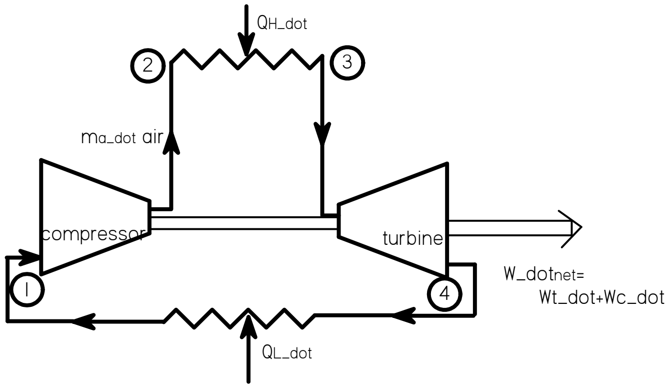
Brayton Cycle Summary

define some units

$kJ := 10^3 \cdot J$



Gas Turbine represented by air standard Brayton cycle

Brayton cycle consists of:
1-2 adiabatic compression
2-3 heat addition ~ constant pressure
3-4 adiabatic expansion in turbine
4-1 heat rejection ~ constant pressure

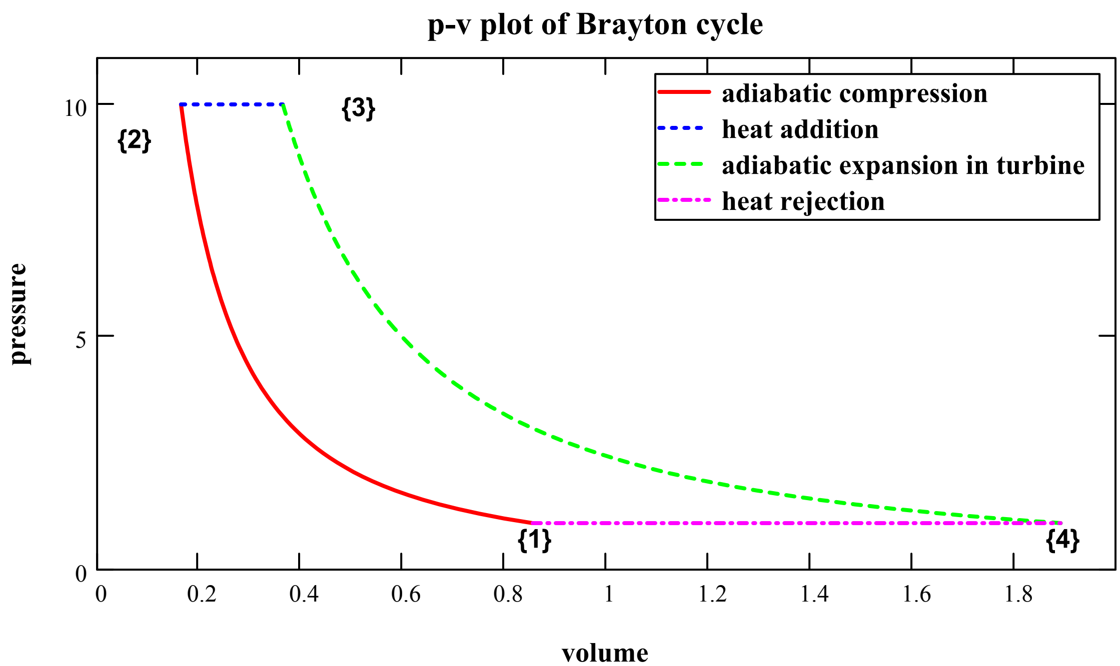
p-v and T - s plots for Brayton cycle shown below for reversible cycle. in irreversible cycle, $p_2 > p_3$ and $p_4 > p_1$, $s_2 > s_1$, $s_4 > s_3$

