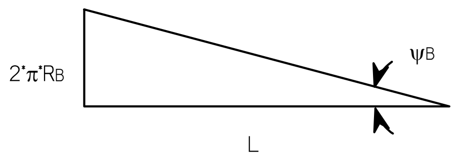
# Helical Gears

ref: Gear Drive Systems; Design and Application, Peter Lynwander

advantages ...
- greater load capacity
- smoother operation
- less sensitivity to tooth errors

teeth are at angle to rotation, contact is a series of oblique lines with several lines in contact simultaneously. total length of contact varies as teeth mesh.

offset adjacent "strings" in involute generator concept on base cylinder by angle $\psi$




$$L = \frac{2\cdot\pi\cdot R_B}{\tan(\psi_B)}$$

$$\tan(\psi_B) = \frac{2\cdot\pi\cdot R_B}{L}$$

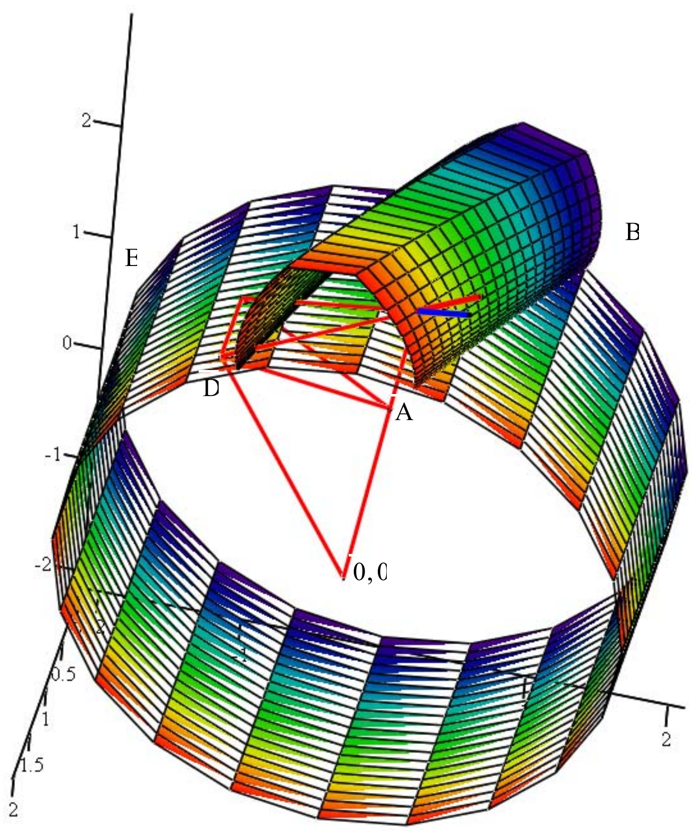
L = lead

$R_B$ = base_radius

$\psi_B$ = base_helix_angle

develop normal at any radius on tooth by considering transverse and normal planes intersecting tooth at that point

▶ geometry development




(Xn, Yn, Zn), (Xn1, Yn1, Zn1), (X_line, Y_line, Z_line), (Xg, Yg, Zg), (X, Y, Z)

point B ... point on gear for normal with helix (shown off gear)
point A ... point on radial line 0,0 to B perpendicular joining tangent
point D ... tangent point
point E ... point on plane perpendicular to tooth at B, connecting with (transverse) tangent point along $R_B$

$$\tan(\phi_N) = \frac{AB}{AE} \qquad \tan(\phi_T) = \frac{AB}{AD} \qquad \cos(\psi) = \frac{AD}{AE}$$

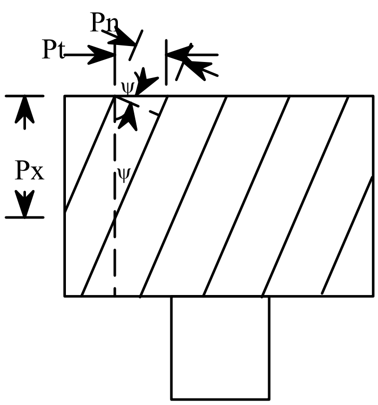
$$\frac{AD}{AE}\cdot\frac{AB}{AD} = \frac{AB}{AE} = \cos(\psi)\cdot\tan(\phi_T) = \tan(\phi_n) \quad \Rightarrow \quad \tan(\phi_T) = \frac{\tan(\phi_n)}{\cos(\psi)} \quad \text{or ...} \quad \phi_T = \operatorname{atan}\left(\frac{\tan(\phi_n)}{\cos(\psi)}\right)$$

other parameters; as in general gear ...

$$P_t = \text{circular\_pitch\_transverse} = \frac{\pi\cdot D}{N_g} = \frac{\pi\cdot d}{N_p}$$

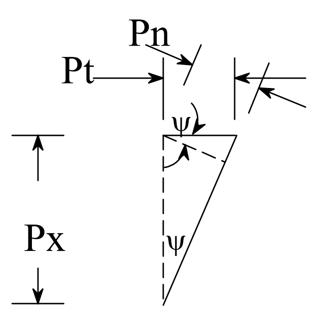
$D$ = diameter_gear   $N_g$ = number_of_teeth_gear   $d$ = diameter_pinion   $N_p$ = number_of_teeth_pinion

considering an expanded view at any radius ...




