Supplement for Repairable System Reliability

Reference: Probability and Reliability for Engineers, Miller, TA340.M648 1985
primarily chapter 15
NIST e-book Engineering Statistics Handbook, sections labelled

$$\mathrm{PDF} = \mathrm{probability\_density\_function} = f(t)$$

$$\mathrm{CDF} = \mathrm{cumulative\_distribution\_function} = F(t) = \int_0^t f(x)\,dx$$

8.1.2.2 Reliability or Survival function

$$\mathrm{Reliability\_function} = \mathrm{probability\_unit\_survives\_beyond\_t}$$

$$R(t) = 1 - F(t) \quad \text{or ...} \quad F(t) = 1 - R(t)$$

8.1.2.3 Failure (or Hazard) rate

$$h(t) = \mathrm{failure\_rate} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}$$

conditional probability

therefore ... $f(t) = R(t)\cdot h(t)$

now ... $R(t) = 1 - F(t)$ $\quad \frac{d}{dt}F(t) = -\frac{d}{dt}R(t) = f(t)$

$$h(t) = \frac{f(t)}{R(t)} = -\frac{\frac{d}{dt}R(t)}{R(t)} = -\frac{d}{dt}\ln(R(t))$$

integrate from 0 to t $\quad \int_0^t h(x)\,dx = -\ln(R(t))$

exponentiate .. $\quad R(t) = e^{-\int_0^t h(x)\,dx}$

therefore ... $\quad f(t) = R(t)\cdot h(t) = h(t)\cdot e^{-\int_0^t h(x)\,dx}$

now .. if assume (observe) failure rate h(t) = constant = λ

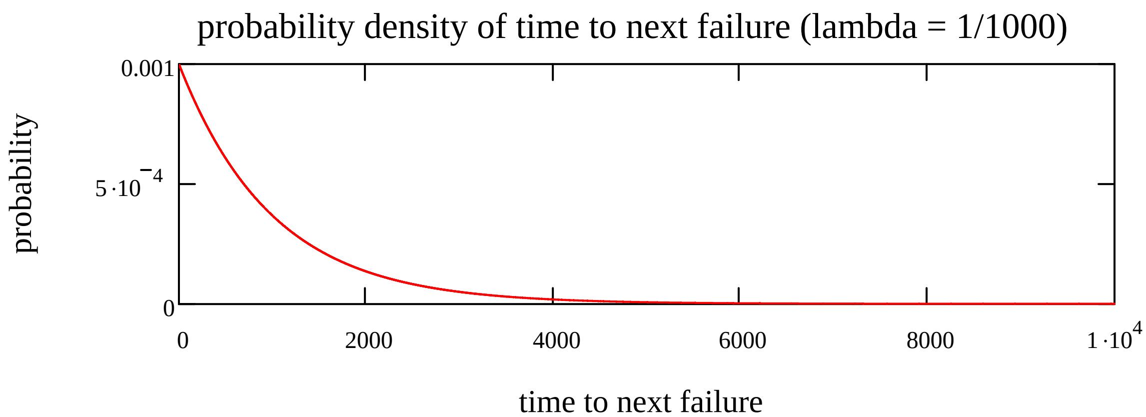
$h(t) := \lambda$

$$f(t) := h(t)\cdot e^{-\int_0^t h(x)\,dx}$$

$$f(t) \rightarrow \lambda\cdot e^{(-\lambda)\cdot} \qquad F(t) := \int_0^t \lambda\cdot e^{(-\lambda)\cdot x}\,dx \rightarrow \left[-e^{(-\lambda)\cdot t}\right] + 1 \qquad F(t) := 1 - e^{-\lambda\cdot t}$$

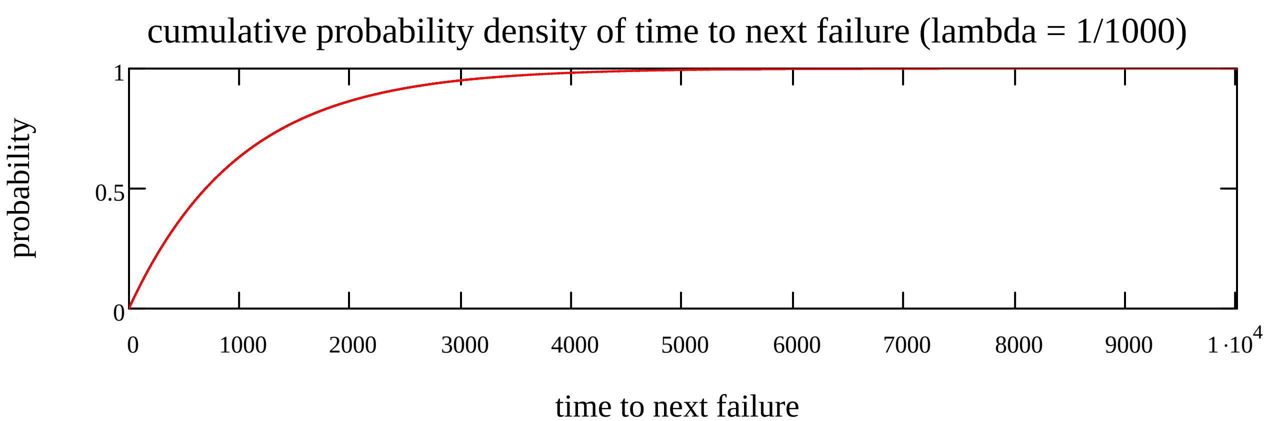
have exponential assumption of probability of failure times exponential pdf and cdf example