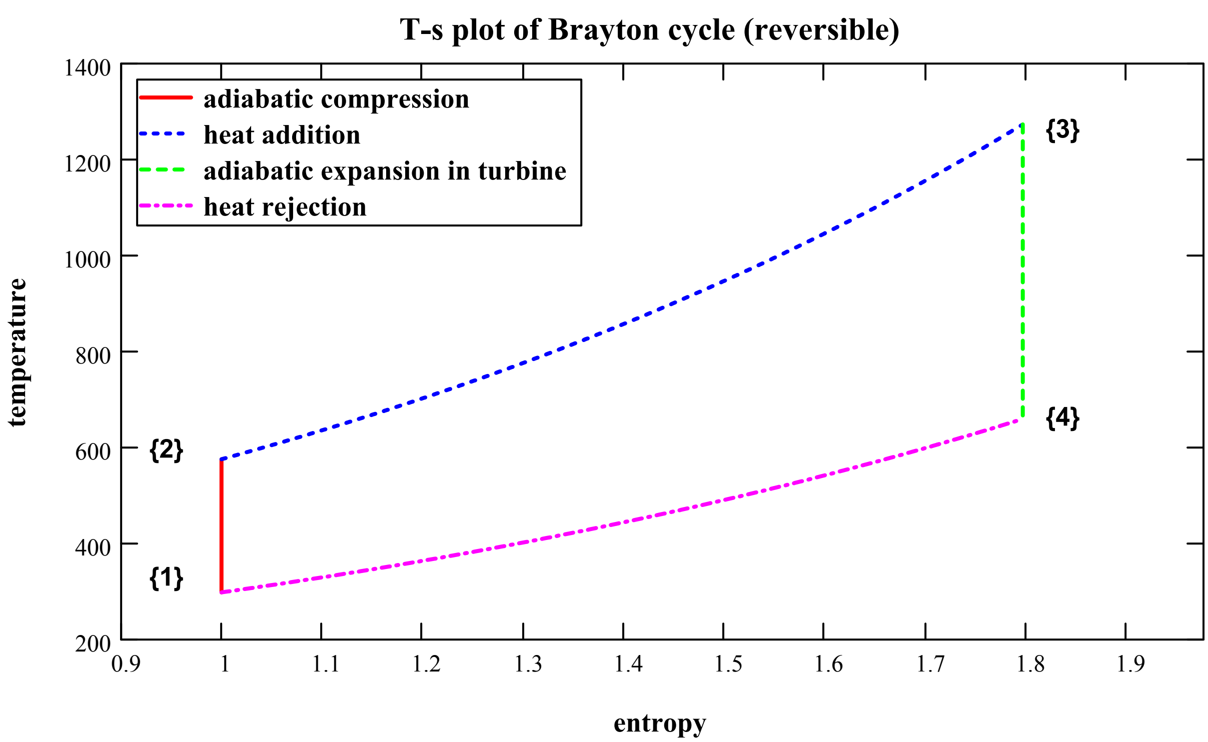
starting conditions $p_{1\_plot} := 1$ $T_{1\_plot} := 25 + 273.15$ $s_{1\_plot} := 1$

after compression $p_{2\_plot} := 10$

max temperature after heat addition $T_{3\_plot} := 1000 + 273.15$

calculations


p-v plot of Brayton cycle
adiabatic compression
heat addition
adiabatic expansion in turbine
heat rejection
{1}
{2}
{3}
{4}
pressure
volume
0
5
10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


T-s plot of Brayton cycle (reversible)
adiabatic compression
heat addition
adiabatic expansion in turbine
heat rejection
{1}
{2}
{3}
{4}
temperature
entropy
200
400
600
800
1000
1200
1400
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9

Ideal (reversible) basic Brayton cycle

compressor work $w_c = -(h_2 - h_1)$

turbine work $w_t = h_3 - h_4$

heat addition $q_H = h_3 - h_2$

heat rejection $q_L = -(h_4 - h_1)$

$$\eta_{th} = \frac{q_H + q_L}{q_H} = 1 + \frac{q_L}{q_H} = \frac{w_t + w_c}{q_H}$$

(2)

$$h_{4s} := C_p \cdot (T_{4s} - T_1) + h_1$$

$$h_3 := C_p \cdot (T_3 - T_{2s}) + h_{2s}$$

assuming perfect gas, constant specific heat.
h only a function of temperature; (5.23) VW &S, Joule's experiment shows u is f(T) only, pv = RT => h=f(T).

