Reduction Gears

need:

propellers	60-300 rpm	
waterjet pump	300-1500 rpm	
low speed diesels	70-250 rpm	
medium speed diesels	350-1200 rpm	
steam turbine	6000-9000 rpm	
gas turbine	3600-15000 rpm	(larger @ lower rpm)

reduction gears make conversion.

some history

ref: Marine Engineering Chapter IX Reduction Gears, by Gary P. Mowers, (SNAME) page 325 ff and others

19th - 20th century (1890-1910) ships propelled by reciprocating steam engines - direct drive
1904 - study by consulting engineers George Melville Adm (Ret.) and John Alpine

George Melville was Chief Bureau of Steam Engineering and in 1899 President of ASME **see**

study: - Problem - steam engine succeeding reciprocating engine:
"If one could devise a means of reconciling, in a practical manner, the necessary high speed of revolution of the turbine with the comparatively low rate of revolution required by an efficient propeller, the problem would be solved and the turbine would practically wipe out the reciprocating engine for the propulsion of ships. The solution of this problem would be a stroke of great genius." Ref: Mar. Eng.

First gear generally attributed to Pierre DeLaval in 1892. Parsons (cavitation) and George Westinghouse developed prototypes and installed gears:

1910 - 15,000 shp with geared turbine drive
1940 - 100,000 shp with geared turbine drive
1917 - double reduction introduced

Development has been evolutionary - few step advances

- single to double
- welding in construction of gear wheel as and casing
- higher hardness pinion and gear materials => higher tooth load

Many types of gears are used (defined) and there is an extensive nomenclature associated with gear definitions. One source: (formerly available free via registration via:

http://www.agma.org/Content/NavigationMenu/EducationTraining/OnlineEducation/default.htm)

AGMA Gear Nomenclature, Definitions of Terms with Symbols
ANSUAGMA 1012-F90
(Revision of AGMA 112.05)

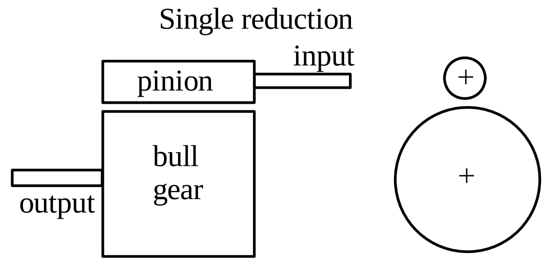
[Tables or other self-supporting sections may be quoted or extracted in their entirety. Credit lines should read: Extracted from AGMA 1012-F90, Gear Nomenclature Terms, Definitions, Symbols and Abbreviations, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia 22314] Availability changed to require registration in course. I have copy from previous registration when it was free.

See: handout from Marine Engineering, (on web site)

Single Reduction

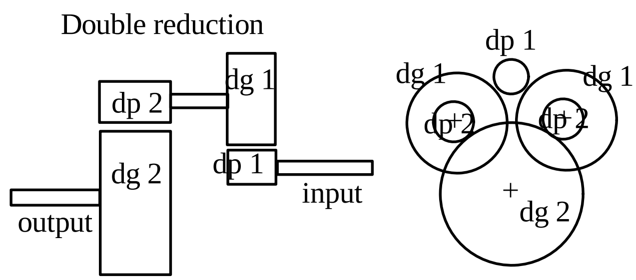
$$gear\_ratio = R = \frac{diameter\_of\_gear}{diameter\_of\_pinion}$$

$$R = \frac{number\_teeth\_gear}{number\_teeth\_pinion} = \frac{rpm\_pinion}{rpm\_gear} = \frac{N_P}{N_G}$$



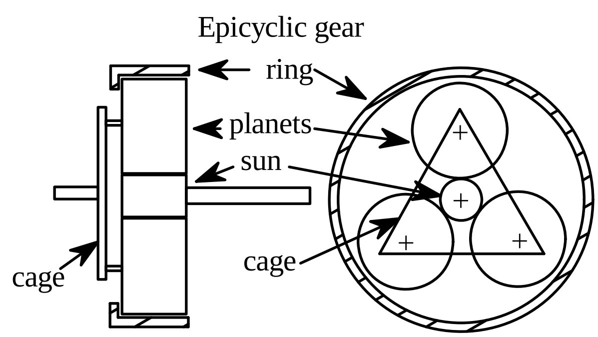
Double Reduction (locked train 50% each drive)

$$R_1 = \frac{dg_1}{dp_1} \qquad R_2 = \frac{dg_2}{dp_2} \qquad R = R_1 \cdot R_2$$



