Electric Motors

Ref: Chapter 9

from electrcal overview Lorentz force...

$M = K_m \cdot \Phi \cdot I$ — K_m = constant_for_given_motor (ref: 2.93) (9.1)

M = torque — $N \cdot m$

Φ = magnetic_flux — $Wb = 1\,weber$

I = current — $A = 1\,amp$ — $1Wb \cdot 1A = 1\,N\cdot$

when rotating, electromotive force induced in rotor given by ..
from electrical overview Faraday's force ...

$E = K_E \cdot \Phi \cdot n$ — K_E = constant_for_given_motor (ref: 2.96) (9.2)

E = induced_electromotive_force — $V = 1\,volt$

Φ = magnetic_flux — $Wb = 1\,weber$

n = rotation_speed — $rpm = 6.283\,\frac{1}{min}$ — $Wb \cdot rpm = 0.105\,V$

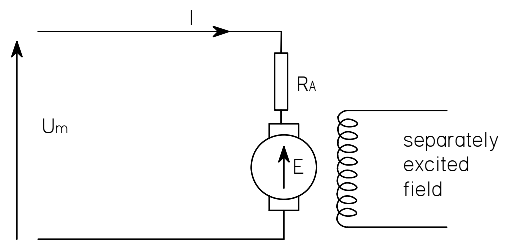
model motor as resistance in series with EMF generator (opposing applied voltage)

$U = E + I \cdot R$ (9.3) $P = E \cdot I + I^2 \cdot R$ (9.4)

with ... $M = K_m \cdot \Phi \cdot I$ (9.1) and ... $E = K_E \cdot \Phi \cdot n$ (9.2)

$I := \frac{M}{K_m \cdot \Phi}$ $E := (U - I \cdot R)$ $n := \frac{E}{K_E \cdot \Phi}$

n collect, $\Phi \rightarrow \frac{U}{K_E \cdot \Phi} - \frac{M}{K_m} \cdot \frac{R}{K_E \cdot \Phi^2}$ $n = \frac{U}{K_E \cdot \Phi} - \frac{M \cdot R}{K_E \cdot K_m \cdot \Phi^2}$ (9.5)



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to see an example of DC motor behavior assume a set of reasonable parameters. Not all are independent.

for fixed magnetic field Φ and rpm at maximum power , maximum current I_m and maximum torque M_m — $M = K_m \cdot \Phi \cdot I$

set Φ, n, R and applied voltage U — maximum current

$\Phi := 1Wb$ $n := 100rpm$ $R := 2\Omega$ $U_m := 400V$ $I_m := 10A$

derived $U = E + I \cdot R$ E $E := U_m - I_m \cdot R$ $E = 380\,V$

$E \cdot I_m = M_m \cdot n \cdot 2 \cdot \pi$ M $M_m := \frac{E \cdot I_m}{n \cdot 2 \cdot \pi}$ $M_m = 57.753\,N \cdot m$ assuming EMF*I converted into mechanical power

$M_m = K_m \cdot \Phi \cdot I_m$ K_m $K_m := \frac{M_m}{\Phi \cdot I_m}$ $K_m = 5.775$ $P = U \cdot I = E \cdot I + I^2 \cdot R = M \cdot n \cdot 2 \cdot \pi + I^2 \cdot R$

$E = K_E \cdot \Phi \cdot n$ K_E $K_E := \frac{E}{\Phi \cdot n}$ $K_E = 36.287$

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$$M = U \cdot \frac{K_m \cdot \Phi}{R} - n \cdot \frac{K_E \cdot K_m \cdot \Phi^2}{R}$$

$$U_m \cdot \frac{K_m \cdot \Phi}{R} - n \cdot \frac{K_E \cdot K_m \cdot \Phi^2}{R} = 57.753\,N \cdot m$$

$$a := \frac{K_m \cdot \Phi}{R} \qquad b := \frac{K_m K_E \cdot \Phi^2}{R}$$

$M(U,n) := (U \cdot a - b \cdot n)$ calculate M when U and n known ...