gear geometry at pitch radius (diameter)

magnified ....




$$\cos(\psi) = \frac{P_n}{P_t}$$

$$P_n = \text{circular\_pitch\_normal} = P_t\cdot\cos(\psi)$$

if radius were $R_B$ ...

$$\tan(\psi_B) = \frac{P_b}{P_x}$$

$P_b$ = base_pitch_transverse

and ... $$\psi_B = \operatorname{atan}\left(\frac{P_b}{P_x}\right) = \operatorname{atan}\left(\frac{P_b}{P_x}\right)$$

3. Pitch $$P_t = \text{circular\_pitch\_transverse} = \frac{\text{pitch\_circumference}}{\text{number\_of\_teeth}} = \frac{\pi\cdot D}{N_g} = \frac{\pi\cdot d}{N_p}$$

$g$ = gear

$p$ = pinion

$$P_n = \text{circular\_pitch\_normal} = P_t\cdot\cos(\psi)$$ from geometry above

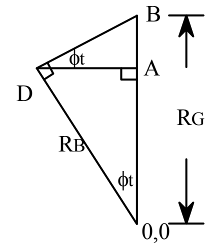
N.B. $$P = \text{diametral\_pitch\_transverse} = \frac{\text{number\_of\_teeth}}{\text{pitch\_diameter}} = \frac{N_g}{D} = \frac{N_p}{d}$$ so ... $P_t = \frac{\pi}{P}$ and ... $P_n = \frac{\pi}{P_N}$

above geometry => $$\tan(\psi) = \frac{P_t}{P_x} \quad \Rightarrow \quad P_x = \frac{P_t}{\tan(\psi)}$$

$P_x$ = axial_pitch = $P_t \cdot \cot(\psi)$

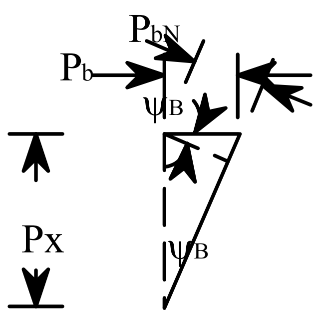
$P_b$ = base_pitch_transverse = $\frac{\text{base\_circumference}}{\text{number\_of\_teeth}} = \frac{\pi \cdot 2 \cdot R_B}{N} = \frac{\pi \cdot 2 \cdot R_G}{N} \cdot \frac{R_B}{R_G}$  $R_G$ = pitch_radius = $\frac{D}{2}$

from geometry way above ... 0.0, A, B, D




$\cos(\phi_t) = \frac{R_B}{R_G}$  =>  $P_b = \frac{\pi \cdot D}{N_g} \cdot \cos(\phi_t) = P_t \cdot \cos(\phi_t)$

$P_{bN}$ = base_pitch_normal




consider geometry at left which is above brought down to base radius
Px is common, not dependent on radius ... PbN forms altitude of triangle with sides Px and Pb and base sqrt(...) calculate area

$$\text{area} = \frac{1}{2} \cdot \text{base} \cdot \text{altitude} = \frac{1}{2} \cdot \sqrt{P_b^2 + P_x^2} \cdot P_{bN}$$

and ...  $\text{area} = \frac{1}{2} \cdot P_x \cdot P_b$  as ... is right triangle ...

=>  $\sqrt{P_b^2 + P_x^2} \cdot P_{bN} = P_x \cdot P_b$  and ...  $P_{bN} = \frac{P_x \cdot P_b}{\sqrt{P_b^2 + P_x^2}}$

also ...  $P_{bN} = \frac{P_x}{\sqrt{P_b^2 + P_x^2}} \cdot P_b = \cos(\psi_B) \cdot P_t \cdot \cos(\phi_t)$  from figure and above ..

also ...  $P_{bN} = P_t \cdot \cos(\psi) \cdot \cos(\phi_n)$  not shown here ...

P = diametral_pitch_transverse = $\frac{N_g}{D}$  shown above ...  $P_t = \frac{\pi}{P}$ and ...  $P_n = \frac{\pi}{P_N}$

$P_N$ = diametral_pitch_normal = $\frac{\pi}{P_n} = \frac{\pi}{P_t \cdot \cos(\psi)} = \frac{P}{\cos(\psi)}$

## ***additional note on tooth loading ...***

$$\frac{hp}{\left(\frac{2 \cdot \pi}{min}\right) \cdot \frac{1 \cdot in}{2} \cdot 1 \cdot in} = 126051 \frac{lbf}{in}$$

$W_t$ = tangential_tooth_load = $126050 \cdot \frac{HP}{RPM_p \cdot d} = 126050 \cdot \frac{HP}{RPM_g \cdot D}$

$W_T$ = total_tooth_load_transverse_plane = $\frac{W_t}{\cos(\phi_t)}$

$W_n$ = tangential_tooth_load_normal_plane = $\frac{W_t}{\cos(\psi)}$

$W_N$ = total_tooth_load_normal_plane = $\frac{W_t}{\cos(\phi_n) \cdot \cos(\psi)}$