Epicyclic gear summary

various combinations can be used with this system



Type	Fixed	Input	Output	Ratio	Normal range
Planetary	ring	sun	cage	$R_R/R_S + 1$	3:1 - 12:1
Star	cage	sun	ring	(-) R_R/R_S	(-) 2:1 - 11:1
Solar	sun	ring	cage	$R_S/R_R + 1$	1.2:1 - 1.7:1

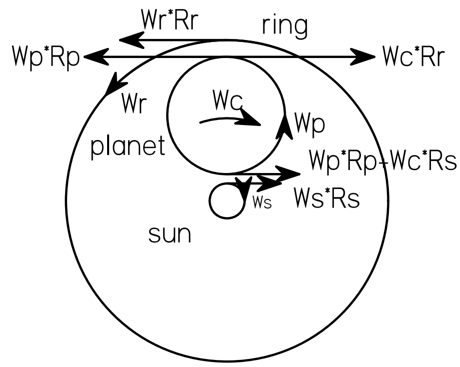
$R_S = radius\_sun$

$R_R = radius\_ring$

$R_P = radius\_planet$

Application: Planetary gears used in high speed as tooth loading reduced by multiplicity of planet gears. Also can provide counter-rotation.

to show above ratios consider:



R = pitch_diameter

W = angular_velocity

W_T = tangential_load

n = rpm

subscripts ...

s = sun

p = planet

r = ring

c = carrier(cage)

for compound systems: 1 = primary, 2 = secondary

ref: Gear Drive Systems; Design and Application, P. Lynwander TJ184.L94 1983
N.B. in this development W_s and W_c are clockwise and W_r and W_p are ccw

1. point on pitch diameter of sun gear has tangential velocity $W_s \cdot R_s$

2. point on sun gear pitch diameter meshing with planet gear pitch diameter has tangential velocity

$$W_p \cdot R_p + W_c \cdot R_s$$

first term from rotation of planet
second from rotation of carrier about center of carrier and sun

mesh => $W_s \cdot R_s = W_p \cdot R_p + W_c \cdot R_s$

3. similarly at ring ... $W_r \cdot R_r = W_p \cdot R_p - W_c \cdot R_r$ or ... $W_r \cdot R_r + W_c \cdot R_r = W_p \cdot R_p$

combining ... $W_s \cdot R_s = W_p \cdot R_p + W_c \cdot R_s = W_r \cdot R_r + W_c \cdot R_r + W_c \cdot R_s = W_r \cdot R_r + W_c \cdot (R_r + R_s)$

$$W_s \cdot R_s = W_r \cdot R_r + W_c \cdot (R_r + R_s)$$

or .. for solving below ...

<u>Planetary arrangement</u> ... input sun, output carrier (cage), fixed ring

$W_r = 0$ $\quad W_s \cdot R_s = W_c \cdot (R_r + R_s)$ $\quad \frac{W_s}{W_c} = \frac{R_r + R_s}{R_s} = \frac{R_r}{R_s} + 1$

<u>Star arrangement</u> ... input sun, output ring, fixed carrier (cage)

$W_c = 0$ $\quad W_s \cdot R_s = W_r \cdot R_r$ $\quad \frac{W_s}{W_r} = \frac{R_r}{R_s}$

but these are in opposite directions
can use for reversing

<u>Solar arrangement</u> ... input ring output carrier (cage) fixed sun

$W_s = 0$ $\quad 0 = W_r \cdot R_r + W_c \cdot (R_r + R_s)$ $\quad \frac{W_r}{W_c} = -\frac{R_r + R_s}{R_r} = -\left(\frac{R_s}{R_r} + 1\right)$

but in figure Wr and Wc are opposite rotation => this is the same actual rotation in result defined mcd here for symbolic calculation, actually below

$$k_1 := \frac{1 - \nu_1^2}{\pi \cdot E_1} \qquad k_2 := \frac{1 - \nu_2^2}{\pi \cdot E_2}$$