$nn(U,M) := \frac{U \cdot a - M}{b}$ calculate n when U and M known - useful at ends of torque range 0 - M_m

$M_0 := 0 N \cdot m$ $\qquad M_m = 57.753\,N \cdot m$ $\qquad nn(U_m, M_m) = 100\,rpm$ derived check $\qquad nn(U_m, 0) = 105.26316\,rpm$

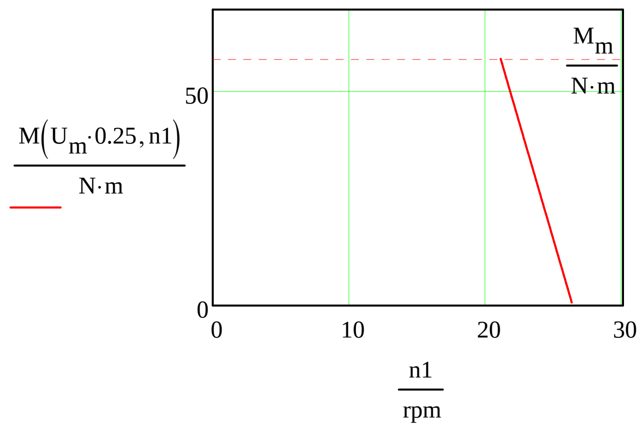
for example at U = 0.25 U_m, calculate n at 0 and maximum torque

$n11 := nn(U_m \cdot 0.25, M_m)$ $\qquad n11 = 21.053\,rpm$ maximum torque

$n12 := nn(U_m \cdot 0.25, M_0)$ $\qquad n12 = 26.316\,rpm$ 0 torque

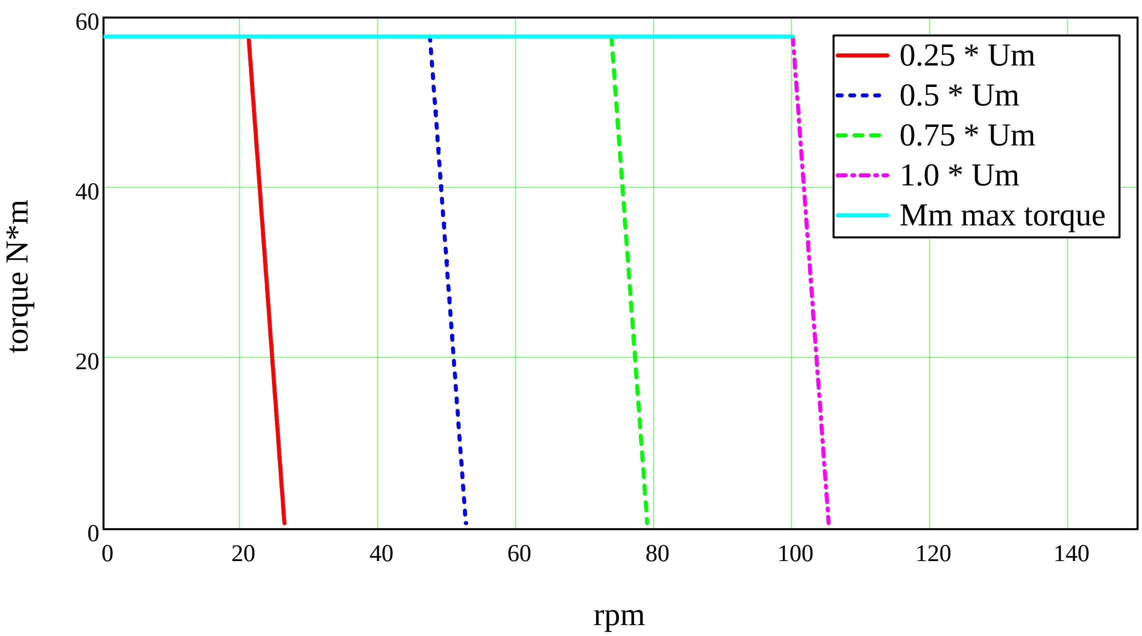
$n1 := n11, n11 + 0.1rpm .. n12$

plot M vs n for U = 0.25*U_m



and if develop similar data for 0.5 * U_m, 0.75 * U_m and U_m obtain the following plot

plot data



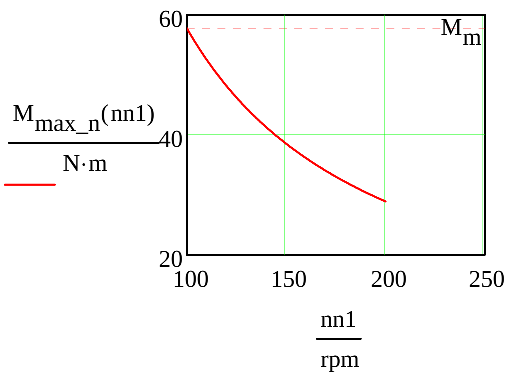
additional operating envelope is available beyond design rpm by reducing the field strength Φ. But the region is limited by the maximum power available

beyond a M limited by P_{max} $\quad P_{max} := M_m \cdot 2 \cdot \pi \cdot n \qquad P_{max} = 3.8 \times 10^3 \, W$