factor out T1 / T2s

(1)

$$\eta_{th} := 1 - \frac{h_{4s} - h_1}{h_3 - h_{2s}} \qquad \eta_{th} \rightarrow 1 - \frac{T_{4s} - T_1}{T_3 - T_{2s}} \qquad \eta_{th} := 1 - \frac{T_1}{T_{2s}} \cdot \frac{\frac{T_{4s}}{T_1} - 1}{\frac{T_3}{T_{2s}} - 1}$$

isentropic compression (and expansion) ...

$$\frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

this is reversible adiabatic process with ideal gas and constant specific heat

since $\frac{p_{2s}}{p_1} = \frac{p_3}{p_{4s}}$

$$\frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_{4s}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_{4s}} \quad \Rightarrow \quad \frac{T_{4s}}{T_1} = \frac{T_3}{T_{2s}}$$

$$\eta_{th} = 1 - \frac{T_1}{T_{2s}} = 1 - \left(\frac{p_1}{p_{2s}}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1}{\left(\frac{p_{2s}}{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}$$

$r = \text{pressure\_ratio}$

example; for 50 % efficiency, and some typical gas constants ...

$$\gamma := \begin{pmatrix} 1.29 \\ 1.4 \\ 1.67 \end{pmatrix}$$

CO2

air

monotonic gasses, He, Ar, Ne, He

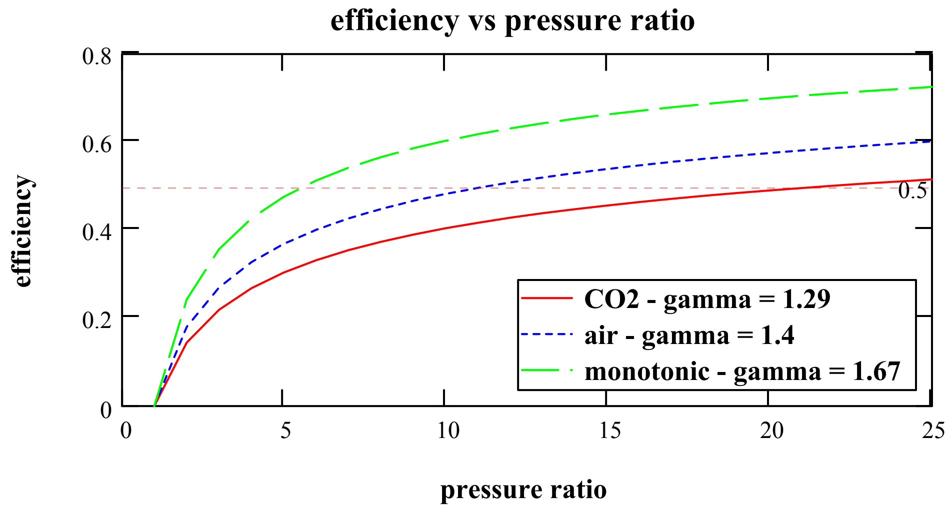
$$\eta_{th} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \qquad \frac{1}{r^{\frac{\gamma-1}{\gamma}}} = 1 - \eta_{th} \qquad r = (1 - \eta_{th})^{\frac{-\gamma}{\gamma-1}}$$

$\eta_{th} := 0.5$ $\quad i := 0 .. 2$

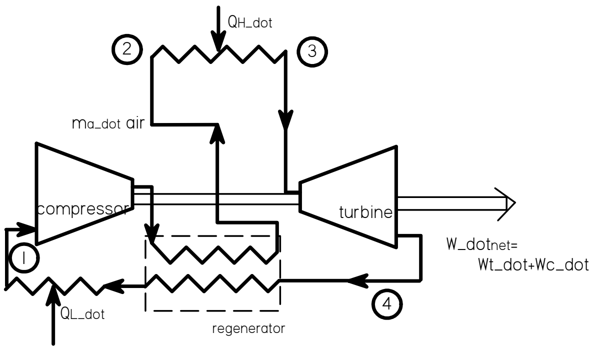
$$r_i := (1 - \eta_{th})^{\frac{-\gamma_i}{\gamma_i - 1}} \qquad r = \begin{pmatrix} 21.83 \\ 11.31 \\ 5.63 \end{pmatrix}$$