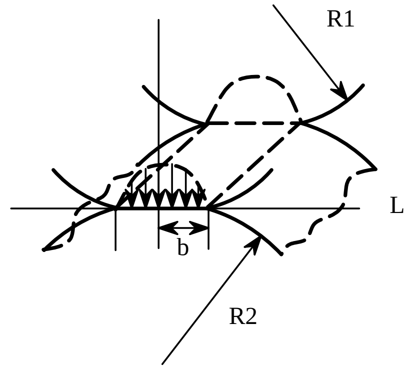
Hertz stress

a brief intro to some details. see Timoshenko, Theory of Elasticity ... page 418 ff for more specifics

It can be shown that the width of contact between two parallel cylinders given: elliptical loading, q_o, etc.

$$b := \sqrt{\frac{4 \cdot P\_p \cdot (k_1 + k_2) \cdot R_1 \cdot R_2}{R_1 + R_2}}$$

this is the solution to the width of contact given: elliptical loading, q_o, etc. (236)



where ... $P\_pr = \frac{1}{2} \cdot \pi \cdot b \cdot q_o$

$q_o = max\_pressure\_elliptical\_distribution$

$b = half\_width\_of\_rectangular\_contact\_area$

$$q_o := \frac{2 \cdot P\_pr}{\pi \cdot b}$$

$P\_pr = P' = \frac{load}{length}$ (235)

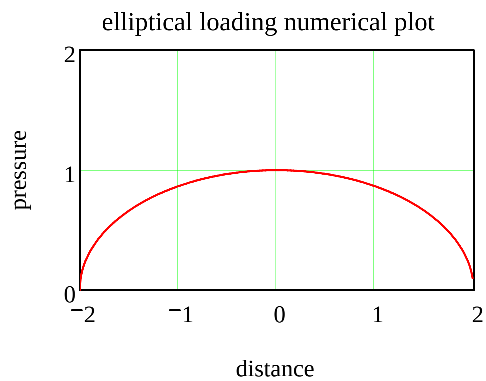
$k_n = \frac{1 - \nu_n^2}{\pi \cdot E_n}$ all n (236)

▼ elliptical loading details

$b := 2 \quad p\_over\_P_{max}(x) := \sqrt{1 - \left(\frac{x}{b}\right)^2} \qquad x := -b, -b + 0.01 .. b$

symbolically

$$p(xx) := P\_max \cdot \sqrt{1 - \left(\frac{xx}{bb}\right)^2}$$



want $\int_{-bb}^{bb} p(xx)\, dxx = load$

let ... $xx(\theta, bb) := bb \cdot \cos(\theta)$ $\qquad \frac{d}{d\theta} xx(\theta, bb) \rightarrow (-bb) \cdot \sin(\theta)$

then ...

$p(\theta) := P\_max \cdot \sin(\theta)$ $\qquad \int_{-bb}^{bb} p(xx)\, dxx = \int_{\pi}^{0} p(xx) \cdot (-bb \cdot \sin(\theta))\, d\theta$

therefore ..

$$\int_{\pi}^{0} p(\theta) \cdot (-bb \cdot \sin(\theta))\, d\theta \rightarrow \frac{1}{2} \cdot \pi \cdot P\_max \cdot bb$$

and with interpretation on per unit length basis $\qquad P\_pr = \frac{1}{2} \cdot \pi \cdot b \cdot q_o$

▲ elliptical loading details

substitution for b and solve for q_o in terms of load per unit length P_pr...

$$q_o \rightarrow \frac{P\_pr}{\left[\pi \cdot \left[P\_pr \cdot \left(\frac{1-\nu_1^{\,2}}{\pi \cdot E_1} + \frac{1-\nu_2^{\,2}}{\pi \cdot E_2}\right) \cdot R_1 \cdot \frac{R_2}{R_1 + R_2}\right]\right]^{\frac{1}{2}}}$$

or ...

