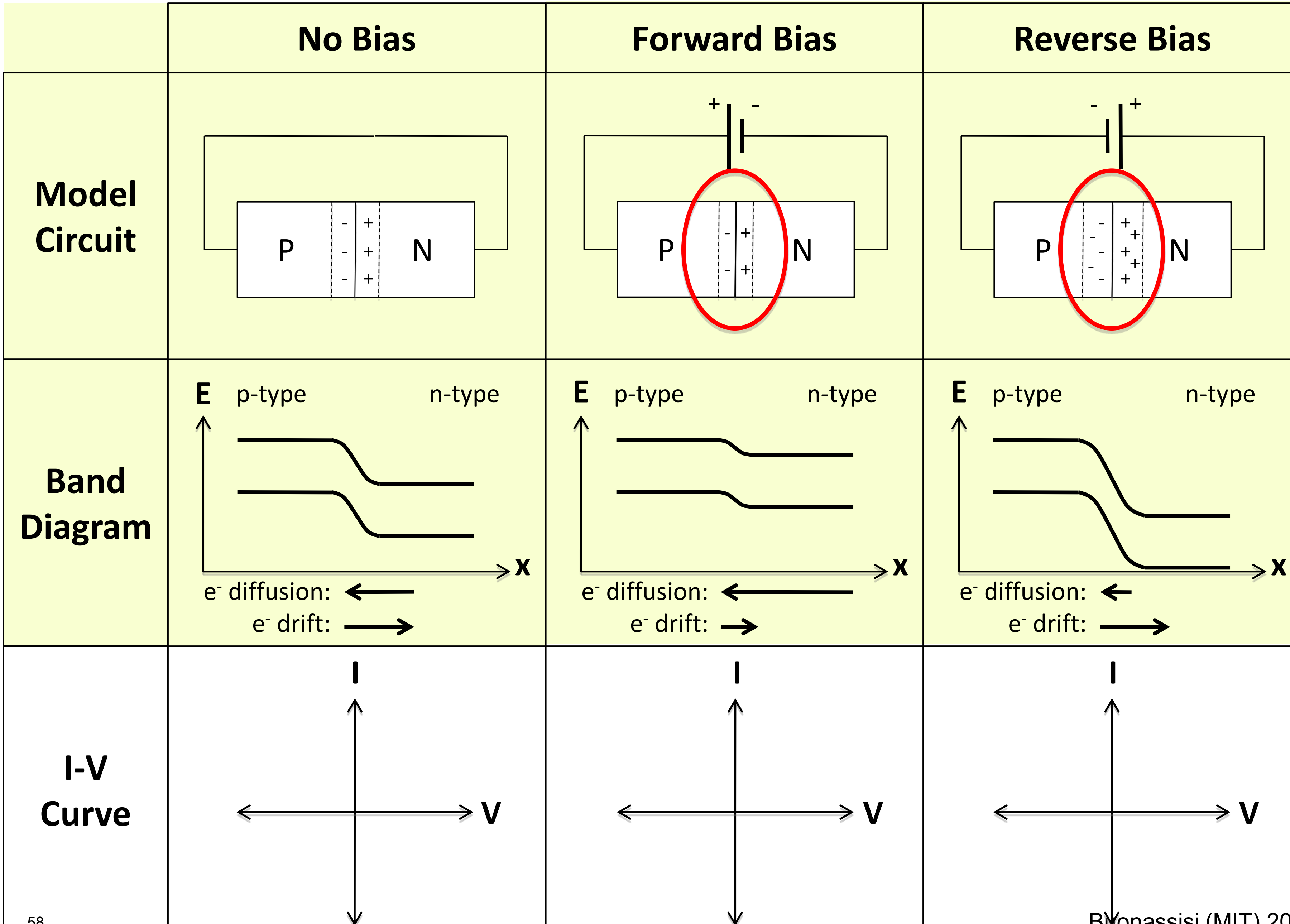
Effect of Bias on Width of Space-Charge Region