above base sped and torque with power consatant at P_{max}, torque is limited inversely with rpm

$$M_{max} \cdot nn \cdot 2 \cdot \pi = P_{max} \qquad M_{max\_n}(nn) := \frac{P_{max}}{nn \cdot 2 \cdot \pi}$$

$$nn1 := n, n + 1rpm .. 2 \cdot n$$



using .. again

$$n = \frac{U}{K_E \cdot \Phi} - \frac{M \cdot R}{K_E \cdot K_m \cdot \Phi^2} \qquad (9.5)$$

$$n = \frac{U}{K_E \cdot \Phi} - \frac{M \cdot R}{K_E \cdot K_m \cdot \Phi^2} \qquad nn = \frac{U}{K_E \cdot \Phi} - \frac{\frac{P_{max}}{nn \cdot 2 \cdot \pi} \cdot R}{K_E \cdot K_m \cdot \Phi^2} \qquad nn^2 = \frac{U}{K_E \cdot \Phi} \cdot nn - \frac{\frac{P_{max}}{2 \cdot \pi} \cdot R}{K_E \cdot K_m \cdot \Phi^2}$$

solved symbolicly on blank sheet

$$nn_{max}(\Phi) := \frac{1}{4 \cdot \pi \cdot K_E \cdot K_m} \cdot \frac{2 \cdot U_m \cdot \pi \cdot K_m + 2 \cdot \left(U_m{}^2 \cdot \pi^2 \cdot K_m{}^2 - 2 \cdot \pi \cdot K_E \cdot K_m \cdot P_{max} \cdot R\right)^{\frac{1}{2}}}{\Phi}$$

calculate n when U and M known - useful at ends of torque range 0 - M_m and in this application a and b are functions of Φ

$$a(\Phi) := \frac{K_m \cdot \Phi}{R} \qquad b(\Phi) := \frac{K_m K_E \cdot \Phi^2}{R} \qquad nn(\Phi, M) := \frac{U_m \cdot a(\Phi) - M}{b(\Phi)}$$

$M(U, n) = U \cdot a - b \cdot n$ $\quad$ calculate M when U and n known ... $\quad MM(\Phi, n) := \left(U_m \cdot a(\Phi) - b(\Phi) \cdot n\right)$

so with Φ = 0.75 Φ base

$n51 := nn_{max}(\Phi \cdot 0.75) \qquad n51 = 133.333\, rpm$

$n52 := nn(0.75 \cdot \Phi, M_0) \qquad n52 = 140.351\, rpm$

$n5 := n51, n51 + 1rpm .. n52$

$MM(\Phi \cdot 0.75, n51) = 43.315\, N \cdot m$

$MM(\Phi \cdot 0.75, n52) = 0\, N \cdot m$

and Φ = 0.6 of Φ base

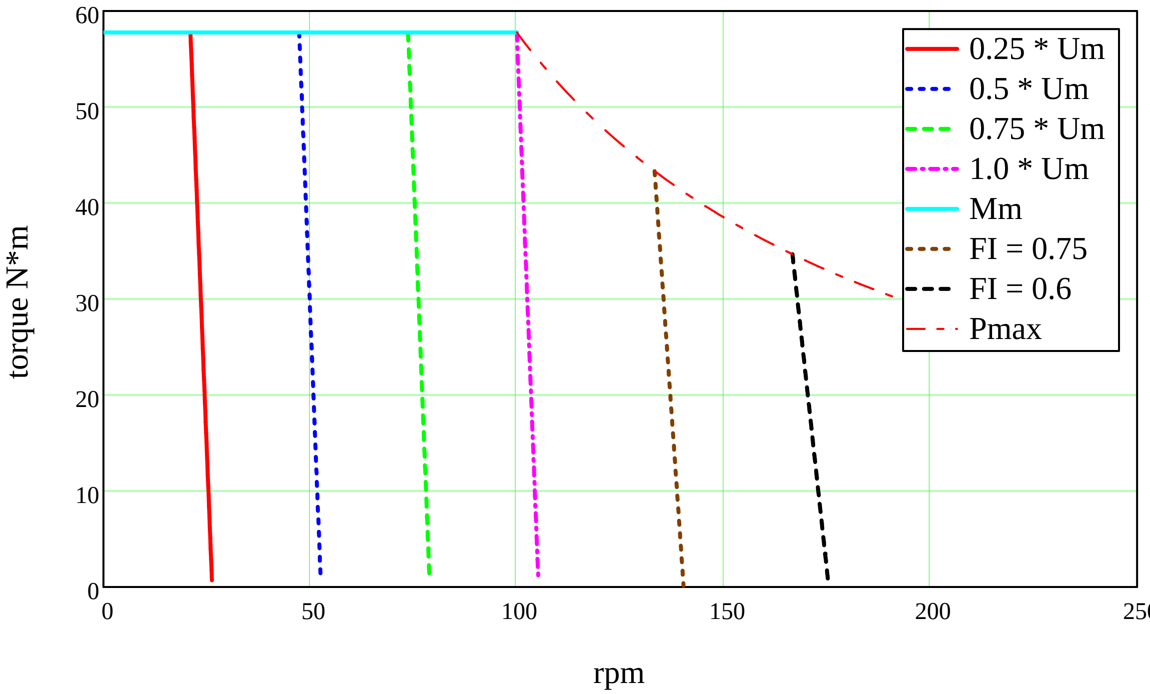
$n61 := nn_{max}(\Phi \cdot 0.6) \qquad n61 = 166.667\, rpm$

