Polytropic Efficiency

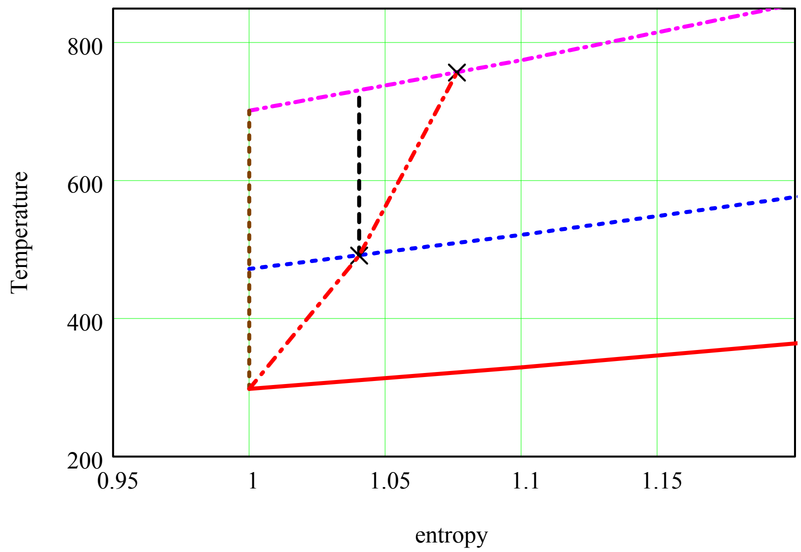
$kJ := 1000J$

consider a two stage compressor with a stage efficiency = 0.9

$p_1 := 1\,bar$ $\quad p_2 := 5\,bar$ $\quad p_3 := 20\,bar$ $\quad s_1 := 1\,\frac{kJ}{kg \cdot K}$ $\quad \eta_{stage} := 0.9$

calculations

the states resulting are plotted ...



using the gas laws and stage effiiciency = η_{stage}, after the first stage the states will be

$T_2 = 491.556\ K$ $\quad s_2 = 1.04\,\frac{kJ}{kg \cdot K}$ $\quad p_2 = 5\,bar$

after the second stage the states will be $\quad T_3 = 756.994\ K$ $\quad s_3 = 1.076\,\frac{kJ}{kg \cdot K}$ $\quad p_3 = 20\,bar$

now if we calculate efficiency of the compressor

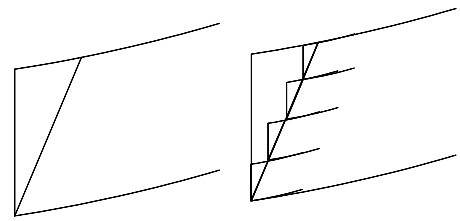
$$\eta_{comp} = \frac{\Delta T_s}{\Delta T} = \frac{T_{3s} - T_1}{T_3 - T_1} \qquad \frac{T_{3s} - T_1}{T_3 - T_1} = 0.88$$

as a check ... $\quad \frac{T_{33s} - T_2}{T_3 - T_2} = 0.9 \qquad \frac{T_{2s} - T_1}{T_2 - T_1} = 0.9$

This effect can be accounted for by using polytropic efficiency - or small stage efficiency.

reset variables T, s, p

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\Delta T_s}{\Delta T}$$



compressor

as the pressure ratio approaches unity

$$\frac{\Delta T_s}{\Delta T} = \frac{d}{dT}T_s \qquad \text{define ...} \qquad \eta_{pc} = \frac{d}{dT}T_s$$

=> $dT_s = \eta_{pc} \cdot dT$ for simple compressible substance $T \cdot ds = dh - v \cdot dp$ (7.7)

$$ds = \frac{dh}{T} - \frac{v}{T} \cdot dp = c_{po} \cdot \frac{dT}{T} - R \cdot \frac{dp}{p}$$ ideal gas and definition of c_{po}

isentropic ds = 0 $$c_{po} \cdot \frac{dT_s}{T} = R \cdot \frac{dp}{p}$$

substitute $dT_s = \eta_{pc} \cdot dT$ $$c_{po} \cdot \eta_{pc} \cdot \frac{dT}{T} = R \cdot \frac{dp}{p}$$ rearrange and integrate ..... $$\ln\left(\frac{T_2}{T_1}\right) = \frac{R}{c_{po} \cdot \eta_{pc}} \cdot \ln\left(\frac{p_2}{p_1}\right)$$