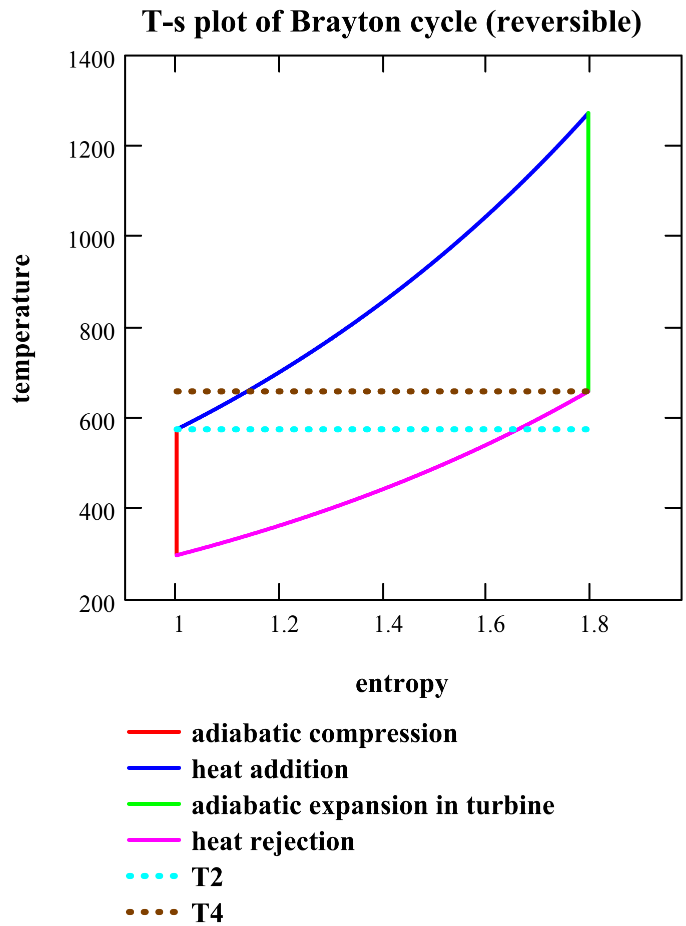
so for air as the working fluid, a pressure ratio of 11.3 will provide 0.5 isentropic efficiency effect of pressure ratio on isentropic efficiency

$$\eta_{th}(r,\gamma) := 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \qquad r := 0 .. 25 \qquad \gamma = \begin{pmatrix} 1.29 \\ 1.4 \\ 1.67 \end{pmatrix}$$



regeneration ...





$$\eta_{th} = \frac{w_{net}}{q_H} = \frac{w_t + w_c}{q_H}$$

$$q_H = c_p \cdot (T_3 - T_x)$$

$$w_t = c_p \cdot (T_3 - T_4)$$

Tx = temperature into regenerator
out of regenerator = T2

max when Tx = T4 then $w_t = q_H$

$$\eta_{th} = 1 + \frac{w_c}{w_t} = 1 - \frac{c_p \cdot (T_2 - T_1)}{c_p \cdot (T_3 - T_4)} = 1 - \frac{T_1 \cdot \left(\frac{T_2}{T_1} - 1\right)}{T_3 \cdot \left(1 - \frac{T_4}{T_3}\right)} = \frac{T_1 \cdot \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_3 \cdot \left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