$n62 := nn(0.6 \cdot \Phi, M_0) \qquad n62 = 175.439\, rpm$

$n6 := n61, n61 + 0.1rpm .. n62$

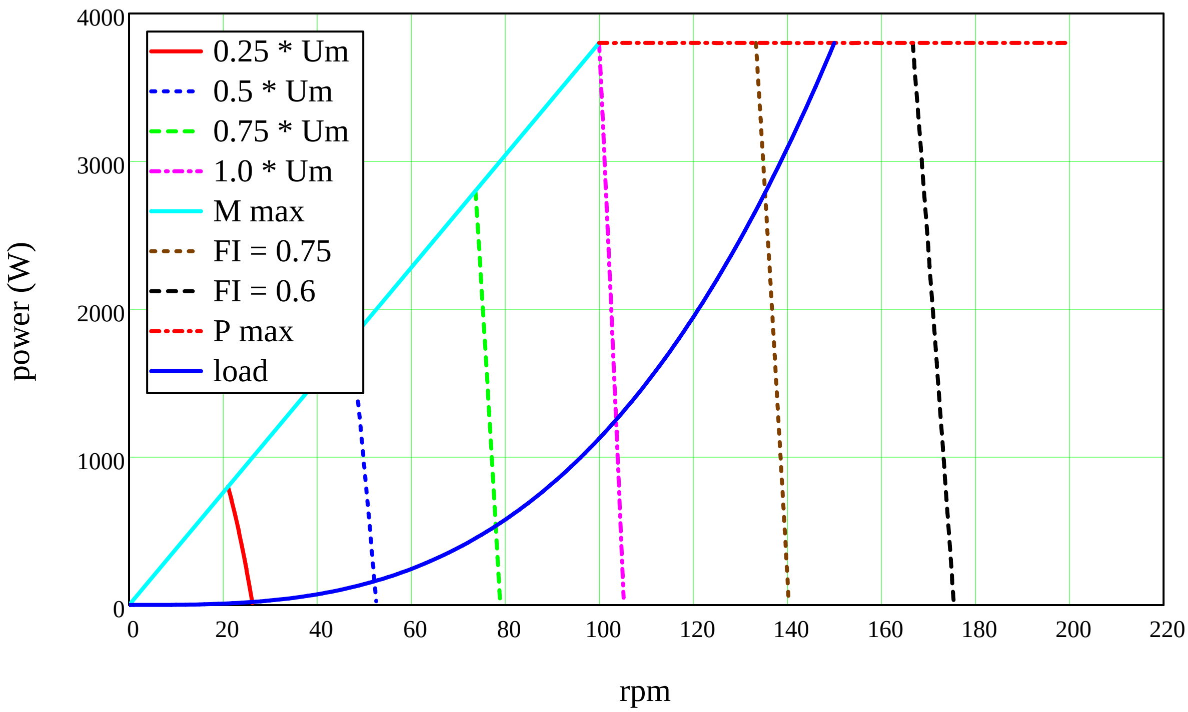
$MM(\Phi \cdot 0.6, n61) = 34.652\, N \cdot m$

$MM(\Phi \cdot 0.6, n62) = 0\, N \cdot m$



now if we plot this data in terms of power, $\text{power} = \text{torque} \cdot \text{rpm} \cdot 2 \cdot \pi$ and superimpose a cubic load curve reaching max power at 1.5 base rpm