$$q_o = \frac{2 \cdot P\_pr}{\pi \cdot \sqrt{\frac{4 \cdot P\_pr \cdot (k_1 + k_2) \cdot R_1 \cdot R_2}{R_1 + R_2}}} = \frac{2}{\pi} \cdot P\_pr \cdot \sqrt{\frac{R_1 + R_2}{4 \cdot P\_pr \cdot (k_1 + k_2) \cdot R_1 \cdot R_2}} = \sqrt{\frac{P\_pr^2 \cdot 4}{\pi^2} \cdot \frac{R_1 + R_2}{4 \cdot P\_pr \cdot (k_1 + k_2) \cdot R_1 \cdot R_2}}$$

$$q_o = \sqrt{\frac{P\_pr}{\pi^2} \cdot \frac{R_1 + R_2}{(k_1 + k_2) \cdot R_1 \cdot R_2}} = \sqrt{\frac{P\_pr}{\pi^2} \cdot \frac{R_1 + R_2}{\left(\frac{1-\nu_1^{\,2}}{\pi \cdot E_1} + \frac{1-\nu_2^{\,2}}{\pi \cdot E_2}\right) \cdot R_1 \cdot R_2}} = \sqrt{\frac{P\_pr}{\pi} \cdot \frac{R_1 + R_2}{\left(\frac{1-\nu_1^{\,2}}{E_1} + \frac{1-\nu_2^{\,2}}{E_2}\right) \cdot R_1 \cdot R_2}} \quad (240)$$

same material E1 = E2, and ... $\nu = 0.3$ ...

$$q_o = \sqrt{\frac{P\_pr}{\pi} \cdot \frac{R_1 + R_2}{2\left(\frac{1-\nu^2}{E}\right) \cdot R_1 \cdot R_2}} = \sqrt{\frac{P\_pr \cdot E}{\pi} \cdot \frac{R_1 + R_2}{2(1-\nu^2) \cdot R_1 \cdot R_2}} = \left[\frac{1}{\pi \cdot 2(1-\nu^2)}\right]^{\frac{1}{2}} \sqrt{\frac{P\_pr \cdot E}{\pi} \cdot \frac{R_1 + R_2}{R_1 \cdot R_2}}$$

$$\nu := 0.3 \quad \left[\frac{1}{\pi \cdot 2(1-\nu^2)}\right]^{\frac{1}{2}} = 0.418 \qquad q_o = 0.418 \cdot \sqrt{\frac{P\_pr \cdot E}{\pi} \cdot \frac{R_1 + R_2}{R_1 \cdot R_2}} \quad (241)$$

Marine Engineering ... $S = \sqrt{0.175 \cdot \frac{P}{L} \cdot E \cdot \frac{r_1 + r_2}{r_1 \cdot r_2}}$ page 330 between (10) and (11)

$S = maximum\_compressive\_stress \cdot psi$ E, r straight forward

$\frac{P}{L} = loading\_per\_inch\_length = P\_pr$ $\quad \frac{1}{\pi} \cdot \frac{1}{2(1-\nu^2)} = 0.175$

maximum stress ~ $\sqrt{\frac{P}{L} \cdot \frac{d_1 + d_2}{d_1 \cdot d_2}}$ $\quad \frac{P}{L} = \frac{W_t}{F_e} = \frac{tangential\_tooth\_load}{effective\_face\_width\_at\_pitch\_diameter} \cdot \frac{lbf}{in}$

$$\frac{W_t}{F_e} = \frac{hp}{\pi \cdot rpm_{pinion} \cdot d_{pinion} \cdot F_e}$$

