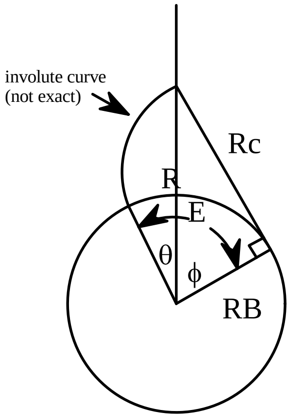
Gear geometry



Consider the curve generated by unwrapping a string from around a disk of radius R_B. The end of the string will trace an involute curve. To mathematically define an involute consider the following:

R_c = length_of_string_unwrapped

tangent with disk at one end

$$\tan(\phi) = \frac{R_C}{R_B}$$

R_B = radius_of_generating_cylinder

ϕ = pressure_angle — direction of loading perpendicular along involute curve

θ = position_paramter_associate_with_involute

$$E = \theta + \phi$$

point at loose end of curve is at polar coordinates R, θ

E = interim_variable_sum_of_angles

length of arc = radius * angle

$$R_C = E \cdot R_B$$

=> $\frac{R_C}{R_B} = E = \theta + \phi$ substitute above ... $\tan(\phi) = \frac{R_C}{R_B} = E = \theta + \phi$ $\tan(\phi) = \theta + \phi$

$\theta = \tan(\phi) - \phi$ basic definition for angular coordinate of involute curve for any ϕ. Curve is generated by setting ϕ to range from 0 to max

from geometry ... $\cos(\phi) = \frac{R_B}{R}$ => $R = \frac{R_B}{\cos(\phi)}$ the other coordinate, R=pitch_radius when ϕ = pressure angle for design

involute curve

$\phi := 40deg$ pressure_angle $\theta1 := 0, 0.01 .. 2\cdot\pi$ $2\cdot\pi$_range_variable

$\theta := \tan(\phi) - \phi$ involute(ϕ) $\theta = 8.077deg$ R_rad := 0, 0.1 .. 2 ϕ1_max := 0.85rad

$R_B := 1$ in this case we will define the base radius

calculate the pitch radius $R_P := \frac{R_B}{\cos(\phi)}$ $R_P = 1.305$ N.B. positive directions for θ and ϕ are opposite

the involute is constructed by varying a dummy pressure angle over a range - equivalent to unwrapping the string from the disk.

$\phi1 := 0, 0.01 .. \phi1\_max$ range_variable_for_construction

$$\theta2(\phi1) := \tan(\phi1) - \phi1 \qquad R2(\phi1) := \frac{R_B}{\cos(\phi1)}$$

a tangent is drawn from the pressure angle thru the involute at the pitch radius (perpendicular to involute)

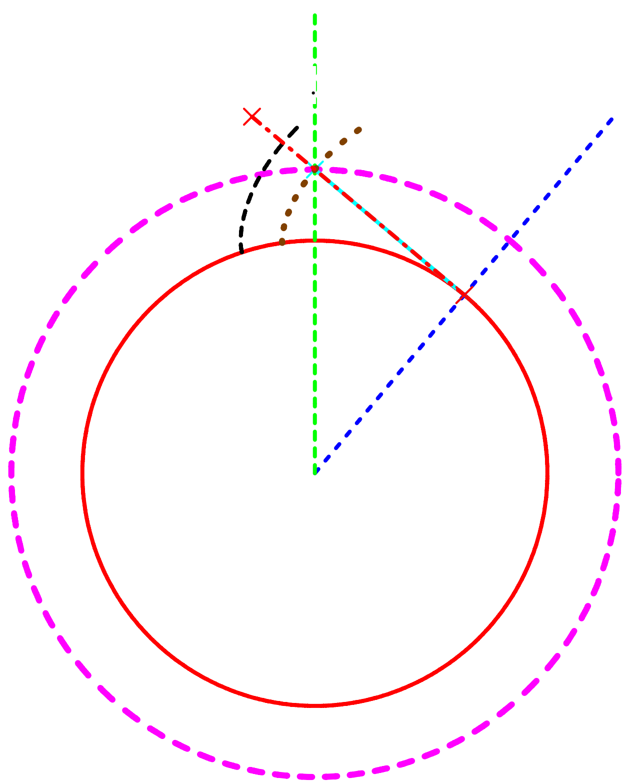
$$R\_tan := \begin{pmatrix} R_P & \frac{\pi}{2} \\ R_B & \frac{\pi}{2} - \phi \end{pmatrix}$$ draws the tangent $$R\_tan = \begin{pmatrix} 1.305 & 1.571 \\ 1 & 0.873 \end{pmatrix}$$