PDF_of_time_to_next_failure = $f(t) = \lambda \cdot e^{-\lambda \cdot t}$ $\lambda := \frac{1}{1000}$ $t := 0 .. 10000$ $f(t) := \lambda \cdot e^{-\lambda \cdot t}$



CDF = $F(t) = 1 - e^{-\lambda \cdot t}$ = CDF_of_waiting_time_to_next_failure $F(t) := \left(1 - e^{-\lambda \cdot t}\right)$

or .. CDF of "interarrival" time between failures



exponential pdf and cdf example

reset variables

interpret time to failure as a waiting time, it can be shown that this can be represented as a Poisson process, if a component which fails is immediately replaced with a new one having the same failure rate λ. Some results from this observation:

mean_waiting_ttime_between_successive_failures = $\frac{1}{\lambda}$ = MTBF

Some results for exponential model

Reliability or Survival function $R(t) := 1 - F(t)$

$$\text{Reliability\_function} = \text{probability\_unit\_survives\_beyond\_t} \qquad R(t) \rightarrow e^{(-\lambda)\cdot t}$$

e.g. if component has a failure rate of 0.05/1000 hours, probability that it will survive at least 10,000 hrs is given by:

$$e^{\frac{-0.05}{1000}\cdot 10000} = 0.607$$

n components in series

if a system consists of n components in series, with respective failure rates $\lambda_1, \lambda_2 \ldots \lambda_n$

$$Rs(t) = \prod_{i=1}^{n} e^{-\lambda_i \cdot t} = e^{-\sum_{i=1}^{n} \lambda_i \cdot t}$$

so it also is an exponential distribution ... and the MTBF for the system is:

$$MTBF_{series\_system} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} \left(\frac{1}{MTBF_i}\right)}$$

for a parallel system

... with respective failure rates $\lambda_1, \lambda_2 \ldots \lambda_n$
in this case we need to deal with "unreliabilities"

$F_i = 1 - R_i$ is probability component i will fail

$$\text{probability\_all\_will\_fail} = \text{unreliability} = F_p = \prod_{i=1}^{n} F_i$$

and probability of survival $=R_p(t)$

$$R_p(t) = 1 - F_p(t) = 1 - \prod_{i=1}^{n} \left[F_i(t)\right] = 1 - \prod_{i=1}^{n} \left[1 - R_i(t)\right]$$

in this case; exponential probability of failure

$$F_p(t) = \prod_{i=1}^{n} \left[F_i(t)\right] = \prod_{i=1}^{n} \left(1 - e^{-\lambda_i \cdot t}\right)$$

this will not show exponential distribution ...

$$h_p(t) = \frac{f_p(t)}{R_p(t)} = \frac{\frac{d}{dt}F_p(t)}{R_p(t)}$$

difficult to evaluate, but notice at least it is f(t).

$R_p(t)$ difficult to obtain in general, but when all components have same failure rate

$$R_p(t) = 1 - \prod_{i=1}^{n} \left[1 - R_i(t)\right] = 1 - \prod_{i=1}^{n} \left(1 - e^{-\lambda \cdot t}\right) = 1 - \left(1 - e^{-\lambda \cdot t}\right)^n$$

$$\text{n\_choose\_k}(n,k) := \frac{n!}{k!\cdot(n-k)!}$$ binomial coefficient

$$1 - \left(1 - e^{-\lambda \cdot t}\right)^n = 1 - \left(1 - \text{n\_choose\_k}(n,1)\cdot e^{-\lambda \cdot t} + \text{n\_choose\_k}(n,1)\cdot e^{-2\lambda \cdot t} - \text{........}\right)$$

$$R_p(t) = \text{n\_choose\_k}(n,1)\cdot e^{-\lambda \cdot t} - \text{n\_choose\_k}(n,2)\cdot e^{-\lambda \cdot t} + \text{.........................} (-1)^{n-1}\cdot e^{-n\cdot\lambda \cdot t}$$

binomial theorem from mathworld.wolfram.com/BinomialTheorem.html

it can be shown see reference page 460

after differentiating to find $f_p(t)$ $f_p(t) = \frac{d}{dt}R_p(t)$

and then calculating the mean ($MBTF_{parallel}$)

$$MBTF_{parallel} = \frac{1}{\alpha}\cdot\left(1 + \frac{1}{2} + \text{............} + \frac{1}{n}\right)$$

e.g. if use two identical components in parallel

$MBTF_{parallel} = \frac{1}{\alpha}\cdot\frac{3}{2}$ increase of 50% not double $\sum_{k=1}^{4} \frac{1}{k} = 2.083$ four to double

another example if time permits on board