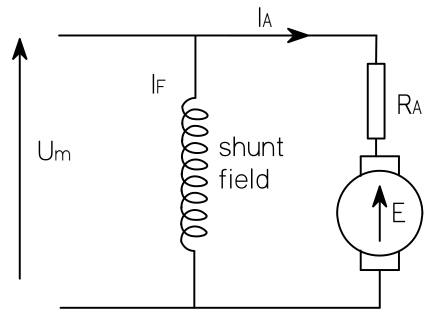
$$\text{load}(\text{nrpm}) := P_{max} \cdot \left(\frac{\text{nrpm}}{1.5 \cdot n}\right)^3 \qquad \text{nrpm} := 0\text{rpm}, 1\text{rpm} .. 1.5 \cdot n \qquad P_{max\_plot} := \begin{pmatrix} 0 & 0 \\ \frac{n}{\text{rpm}} & \frac{P_{max}}{W} \end{pmatrix}$$



So ... as observed in the text: "The operational envelopes show that a DC motor is very suited to drive a propeller for ship propulsion."

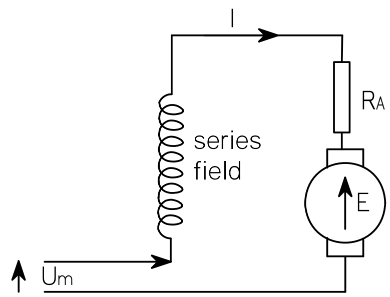
practical aspects ...

shunt motor: field in parallel with armature



operates same as separate excitation ...
field excitation constant

series motor: field in series with armature



now the field current and armature current is the same
using the same relationships from above ...

$M := K_m \cdot \Phi \cdot I$ (9.1)

$E = K_E \cdot \Phi \cdot n$ (9.2)

$U = E + I \cdot R$ (9.3)

$\Phi := K_F \cdot I$ $\qquad K_F = constant\_for\_given\_motor$ $\qquad I = current$ (9.7)

$M := K_m \cdot \Phi \cdot I \qquad M \rightarrow K_m \cdot K_F \cdot I^2 \qquad M := K_m \cdot K_F \cdot I^2$

eliminate I ... $E := K_E \cdot \Phi \cdot n \qquad E \rightarrow K_E \cdot K_F \cdot I \cdot n \qquad E := K_E \cdot K_F \cdot I \cdot n$

$$U = E + I \cdot R \qquad I := \frac{U - E}{R} \qquad I \rightarrow \frac{U - K_E \cdot K_F \cdot I \cdot n}{R} \qquad I = \frac{U - K_E \cdot K_F \cdot I \cdot n}{R}$$

$$I \cdot R = U - K_E \cdot K_F \cdot I \cdot n \qquad I \cdot \left(R + K_E \cdot K_F \cdot n\right) = U \qquad I = \frac{U}{R + K_E \cdot K_F \cdot n} \qquad I := \frac{U}{R + K_E \cdot K_F \cdot n}$$

$$M := K_m \cdot K_F \cdot I^2 \qquad M \rightarrow K_m \cdot K_F \cdot \frac{U^2}{\left(R + K_E \cdot K_F \cdot n\right)^2} \qquad M := K_m \cdot K_F \cdot \frac{U^2}{\left(R + K_E \cdot K_F \cdot n\right)^2} \qquad (9.8)$$

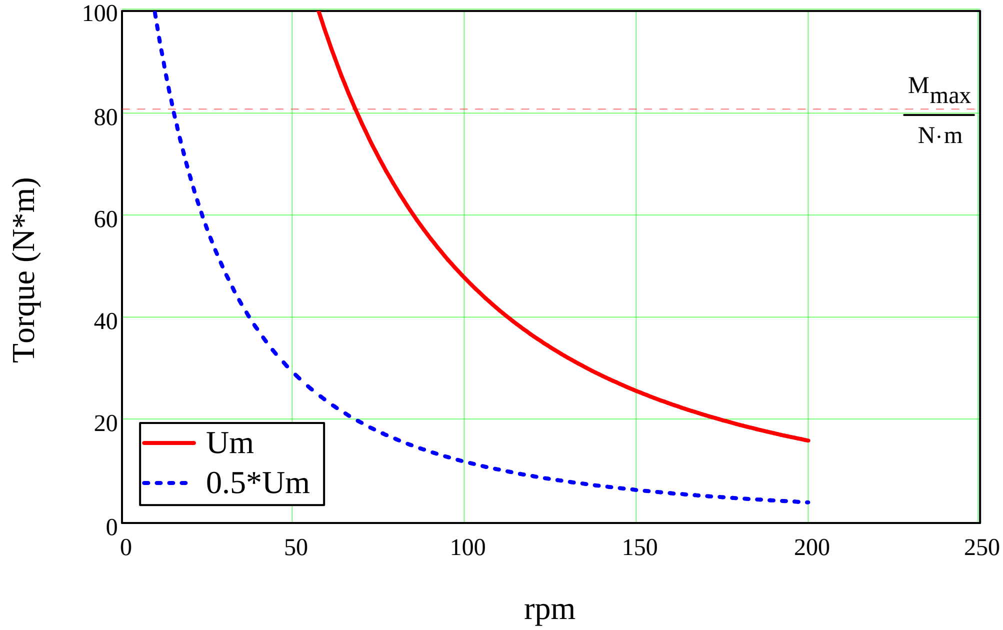
some numerical values for a plot ...