add in an involute at a nominal pressure angle of 50 deg and then rotate it by the difference between pressure angles. Notice it overlays the first tangent.

$\phi 4 := 50\text{deg}$ $\theta 4 := \tan(\phi 4) - \phi 4$ $\theta 4 = 18.282\,\text{deg}$ $(\phi 4 - \phi)\cdot k4$ does the rotation with k4 = 1

$$R\_tan1 := \begin{bmatrix} \frac{R_B}{\cos(\phi 4)} & \frac{\pi}{2} + (\phi 4 - \phi)\cdot k4 \\ R_B & \frac{\pi}{2} - \phi 4 + (\phi 4 - \phi)\cdot k4 \end{bmatrix} \qquad R\_tan1 = \begin{pmatrix} 1.556 & 1.745 \\ 1 & 0.873 \end{pmatrix}$$

the resulting figure is as follows:



tooth construction (design)

at this point we know ... R_B = radius_of_generating_cylinder

ϕ = pressure_angle

$R = \frac{R_B}{\cos(\phi)}$ radius as function of pressure angle
= pitch radius at design pressure angle

define $CP = \text{circular\_pitch} = \frac{\text{circumference\_of\_pitch\_diameter}}{\text{number\_of\_teeth}}$

set pressure angle $\phi := 25deg$ pressure_angle

$DP := 10$ $diametral\_pitch = DP = \frac{number\_of\_teeth}{pitch\_diameter} = \frac{N_G}{2 \cdot R_G} = \frac{N_P}{2 \cdot R_P}$ $CP \cdot DP = \pi$ an aside ...

$N_P := 20$ number_of_pinion_teeth $N_G := 30$ number_of_gear_teeth

$BL := 0.01$ backlash = 0.01 beyond scope, depends on DP $CTT_P := \frac{\pi}{DP \cdot 2} - \frac{BL}{2}$ circular_tooth_thickness

calculate pitch and base radii $CTT_G := CTT_P$ same on pitch diameter

$R_G := \frac{N_G}{DP} \cdot \frac{1}{2}$ $R_G = 1.5$ pitch_radius_gear $R_{BG} := R_G \cdot \cos(\phi)$ $R_{BG} = 1.359$ base_diameter_gear

$R_P := \frac{N_P}{DP} \cdot \frac{1}{2}$ $R_P = 1$ pitch_radius_pinion $R_{BP} := R_P \cdot \cos(\phi)$ $R_{BP} = 0.906$ base_diameter_pinion

$C := R_G + R_P$ $C = 2.5$ center_distance

$R := \frac{R_G}{R_P}$ $R = 1.5$ gear_ratio i.e. gear ration is ratio of pitch radii (or diameters or number of teeth)

$$CTT_{P2} = 2 \cdot R_{P2} \cdot \left(\frac{CTT_P}{2 \cdot R_{P1}} + inv(\phi 1) - inv(\phi 2) \right)$$ derived from involute geometry

defining function inv

at R_2 point on thickness of tooth B is $B = \theta 1 + \frac{1}{2} \cdot \frac{CTT_1}{R_1} - \theta 2$ $inv(\phi) := \tan(\phi) - \phi$

derived below ...