$$\gamma = \frac{c_{po}}{c_{vo}} \qquad c_{vo} := \frac{c_{po}}{\gamma} \qquad R := c_{po} - c_{vo} \qquad \frac{R}{\eta_{pc} \cdot c_{po}} \text{ simplify } \rightarrow \frac{\gamma - 1}{\gamma \cdot \eta_{pc}}$$

raise to exponents $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma \cdot \eta_{pc}}}$$ use in definition of η_c $$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\frac{T_{2s}}{T_1} - 1}{\frac{T_2}{T_1} - 1} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma \cdot \eta_{pc}}} - 1}$$

example polytropic efficiency = 0.9; calculate isentropic efficiency for p_2/p_1 = 2, 16, 30; use air as working fluid

$i := 0 .. 2$

$$\eta_{pc} := 0.9 \qquad \gamma := 1.4 \qquad r := \begin{pmatrix} 2 \\ 16 \\ 30 \end{pmatrix} \qquad \eta_{c_i} := \frac{\left(r_i\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(r_i\right)^{\frac{\gamma-1}{\gamma \cdot \eta_{pc}}} - 1} \qquad \eta_c = \begin{pmatrix} 0.89 \\ 0.856 \\ 0.845 \end{pmatrix}$$

if T1 were 25 deg C $T_1 := 25 + 273.15$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma \cdot \eta_{pc}}} \qquad T_2 := T_1 \cdot r^{\frac{\gamma-1}{\gamma \cdot \eta_{pc}}} \qquad T_2 = \begin{pmatrix} 371.535 \\ 718.944 \\ 877.733 \end{pmatrix}$$

whereas T2 calculated using η_{pc} as η_c $$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} \qquad T_{2s} := T_1 \cdot r^{\frac{\gamma-1}{\gamma}}$$

$$T_{2s} = \begin{pmatrix} 363.449 \\ 658.369 \\ 787.897 \end{pmatrix} \qquad T_2 := T_1 + \frac{T_{2s} - T_1}{\eta_{pc}} \qquad T_2 = \begin{pmatrix} 370.704 \\ 698.393 \\ 842.313 \end{pmatrix}$$

temperature above is higher indicating more energy required for compressor consistent with lower efficiency another observation ... if we say the two stages have a polytropic efficiency of 0.9 then using ...

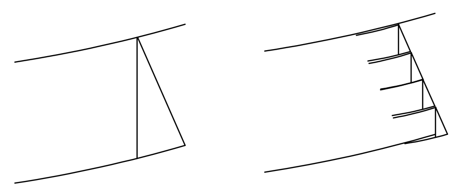
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}} \quad \text{and ...} \quad \frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}} \quad \Rightarrow \quad \frac{T_2}{T_1}\cdot\frac{T_3}{T_2} = \frac{T_3}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}}\cdot\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}}$$

$$\frac{T_3}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}}$$

the polytropic efficiency of the compressor is identical, i.e T_3 is determined from the polytropic efficiency that is the same as the two stages
since polytropic efficiency approaches isentropic efficiency for pressure ratio ~ 1, this is the same as saying that for a compressor with a large number of stages each with pressure ration near 1, the polytropic efficiency of the compressosor is isentropic efficiency of the individual stages

turbine would be similar with exception of inversion of relationship

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}} \quad \Rightarrow \quad \eta_{pt} = \frac{dT}{dT_s} \qquad dT_s = \frac{dT}{\eta_{pt}}$$



turbine

$$c_{po}\cdot\frac{dT_s}{T} = R\cdot\frac{dp}{p} \quad \Rightarrow \quad \frac{c_{po}}{\eta_{pt}}\cdot\frac{dT}{T} = R\cdot\frac{dp}{p} \quad \Rightarrow \quad \frac{dT}{T} = \frac{R\cdot\eta_{pt}}{c_{po}}\cdot\frac{dp}{p} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\eta_{pc}\cdot\frac{\gamma-1}{\gamma}}$$

could do by direct analogy if write

$$\frac{1}{\eta_t} = \frac{T_{4s} - T_3}{T_4 - T_3} \quad \Rightarrow \quad \begin{matrix} 4 = 2 \\ 3 = 1 \end{matrix} \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_{pc}}}$$

morphs to ...