$U_m := 100V \quad K_E := 1 \qquad K_m := 1 \qquad R := 4\Omega \qquad K_F := 1\frac{Wb}{A} \qquad I_{max} := 9A$

$$M(U,n) := K_m \cdot K_F \cdot \frac{U^2}{\left(R + K_E \cdot K_F \cdot n\right)^2} \qquad M\left(U_m, 100rpm\right) = 47.747\,N \cdot m \qquad M_{max} := K_m \cdot K_F \cdot I_{max}^{\;2}$$

$M_{max} = 81\,N\cdot$

$n := 1rpm, 2rpm .. 200rpm$



motor suitable for traction purposes - high torque at low rpm

Induction motors (AC)

$I_F := 1$ $\omega := \frac{1}{s}$ $t := FRAME \cdot \frac{4 \cdot \pi}{100} \cdot sec$ t to go from 0 to 4*π in 100 steps

current sinusoidal (cos) with time

$I_a(t) := I_F \cdot \cos(\omega \cdot t)$ $I_b(t) := I_F \cdot \cos\left(\omega \cdot t - 2 \cdot \frac{\pi}{3}\right)$ $I_c(t) := I_F \cdot \cos\left(\omega \cdot t - 4 \cdot \frac{\pi}{3}\right)$

field vector displaced by 2*π/3 and 4*p/3, and current at appropriate phase shift applied

$Bz_a(t) := I_F \cdot \cos(\omega \cdot t)$

$Bz_b(t) := I_F \cdot \cos\left(\omega \cdot t - 2 \cdot \frac{\pi}{3}\right) \cdot \left(\cos\left(2 \cdot \frac{\pi}{3}\right) + \sin\left(2 \cdot \frac{\pi}{3}\right) \cdot i\right)$

$Bz_c(t) := I_F \cdot \cos\left(\omega \cdot t - 4 \cdot \frac{\pi}{3}\right) \cdot \left(\cos\left(4 \cdot \frac{\pi}{3}\right) + \sin\left(4 \cdot \frac{\pi}{3}\right) \cdot i\right)$

$I_F \cdot \cos(\omega \cdot t) = 1$

$I_F \cdot \cos\left(\omega \cdot t - 2 \cdot \frac{\pi}{3}\right) = -0.5$

$I_F \cdot \cos\left(\omega \cdot t - 4 \cdot \frac{\pi}{3}\right) = -0.5$

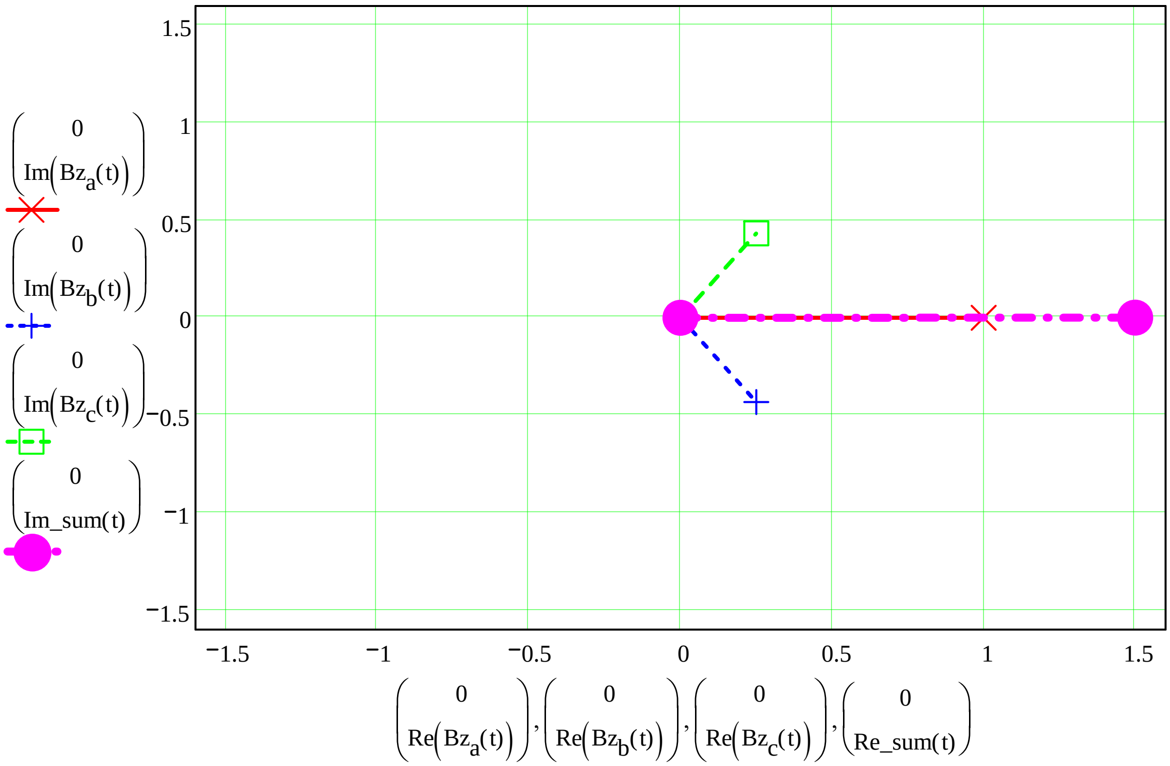
$Re\_sum(t) := Re(Bz_a(t)) + Re(Bz_b(t)) + Re(Bz_c(t))$