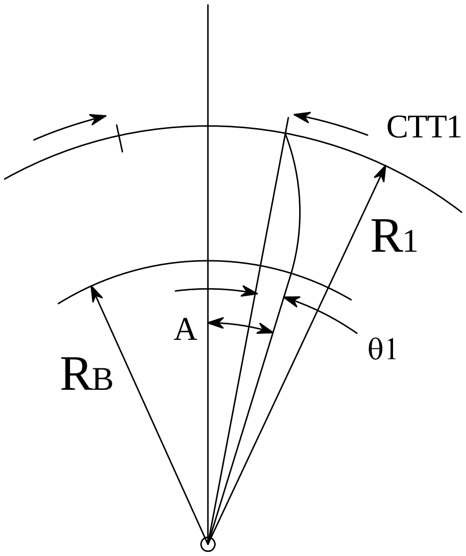


figure 2.10 page 31 Lynwander
reversed and rotated - values at pitch radius

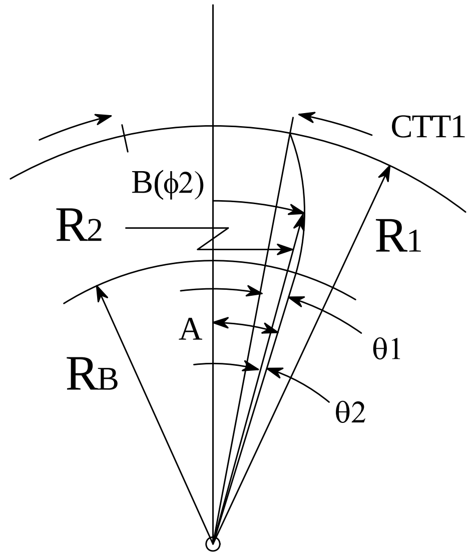


figure 2.10 page 31 Lynwander
reversed and rotated

$$A = \theta 1 + \frac{1}{2} \cdot \frac{CTT_1}{R_1}$$

CTT_1 = circular_tooth_thickness

ϕ = pressure_angle_design

$\theta 1$ = involute_of_design_pressure_angle

$$R_1 = \text{pitch\_radius} = \frac{R_B}{\cos(\phi)}$$

here consider varying ϕ from 0
to a value > design angle = $\phi 2$

$\theta 2$ = involute_of_$\phi 2$

$$B(\phi 2) = A - \theta 2$$

$$R_2 = \frac{R_B}{\cos(\phi 2)}$$

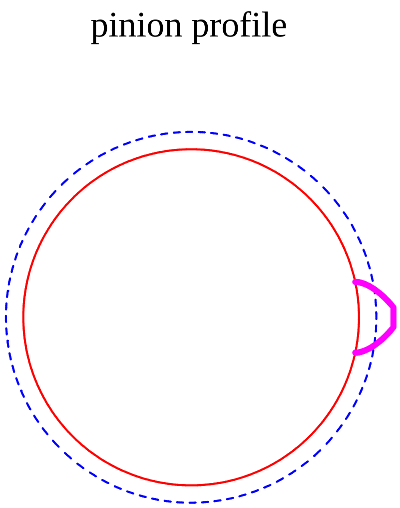
so ..
$$B = \theta 1 + \frac{1}{2} \cdot \frac{CTT_1}{R_1} - \theta 2$$

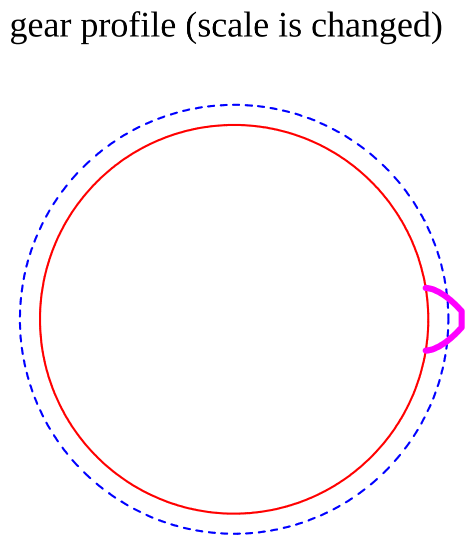
and points on tooth surface are R2,B

additional definitions addendum dedendum root_diameter

tooth profile ... with pitch radius and base radius shown ...

▶ plot set up





move the pinion out to C, rotating it by π and offsetting both by half tooth thickness

$\theta\_plot_G(R_G)$

▶ geometry to shift circle

▶ plot set up