$$q(\psi_o - V_A) = k_b T \ln\left(\frac{N_A N_D}{n_i^2}\right) - V_A$$

pn-junction, under dark conditions



Learning Objectives: Diode

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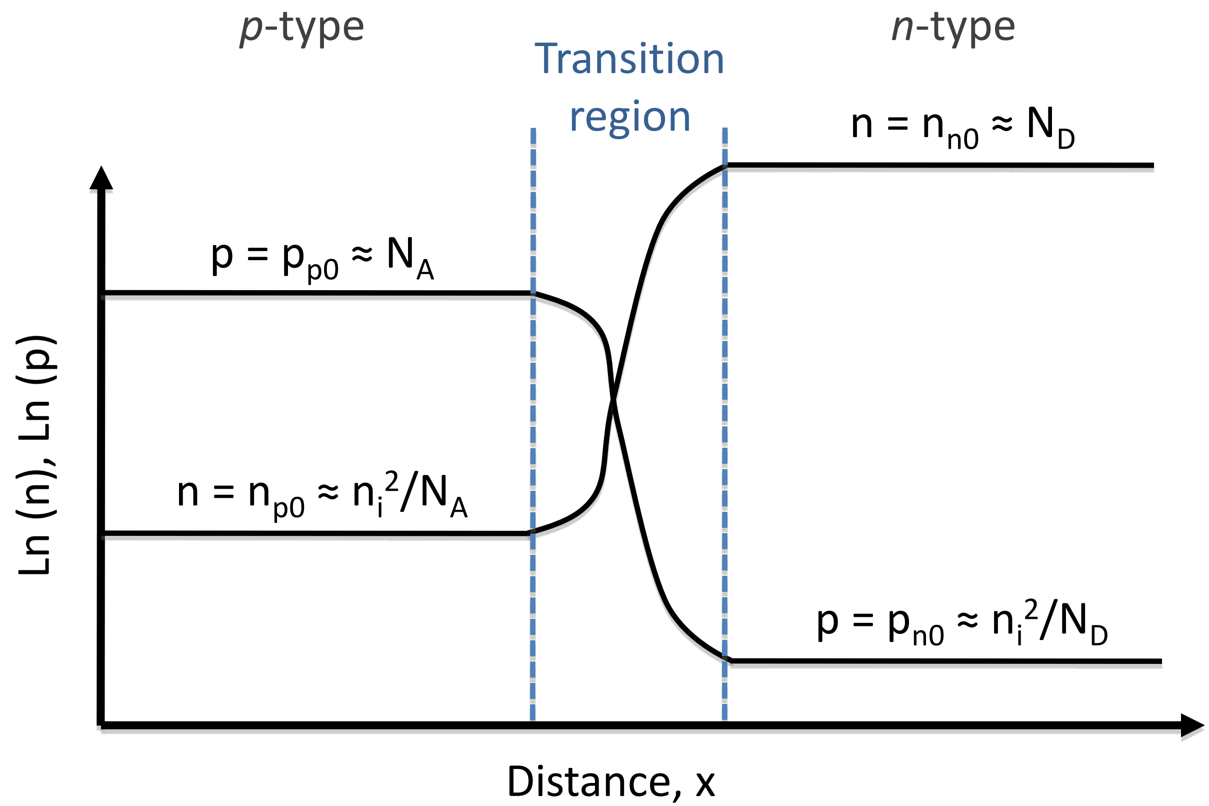
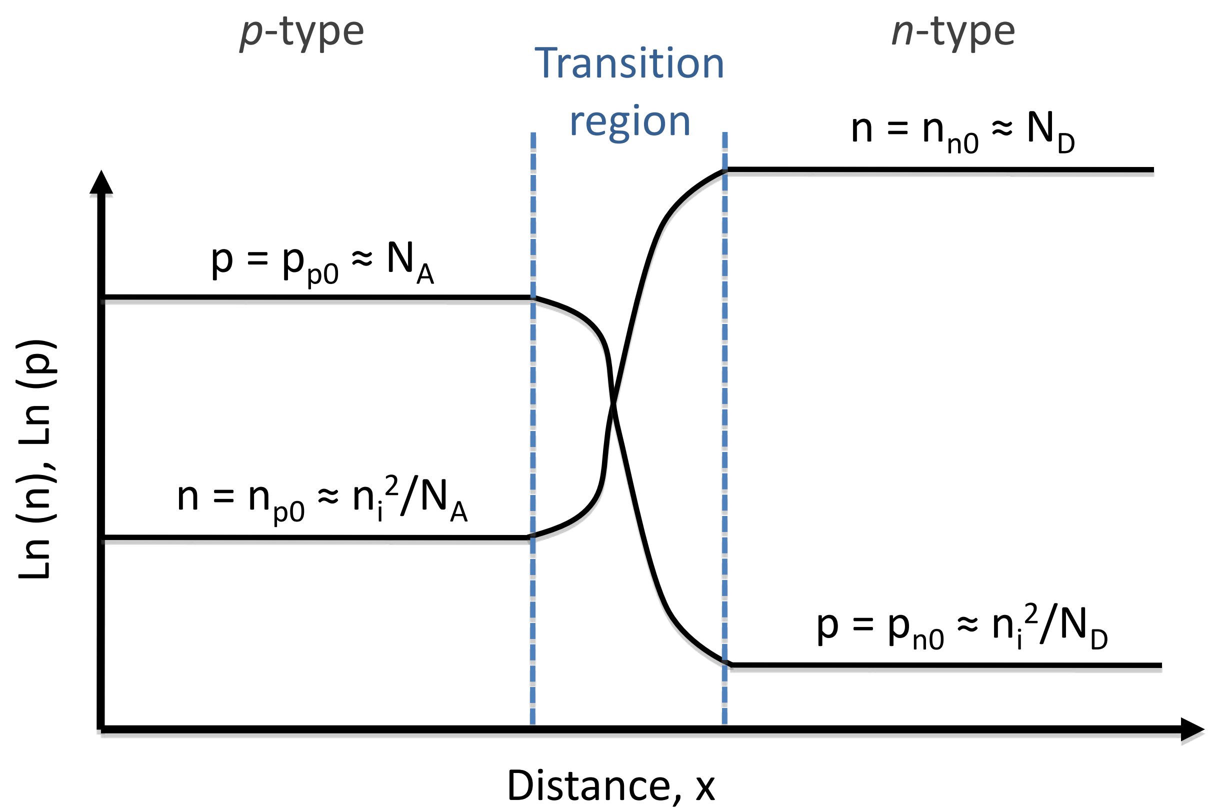
Carrier Concentrations Across a pn-Junction



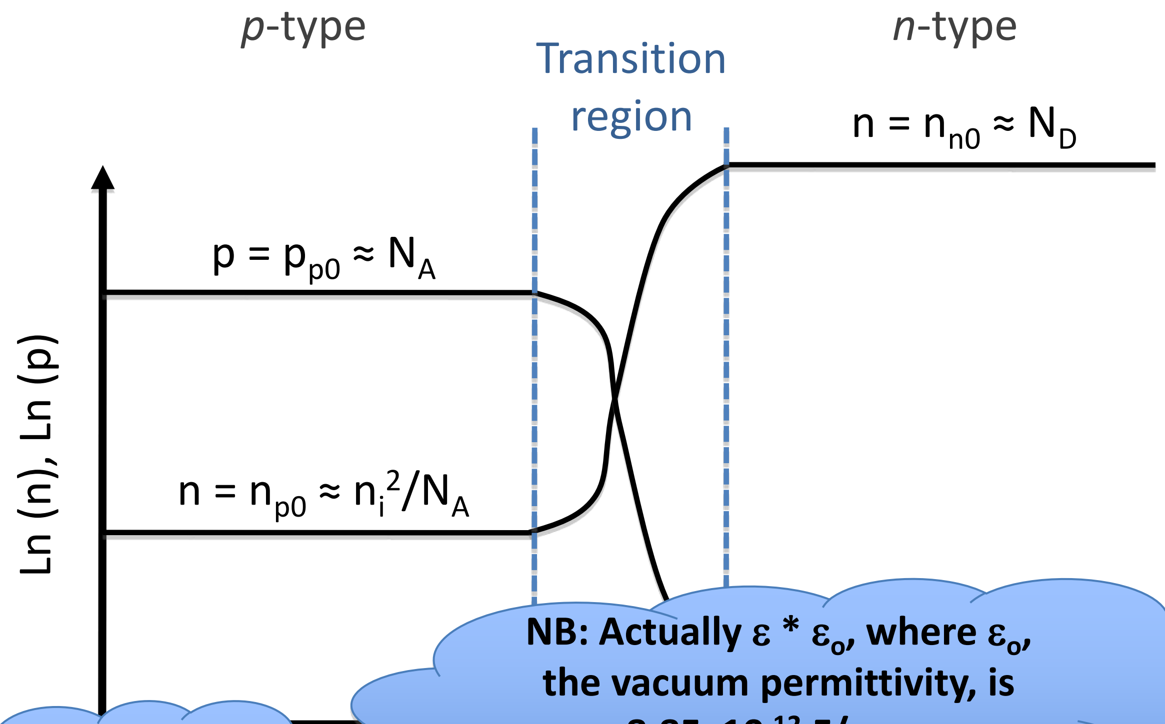
Approximation 1: Device can be split into two types of region: quasi-neutral regions (space-charge density is assumed zero) and the depletion region (where carrier concentrations are small, and ionized dopants contribute to fixed charge).

Width of space charge region



$$W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q}(\psi_o - V_a)\cdot\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