$Im\_sum(\,) := Im(Bz_a(t)) + Im(Bz_b(t)) + Im(Bz_c(t))$

$Re\_sum(t) = 1.5$

$Im\_sum(t) = 0$

$\sqrt{Re\_sum(t)^2 + Im\_sum(t)^2} = 1.5$



$t1 := \frac{\pi}{3}\text{sec}$

$I_F \cdot \cos(\omega \cdot t1) = 0.5$

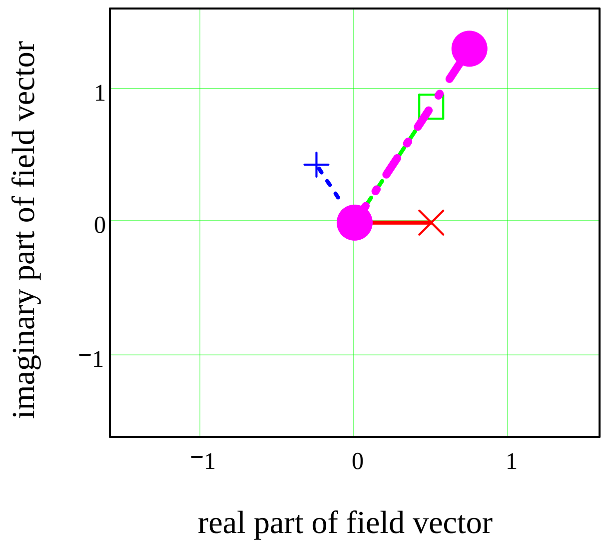
$\sqrt{\text{Re}(Bz_a(t1))^2 + \text{Im}(Bz_a(t1))^2} = 0.5$

$I_F \cdot \cos\left(\omega \cdot t1 - 2 \cdot \frac{\pi}{3}\right) = 0.5$

$\sqrt{\text{Re}(Bz_b(t1))^2 + \text{Im}(Bz_b(t1))^2} = 0.5$

$I_F \cdot \cos\left(\omega \cdot t1 - 4 \cdot \frac{\pi}{3}\right) = -1$

$\sqrt{\text{Re}(Bz_c(t1))^2 + \text{Im}(Bz_c(t1))^2} = 1$



speed of rotation of this machine = frequency of the supplied AC. as shown, there are two poles (one pair) N-S
with multiple pairs the speed of rotation is reduced proportional to the number of poles

with AC frequency ω and two poles $n_s = f = \frac{\omega}{2\cdot\pi}$ n_s = rotation_speed rpm

f = frequency Hz

ω = frequency $\frac{1}{s}$

$\mathrm{Hz} = 1\frac{1}{s}$ $\mathrm{Hz} = 9.549\,\mathrm{rpm}$

Hz assumes radians

$2\cdot\pi\cdot\mathrm{Hz} = 60\,\mathrm{rpm}$

for p poles $n_s = \frac{2\cdot f}{p}$ (9.10)

one stator winding ...