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma\cdot\left(\frac{1}{\eta_{pt}}\right)}}$$

and ... $$\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\eta_{pt}\cdot\frac{\gamma-1}{\gamma}}$$

same example polytropic efficiency = 0.9; calculate isentropic efficiency for p_3/p_4 = 2, 16, 30; use air as working fluid

$i := 0..2$

$\eta_{pt} := 0.9 \qquad \gamma := 1.4 \qquad r := \begin{pmatrix} 2 \\ 16 \\ 30 \end{pmatrix}$

$$\eta_{t_i} := \frac{1 - \left(\frac{1}{r_i}\right)^{\eta_{pt}\cdot\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{1}{r_i}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \eta_t = \begin{pmatrix} 0.909 \\ 0.932 \\ 0.938 \end{pmatrix}$$

if T3 were 700 deg C $\quad T_3 := 700 + 273.15$

$$\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\eta_{pt}\cdot\frac{\gamma-1}{\gamma}} \qquad T_4 := T_3\cdot\left(\frac{1}{r}\right)^{\eta_{pt}\cdot\frac{\gamma-1}{\gamma}} \qquad T_4 = \begin{pmatrix} 814.277 \\ 477.034 \\ 405.834 \end{pmatrix}$$

whereas T_4 calculated using η_{pt} as η_t

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}} = \frac{T_4 - T_3}{T_{4s} - T_3} \qquad T_4 = T_3 + \eta_{pt}\cdot(T_{4s} - T_3) \qquad T_{4s} := T_3\cdot\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{4s} = \begin{pmatrix} 798.309 \\ 440.702 \\ 368.252 \end{pmatrix} \qquad T_4 := T_3 + \eta_{pt}\cdot(T_{4s} - T_3) \qquad T_4 = \begin{pmatrix} 815.793 \\ 493.947 \\ 428.742 \end{pmatrix}$$

above temperature is lower indicating more energy extracted from fluid, consistent with higher efficiency

direct approach for calculating T_2 modeling as discrete multiple stages. increasing number_of_stages should make η_{c_1} approach η_c (back to compressor for calculations)

$$\text{number\_of\_stages} := 4 \qquad j := 0..2 \qquad TT_{0,j} := 25 + 273.15 \qquad \gamma := 1.4 \qquad r = \begin{pmatrix} 2 \\ 16 \\ 30 \end{pmatrix}$$

$$\text{r\_per\_stage} := r^{\frac{1}{\text{number\_of\_stages}}} \qquad \text{r\_per\_stage}^T = (1.189 \quad 2 \quad 2.34) \qquad \text{power} := \frac{\gamma - 1}{\gamma}$$

temperature after each stage

$$TT_{1,ns} = TT_{0,ns} + \frac{\left(\text{r\_per\_stage}_j\right)^{\text{power}}\cdot TT_{0,j} - TT_{0,j}}{\eta_{pc}} \qquad \eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

$$n := 0..\text{number\_of\_stages} \qquad T_{2s} = T_1\cdot r^{\text{power}} \qquad TT_{n+1,j} := TT_{n,j} + \frac{\left(\text{r\_per\_stage}_j\right)^{\text{power}}\cdot TT_{n,j} - TT_{n,j}}{\eta_{pc}}$$

$$\eta_{c\_1_j} := \frac{TT_{0,j}\cdot\left(r_j\right)^{\text{power}} - TT_{0,j}}{TT_{\text{number\_of\_stages},j} - TT_{0,j}}$$

number_of_stages = 4	isentropic continuous model	10 stages	20 stages	50 stages
$\eta_{c\_1} = \begin{pmatrix} 0.892 \\ 0.869 \\ 0.862 \end{pmatrix}$	$\eta_c = \begin{pmatrix} 0.89 \\ 0.856 \\ 0.845 \end{pmatrix}$	$\eta_{c\_1} = \begin{pmatrix} 0.891 \\ 0.862 \\ 0.852 \end{pmatrix}$	$\eta_{c\_1} = \begin{pmatrix} 0.89 \\ 0.859 \\ 0.849 \end{pmatrix}$	$\eta_{c\_1} = \begin{pmatrix} 0.89 \\ 0.857 \\ 0.846 \end{pmatrix}$

further evidence that for a compressor with a large number of stages each with pressure ration near 1, the polytropic efficiency of the compressosor is isentropic efficiency of the individual stages. this follows through to determine isentropic efficiency for the compressor based on equating polytropic efficiency of small stages to the isentropic efficiency of the (small) stage.