Width of space charge region

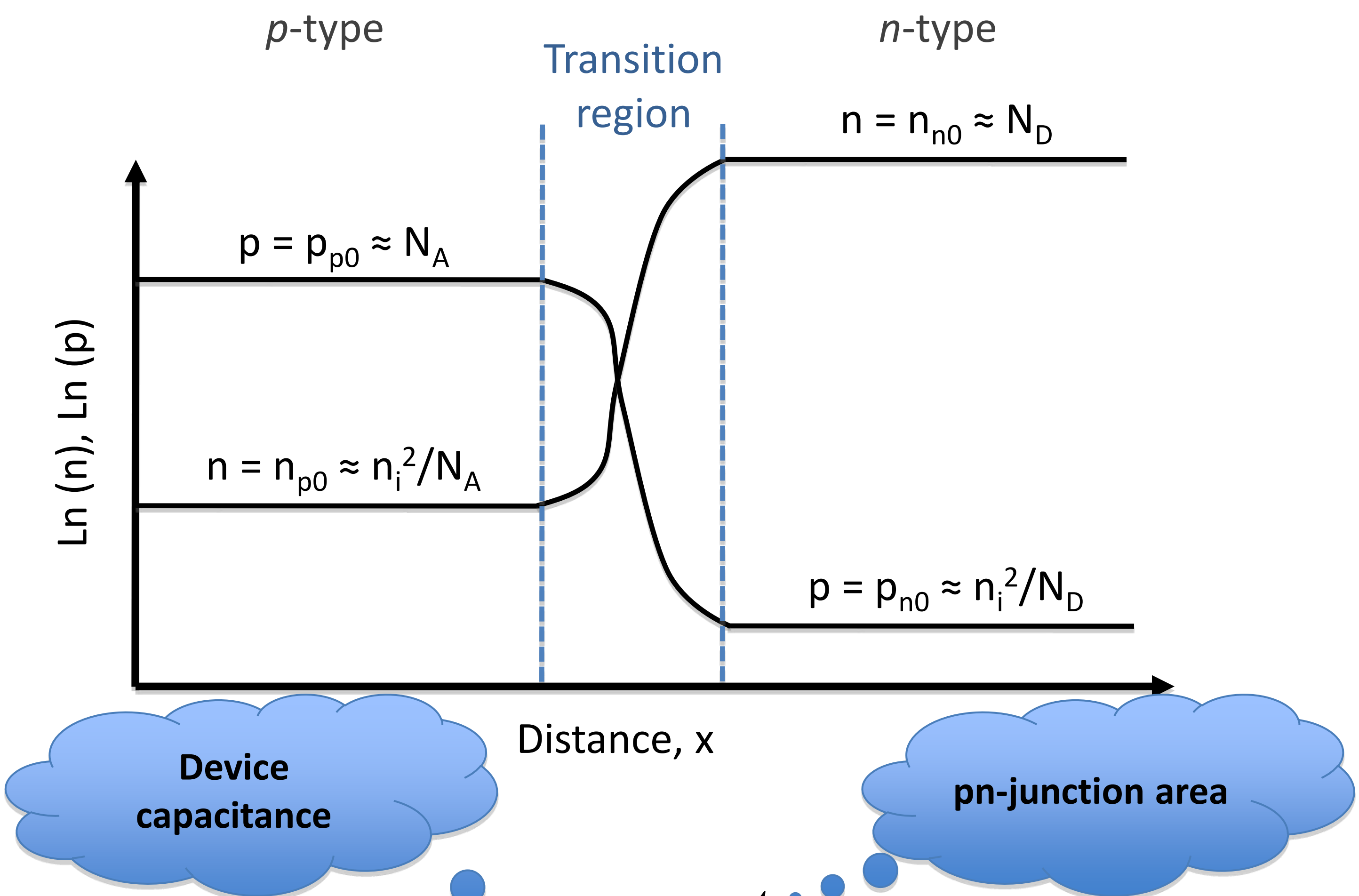


NB: Actually ε * ε$_o$, where ε$_o$, the vacuum permittivity, is 8.85x10^{-12} F/m or 5.53x10^7 e/(V*m)

Width of the space-charge region

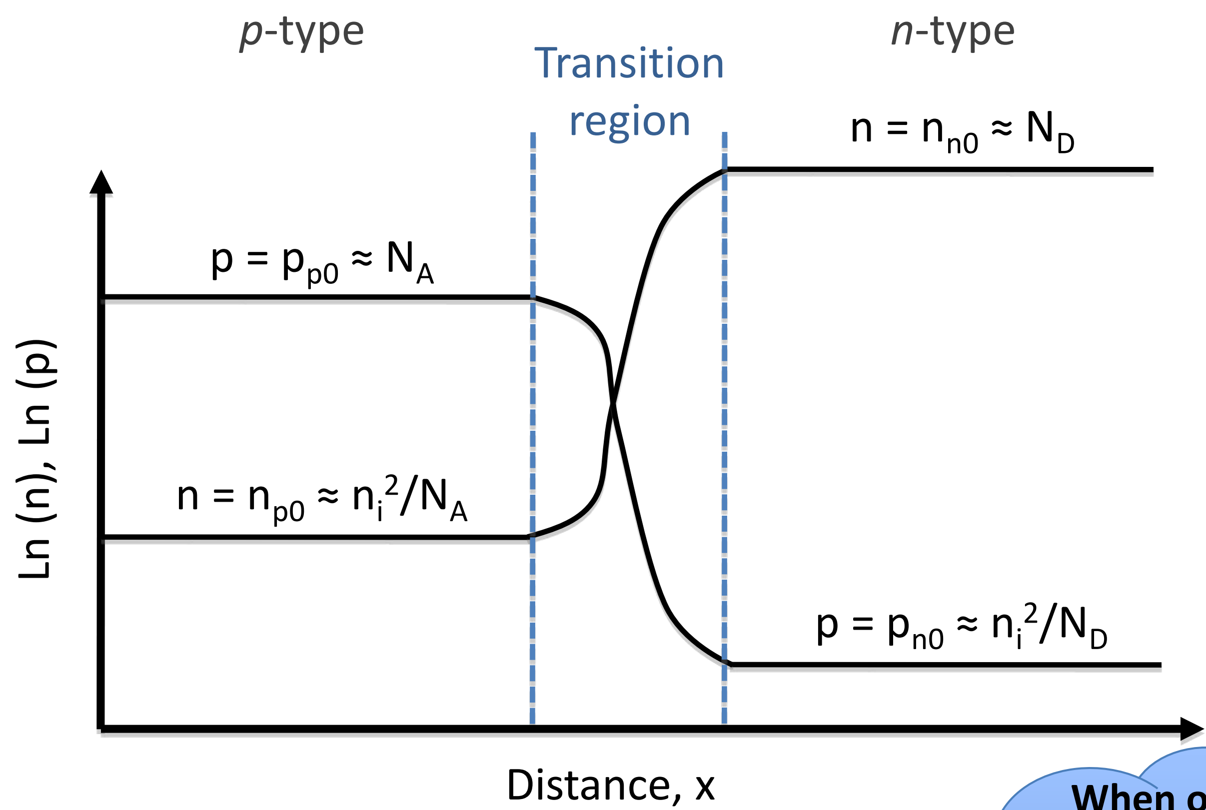
$$W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q}\left(\psi_o - V_a\right)\cdot\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$$