$E = -N\cdot\frac{d}{dt}\Phi$ (2.95) $\frac{d}{dt}\Phi = -\mathrm{constant}\cdot\Phi\cdot f$ => $E = \frac{\Phi\cdot f}{K_F}$

small ...

$U = I_F\cdot R + E$ $R < 1$ $U = E = \frac{\Phi\cdot f}{K_F}$ => $\Phi = K_F\cdot\frac{U}{f}$ K_F = constant_for_given_motor

now consider the rotor,if it is turning at the same speed as the rotating magnetic field of the stator, there is no EMF
the current induced in the rotor is strongly dependent on the *relative* speed

define ... $s = \frac{n_s - n}{n_s}$ (9.14) s = slip n_s = rotation_speed_stator n = rotation_speed_rotor

at low slip 0% to .. 10% $f_{R\_EMF} = n_s - n$ f_{R_EMF} = frequency_of_rotor_EMF (9.15)

because the rotor induced current will be at slip frequency, EMF is low so reactance (L) will be low. I_A depends primarily (only) on rotor (armature) resistance R_A and is in phase with the flux pattern. The net result is torque is ~ directly proportional to slip

$M = K\cdot s$ (9.16)

some typical curves from text for discussion

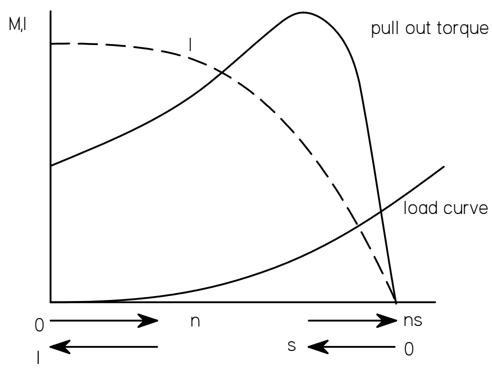


fig. 9.18 Torque - speed curve induction motor from Woud

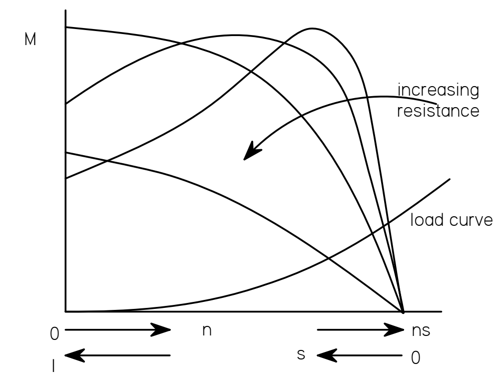


fig. 9.20A Torque - speed curve varying rotor resistance from Woud

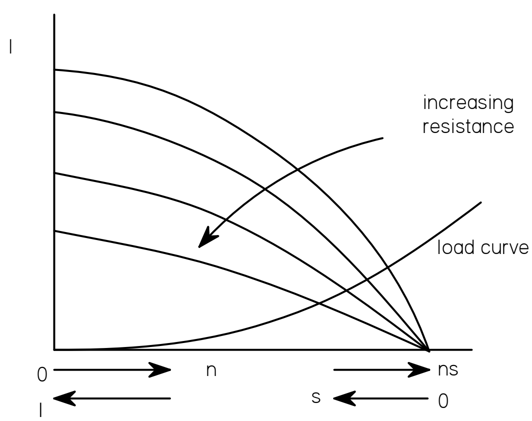


fig. 9.20B Current - speed curve varying rotor resistance from Woud

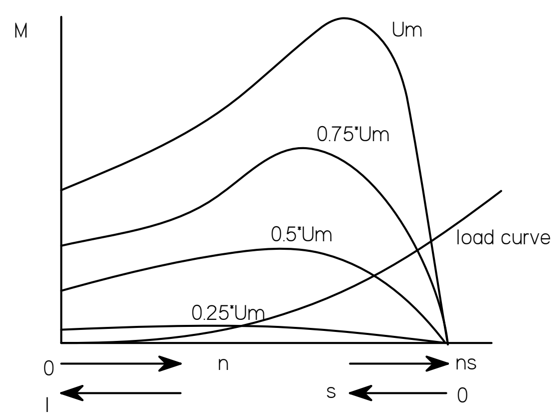


fig. 9.21 Torque - speed curve induction motor varying supply voltage, from Woud

starting has challenges and text reviews some alternatives

There is much more to this subject and text covers quite well. Next lecture will review ship applications.

One comment regarding opinion in text regarding DC motor drive. Dc drives not typical as commutation brushes require significant maintenance. DDX motor has innovative new brush technology.

see: **http://www.globalsecurity.org/military/library/report/2002/mil-02-04-wavelengths02.htm**