as ... p1/p2 = p4/p3

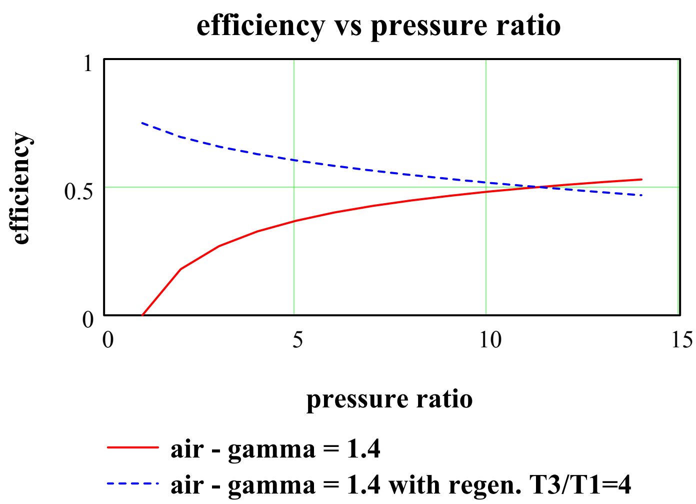
form is ... $\frac{a^b - 1}{1 - \frac{1}{a^b}} = \frac{a^b - 1}{\frac{a^b - 1}{a^b}} = a^b$

$$\eta_{th} = 1 - \frac{T_1}{T_3} \cdot \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Q.E.D.

for example, plot η_{th} vs pr for γ = 1.4 (air) with regeneration and T1/T3 = 0.25   figure 9.27

$\gamma := 1.4$ $r := 1 .. 14$ T1_over_T3 := 0.25 $\eta_{th\_reg}(r, \gamma, T1\_over\_T3) := 1 - T1\_over\_T3 \cdot r^{\frac{\gamma-1}{\gamma}}$



$r := 2$

solve for pressure ratio at intersection

Given

$$\eta_{th\_reg}(r, \gamma, T1\_over\_T3) = \eta_{th}(r, \gamma)$$

$r\_intersect := Find(r)$ $r\_intersect = 11.314$

say ... $T_1 := 300$ $T_3 := 1200$

at this pressure ratio

$$T_{2\_intersect} := T_1 \cdot r\_intersect^{\frac{\gamma - 1}{\gamma}}$$

$$T_{2\_intersect} = 600$$

$$T_{4\_intersect} := T_3 \cdot \left(\frac{1}{r\_intersect}\right)^{\frac{\gamma-1}{\gamma}} \qquad T_{4\_intersect} = 600$$

at the r_intersect the temperature out of the turbine matches the temperature out of the compressor, hence regeneration is infeasible

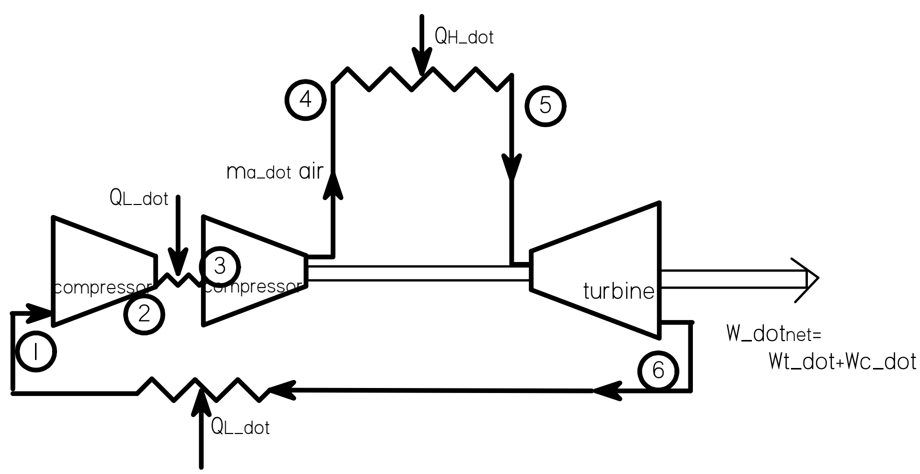
air-standard cycles ...
1. air as ideal gas is working fluid throughout cycle -no inlet or exhaust process
2. combustion process replaced by heat transfer process
3. cycle is completed by heat transfer to surroundings
4. all processes internally reversible
5. usually constant specific heat (page 311)

these are our assumptions for this analysis

reset variables

Intercooled Brayton cycle

figure later
T3 = low temperature from first intercooler, T4 second compressor. additional stages replicated at T3 and T4 which = T1 and T2 respectively. T5 is turbine inlet



example plot of intercooled Brayton cycle

parameters for plot. to retain states 2, 3 & 4 as previously defined two points 1a and 1b are inserted rather than renumbering. for intercooling, T_1 => T_{1a} => T_{1b} =>T_2

p_1 => p_{1a} => p_{1b} =>p_2
s_1 => s_{1a} => s_{1b} =>s_2

starting conditions $p_{1\_plot} := 1$ $T_{1\_plot} := 25 + 273.15$ $s_{1\_plot} := 1$