Capacitance



$$C = \frac{\varepsilon A}{W}$$

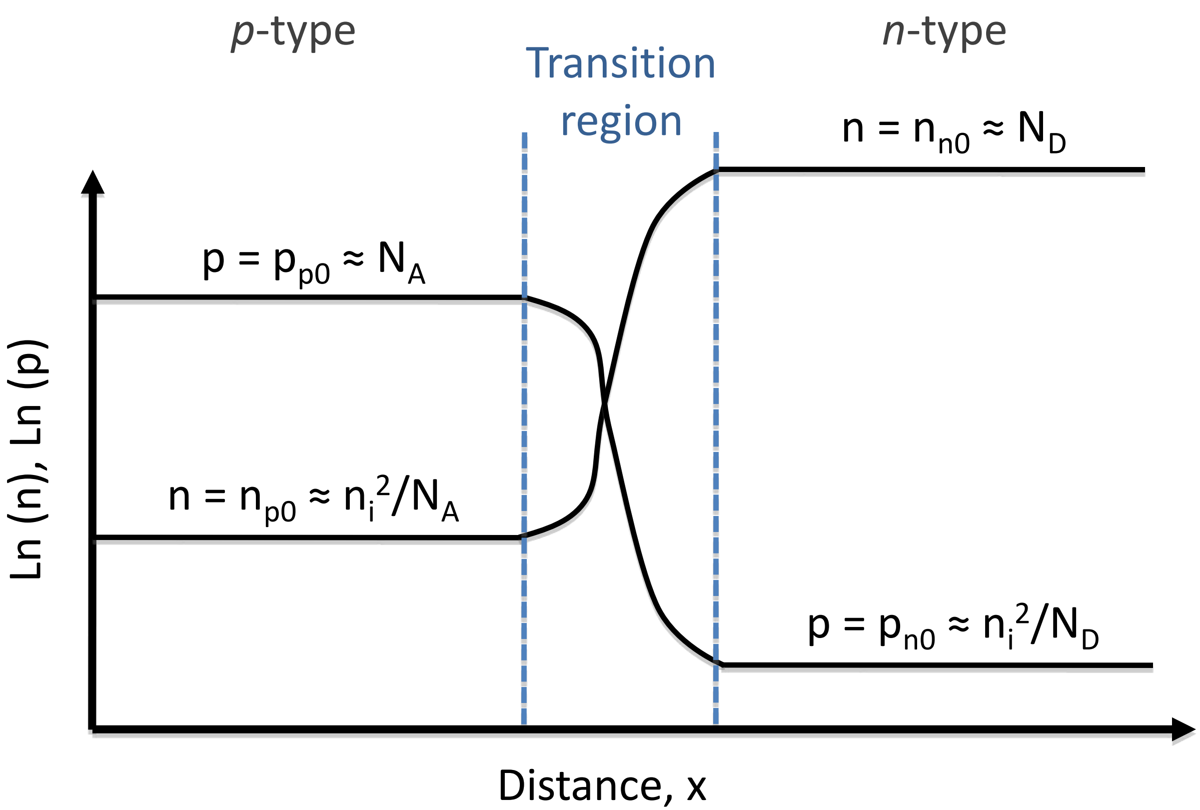
Capacitance



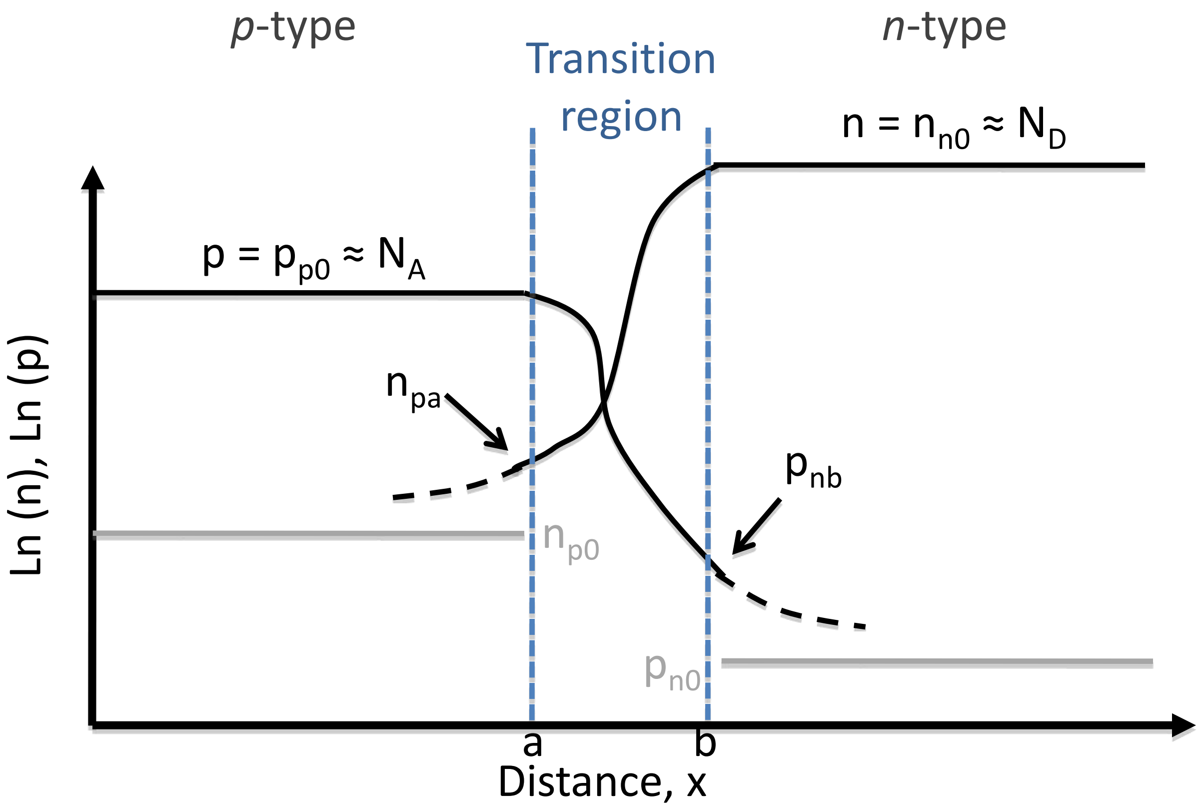
$$\frac{C}{A} = \sqrt{\frac{q\varepsilon N}{2(\psi_o - V_a)}}$$

When one side of the pn-junction is heavily doped, the capacitance reduces to this expression