$hp = horse\_power\_transmitted\_per\_mesh$

hp in horsepower

rpm in min^{-1}

d_{pinion} in inches

Fe in length; carries to result

unit version $\frac{W_t}{F_e} = 126051 \frac{hp}{rpm_{pinion} \cdot d_{pinion} \cdot F_e}$

result ... $\frac{lbf}{Fe\_unit}$

replace ... $d_1 = d_g$ $d_2 = d_p$

maximum stress ~ $\sqrt{\frac{W_t}{F_e}\cdot\frac{d_g + d_p}{d_g \cdot d_p}} = \sqrt{\frac{W_t}{F_e}\cdot\frac{\frac{d_g}{d_p} + 1}{d_g}} = \sqrt{\frac{W_t}{F_e}\cdot\frac{R + 1}{d_g}} = \sqrt{\frac{W_t}{F_e}\cdot\frac{R + 1}{d_p \cdot R}}$ as ... $R = \frac{d_g}{d_p}$

motivates parameter K factor $K = \frac{W_t}{F_e}\cdot\frac{1}{d_p}\cdot\frac{R + 1}{R}$ units of pressure: kPa. psi, N/m^2 etc.

maximum stress ~ $\sqrt{K}$

some observations ... $\frac{W_t}{F_e} = K \cdot d_p \cdot \frac{R}{R + 1} = \frac{hp}{\pi \cdot n_p \cdot d_p \cdot F_e}$ => $d_p^{\,2} = \frac{hp}{\pi \cdot K \cdot n_p \cdot F_e}\cdot\left(\frac{R + 1}{R}\right)$

for double helical pinion gear $\left(\frac{F_e}{d_p}\right)_{max} = C$ $2 \le C \le 2.5$ 1.5 for epicyclic

substitute $F_e = C{*}d_p$ into above ...

$$d_p^{\,3} = \frac{hp}{\pi \cdot K \cdot C \cdot n_p}\cdot\left(\frac{R + 1}{R}\right)$$

$$d_p = \left[\frac{hp}{\pi \cdot K \cdot C \cdot n_p}\cdot\left(\frac{R + 1}{R}\right)\right]^{\frac{1}{3}}$$

unit conversion $\frac{550}{\pi}\cdot 60 \cdot 12 = 126051$

$$\frac{lbf \cdot ft}{\frac{sec}{hp}}\cdot\frac{60sec}{min}\cdot\frac{12in}{ft}$$

K used in design 100 - 1000 psi
highest in hardened and ground aircraft engine gears

$100psi = 689.5\,\frac{kN}{m^2}$ $1000psi = 6895\,\frac{kN}{m^2}$

classification societies limit to 300 psi $300psi = 2068\,\frac{kN}{m^2}$

Marine Engineering suggests the first approximation for the gear ratio for the second reduction be taken as ...

$\sqrt{R_{overall}} - 1$ articulated $\sqrt{R_{overall}} + 3$ locked train

another Navy study (ref: Prof Carmichael) suggests $\sqrt{R_{overall}}$ for double reduction $R_{overall}^{\frac{1}{3}}$ for triple reduction

volume ... for solid gear $vol = \frac{\pi}{4}\cdot d^2 \cdot F_e$

$$d_p^{\,2} \cdot F_e = d_p^{\,3} \cdot C = \frac{hp}{\pi \cdot K \cdot n_p}\cdot\left(\frac{R + 1}{R}\right)$$

$$volume\_bull\_gear = \frac{\pi}{4}\cdot d_g^{\,2} \cdot F_e = \frac{\pi}{4}\cdot R^2 \cdot d_p^{\,2} \cdot F_e = \frac{\pi}{4}\cdot R^2 \cdot \frac{hp}{\pi \cdot K \cdot n_p}\cdot\left(\frac{R + 1}{R}\right)$$

$$total\_volume = \frac{\pi}{4}\cdot d_p^{\,2} \cdot F_e \cdot \left(R^2 + 1\right) = \frac{\pi}{4}\cdot\frac{hp}{\pi \cdot K \cdot n_p}\cdot\left(\frac{R + 1}{R}\right)\cdot\left(R^2 + 1\right)$$

in terms of diameter of pinion

or substituting $n_p = R \cdot n_g$

$$total\_volume = \frac{\pi}{4} \cdot \frac{hp}{\pi \cdot K \cdot n_g} \cdot \left(\frac{R+1}{R^2}\right) \cdot \left(R^2 + 1\right)$$

in terms of diameter of bull gear

overall volume ... increases with

1. power increases (per torque path
2. rpm_{gear} decreases
3. K decreases
4. R increases

data for plot