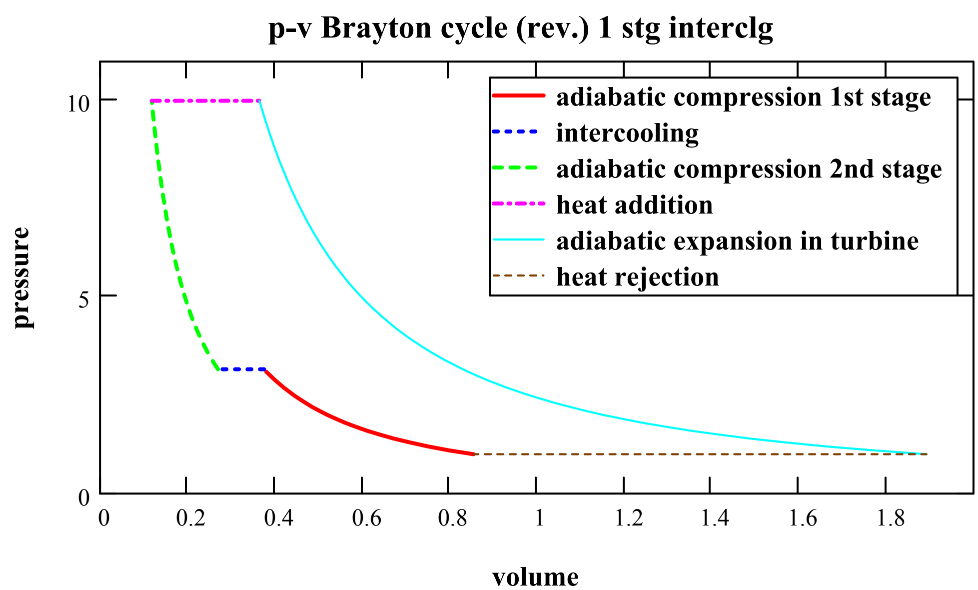
after first stage compression $p_{1a\_plot} := \sqrt{10}$

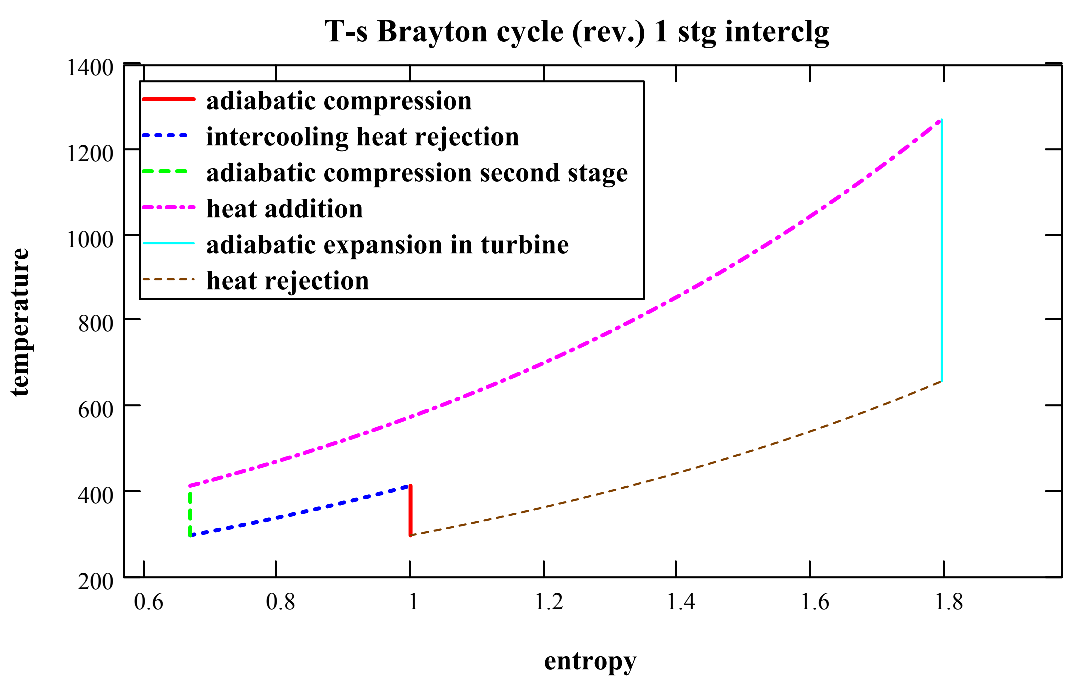
intercooler final temperature $T_{1b\_plot} := T_{1\_plot}$