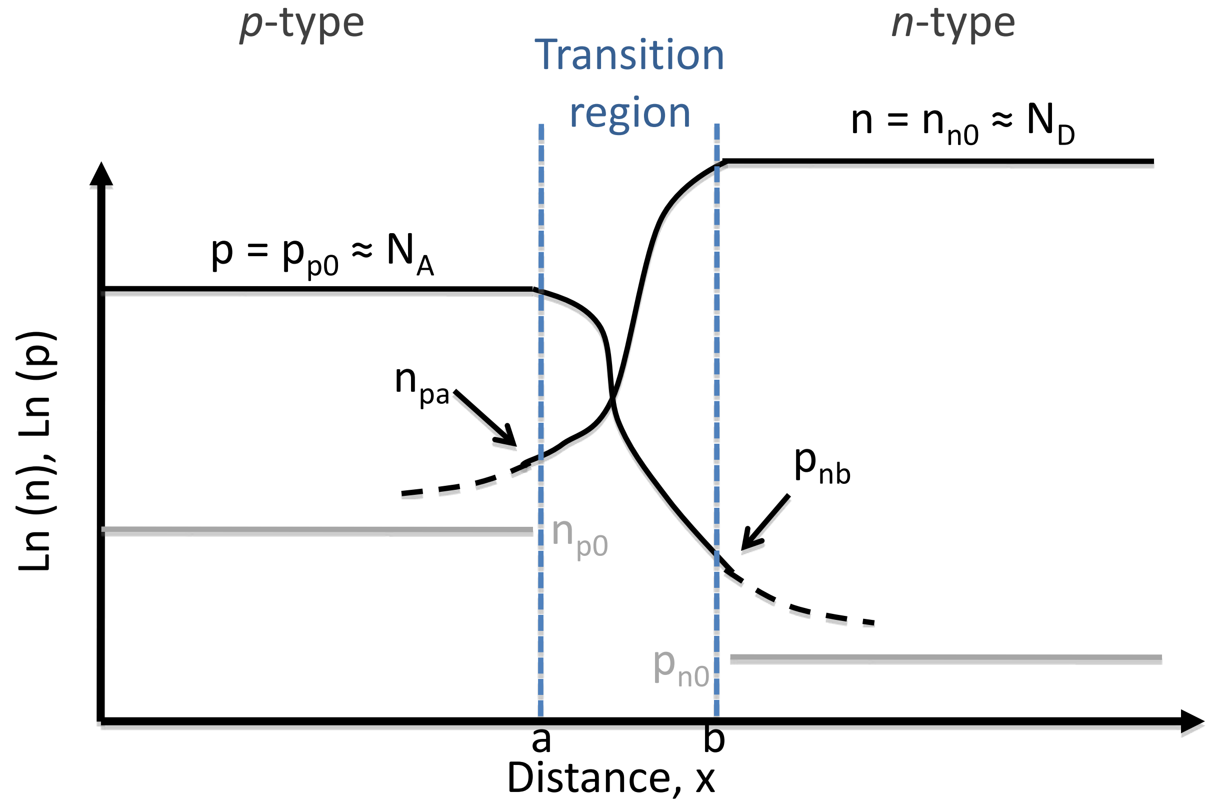
Pn-junction under zero bias



Pn-junction under forward bias



Pn-junction under forward bias



At zero bias:

$$p_{nb} = p_{n0} = p_{p0} \cdot \exp\left(-\frac{q\psi_o}{kT}\right) \approx \frac{n_i^2}{N_D}$$

$$n_{pa} = n_{p0} = n_{n0} \cdot \exp\left(-\frac{q\psi_o}{kT}\right) \approx \frac{n_i^2}{N_A}$$

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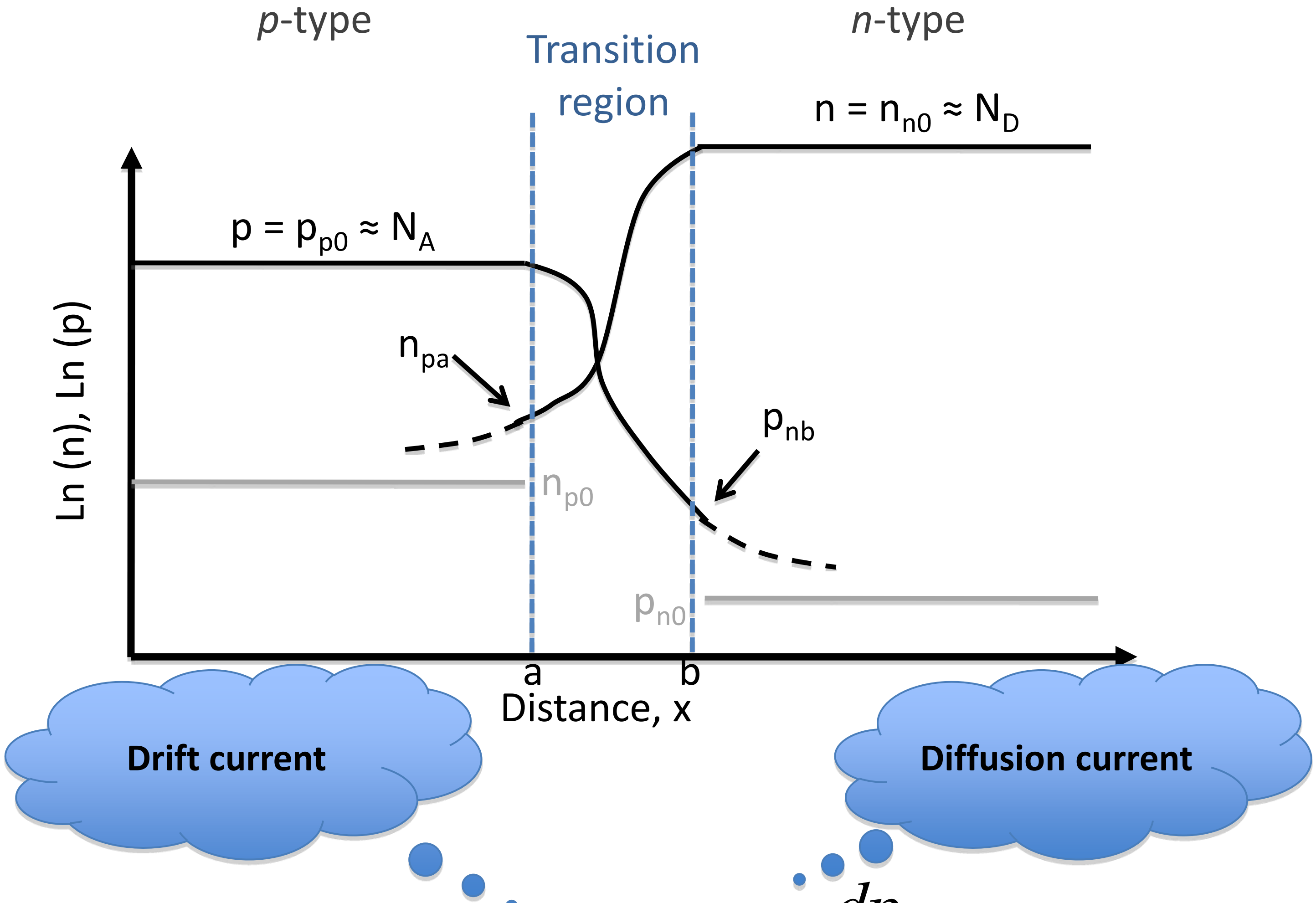
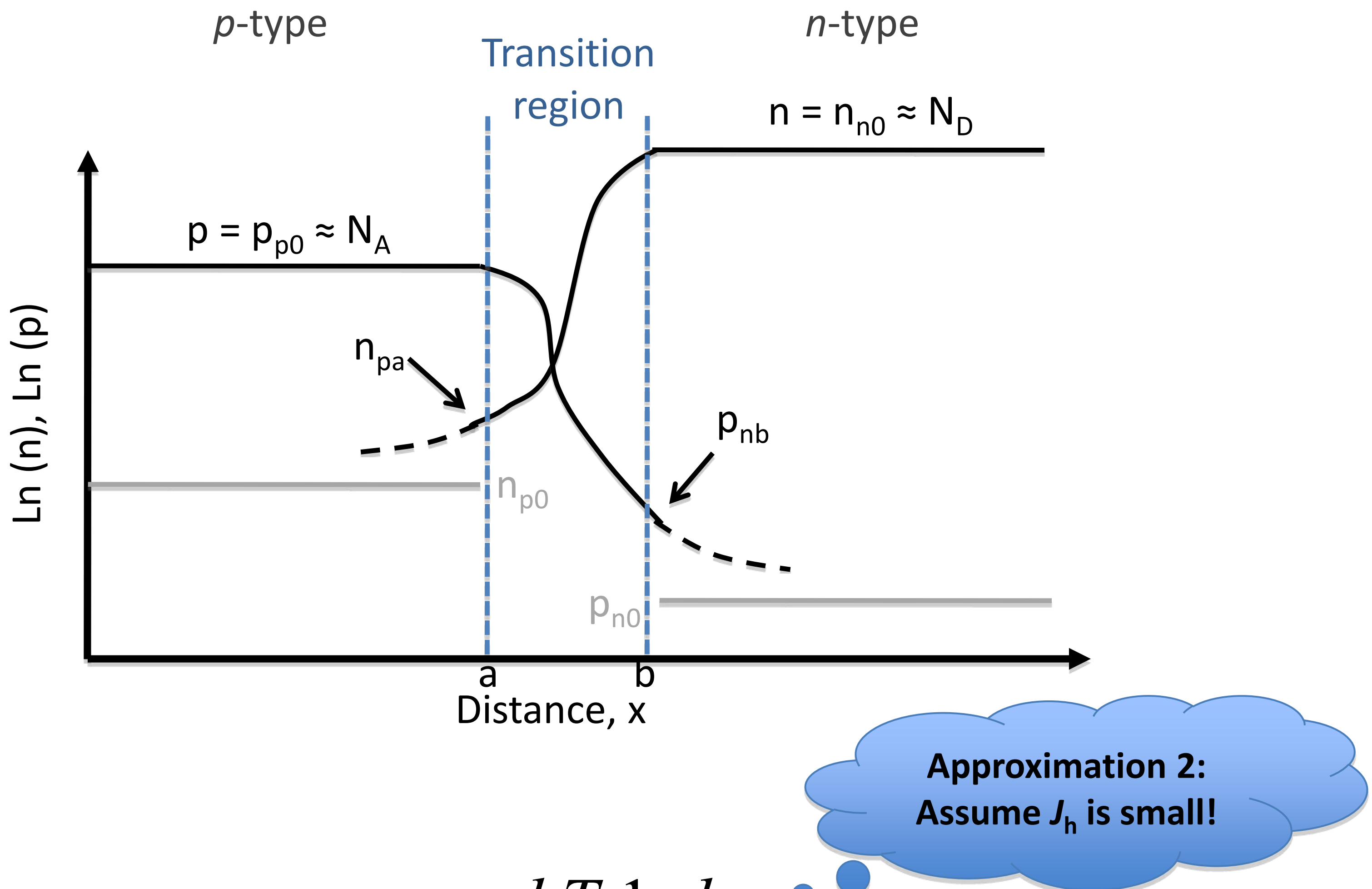
Current flow through the depletion region



For holes:

$$J_h = q\mu_h p\xi - qD_h \frac{dp}{dx}$$

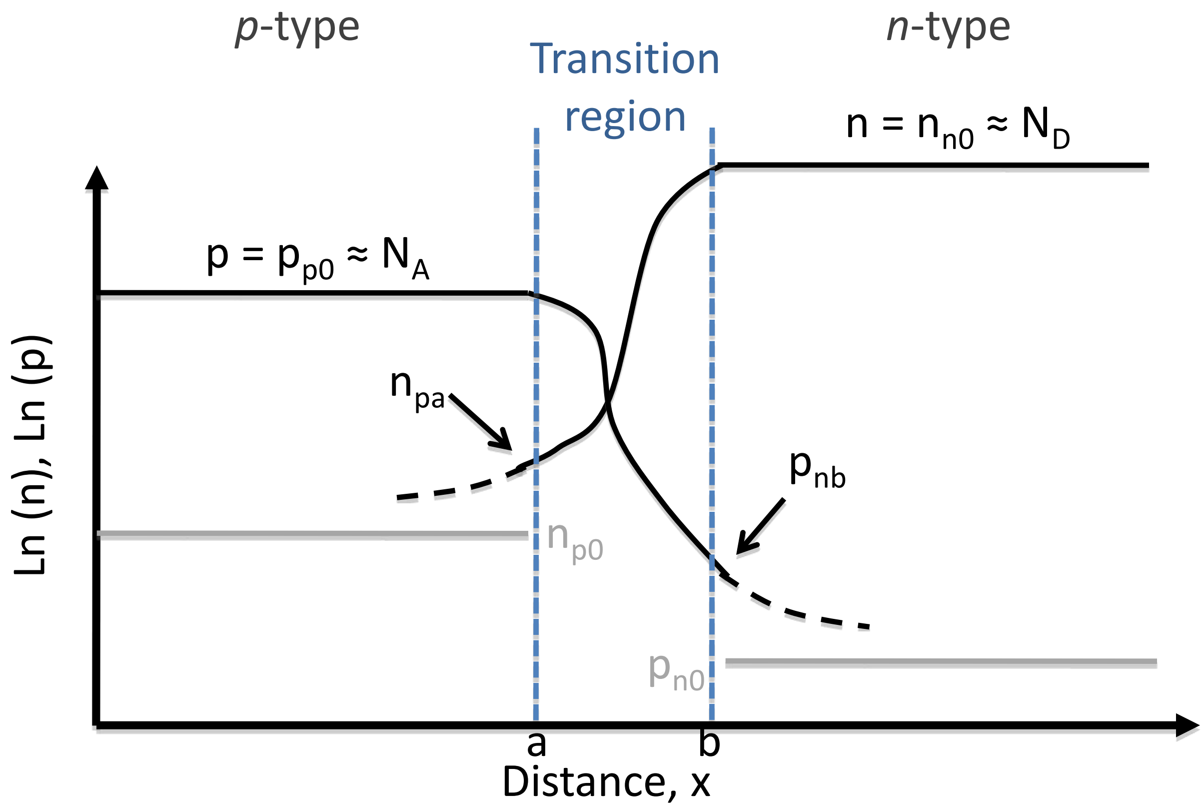
Current flow through the depletion region



For holes:

$$\xi \approx \frac{kT}{q}\frac{1}{p}\frac{dp}{dx}$$

Current flow through the depletion region

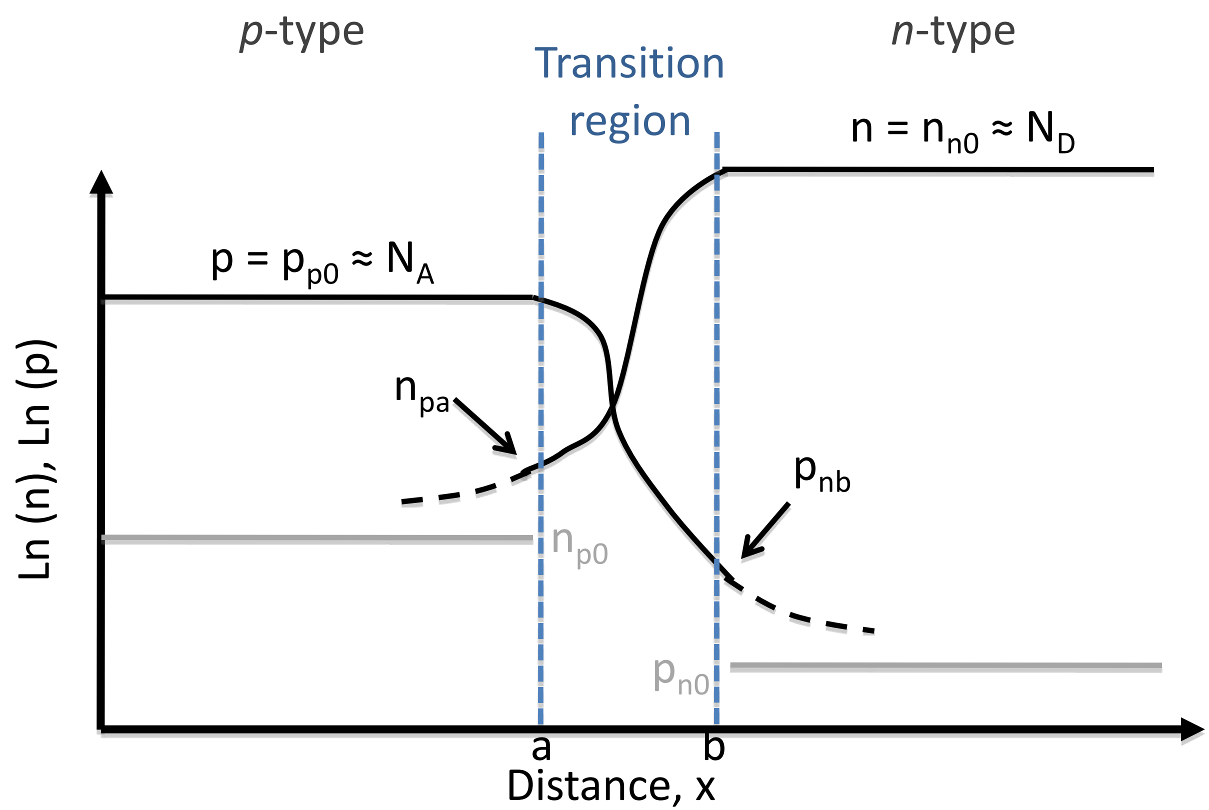


Integrating…

$$\psi_o - V_a = -\frac{kT}{q}\ln(p)\Big|_a^b$$

$$= \frac{kT}{q}\ln\left(\frac{p_{pa}}{p_{nb}}\right)$$

Current flow through the depletion region



Approximation 3: Only cases where minority carriers have a much lower concentration than majority carriers will be considered,
i.e., $p_{pa} >> n_{pa}$, $n_{na} >> p_{na}$

$$p_{pa} = N_A + n_{pa}$$

Current densities

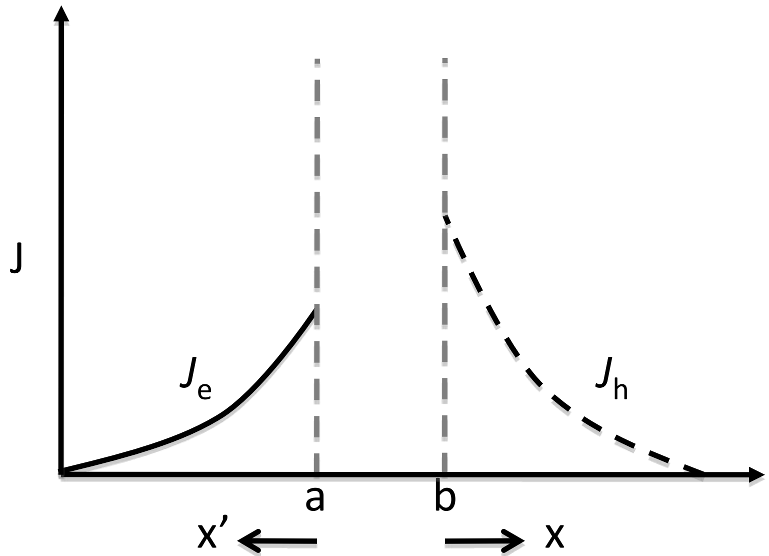
Calculate (diffusive) currents in quasi-neutral region:

$$J_h = -qD_\mathrm{h}\frac{dp}{dx}$$

... from previous slide ...

$$J_h(x) = \frac{qD_h p_{n0}}{L_h}\left(e^{qV/kT}-1\right)e^{-x/L_h}$$

$$J_e(x') = \frac{qD_e n_{n0}}{L_e}\left(e^{qV/kT}-1\right)e^{-x/L_e}$$



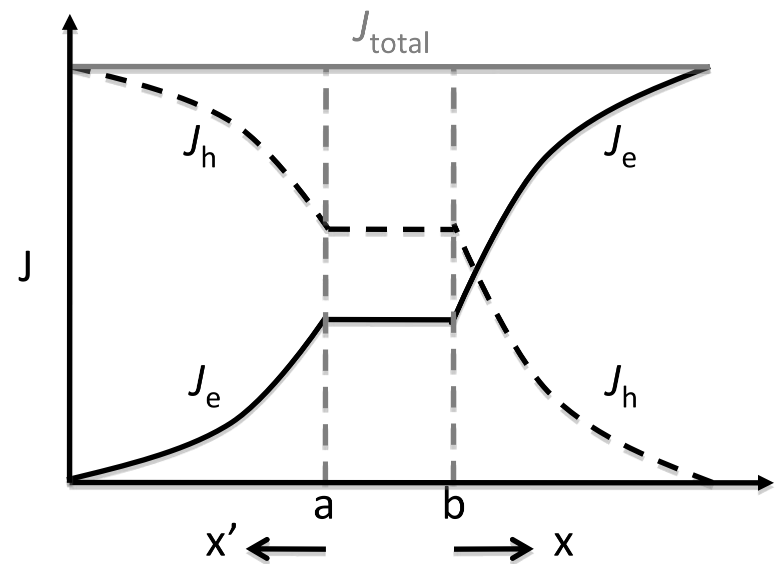
Current densities

$$\frac{1}{q}\frac{dJ_e}{dx} = U - G = -\frac{1}{q}\frac{dJ_h}{dx}$$

Magnitude of the change in current across the depletion region:

$$\delta J_e = |\delta J_h| = q\int_{-W}^{0}(U-G)dx \approx 0$$

Key assumption: W is small compared to L_e and L_h. Therefore, integral is negligible. It follows that the current J_e and J_h are essentially constant across the depletion region, as shown below.



Ideal Diode Equation

Since J_e and J_h are known at all points in the depletion region, we can calculate the total current:

$$J_{\text{total}} = J_e\big|_{x'=0} + J_h\big|_{x=0} = \left(\frac{qD_e n_{p0}}{L_e} + \frac{qD_h p_{n0}}{L_h}\right)\left(e^{qV/kT} - 1\right)$$

This leads to the ideal diode law:

$$I = I_o\left(e^{qV/kT} - 1\right), \text{ where}$$

$$I_o = A\left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D}\right)$$

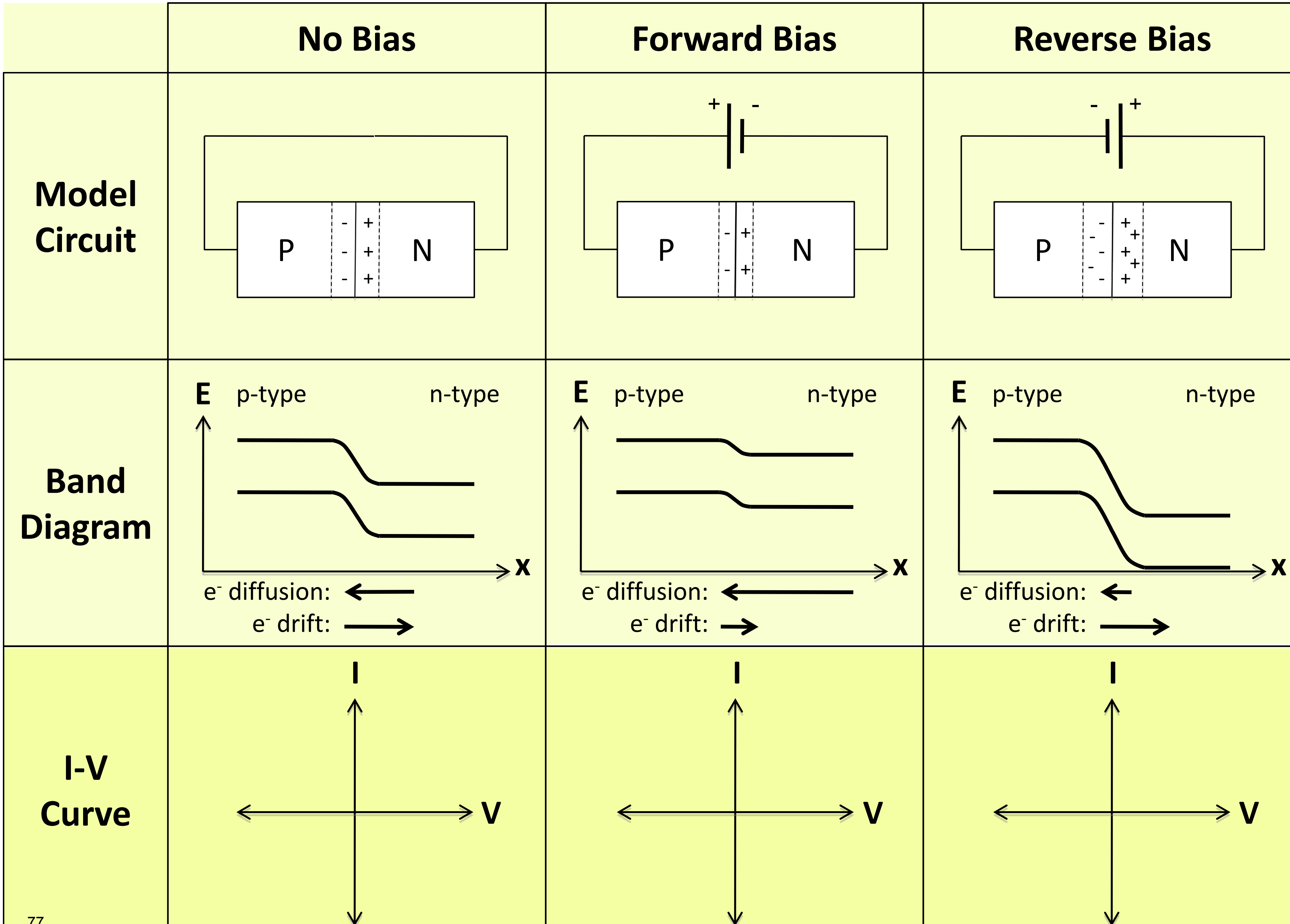
Key Point

- The IV response of a pn-junction is determined by changes in ***minority carrier current*** at the edge of the space-charge region.

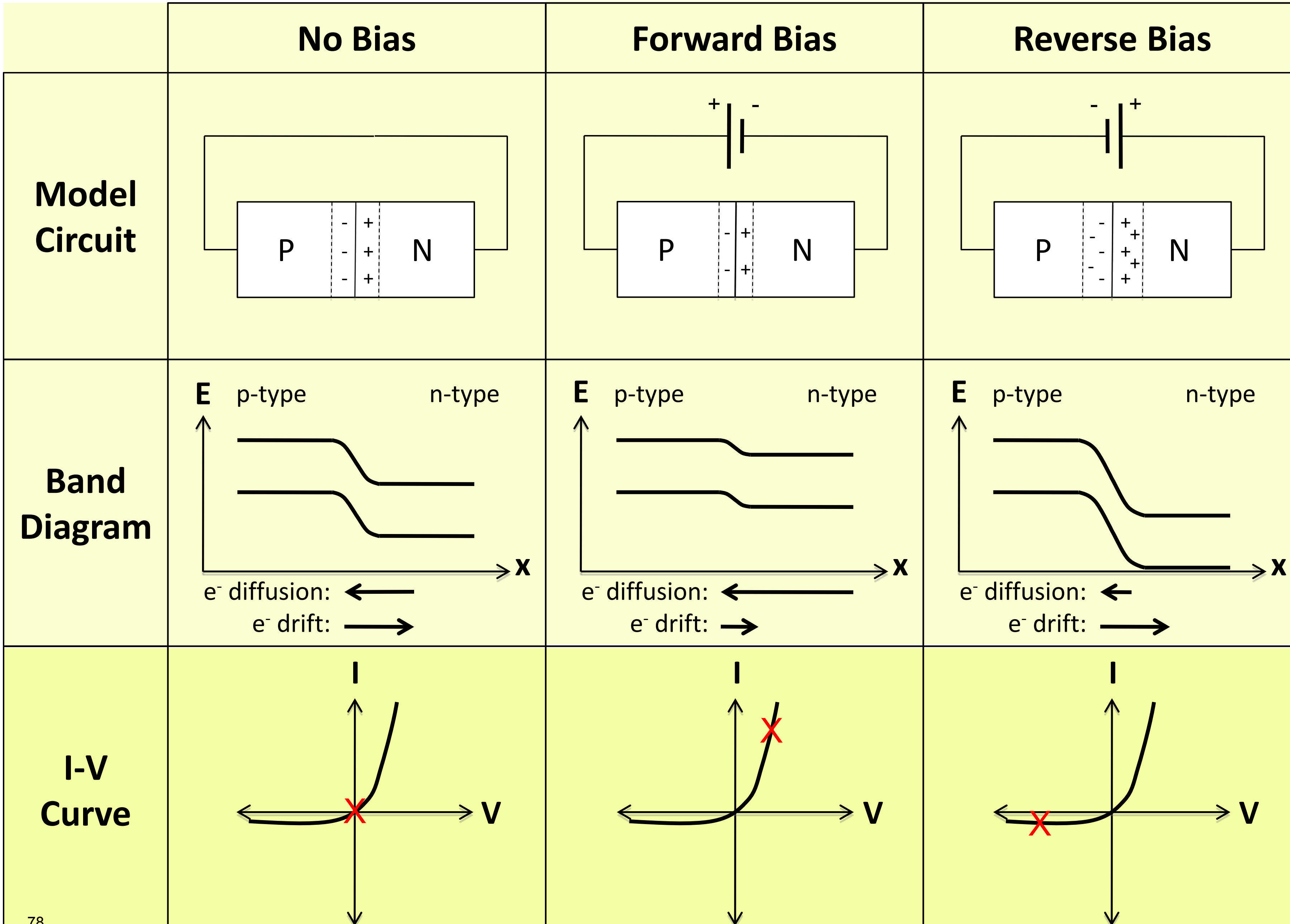
Readings are strongly encouraged

- Green, Chapter 4
- http://www.pveducation.org/pvcdrom/, Chapters 3 & 4.

pn-junction, under dark conditions



pn-junction, under dark conditions



Hands-On: Measure Solar Cell IV Curves

2.627 / 2.626 Fundamentals of Photovoltaics
Fall 2013