after second stage compression $p_{2\_plot} := 10$

max temperature after heat addition $T_{3\_plot} := 1000 + 273.15$

calculations





$\gamma := 1.667$ for these calculations $T_1 := 300$ $T_5 := 1200$ maximum

$$\eta_{th\_ic} = 1 + \frac{Q_L}{Q_H}$$

as ... assume ... $T_4 := T_2$

taking advantage of constant c_{po}

observe .. $T_3 := T_1$

for all intercooled stages $pr := 1 .. 5$

$$\eta_{th\_ic} = 1 - \frac{T_6 - T_1 + N \cdot (T_2 - T_1)}{T_5 - T_2}$$

$N := 1$

$$power := \frac{\gamma - 1}{\gamma}$$

$$T_6(pr) := T_5 \cdot \left(\frac{1}{pr}\right)^{power}$$

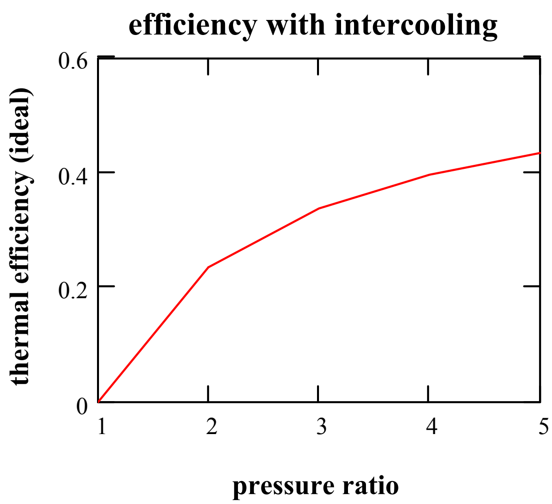
$$r_c(pr, N) := pr^{\frac{1}{N+1}}$$

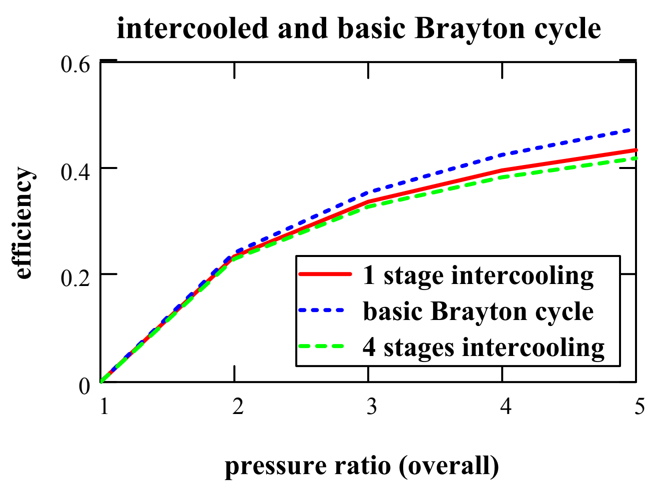
$$T_2(pr, N) := r_c(pr, N)^{power} \cdot T_1$$

$$\eta_{th\_ic}(pr, N) := 1 - \frac{T_6(pr) - T_1 + N \cdot (T_2(pr, N) - T_1)}{T_5 - T_2(pr\quad)}$$

$N = 1$ stages of intercooling $\gamma = 1.667$

1 and 4 stages of intercooling $\gamma = 1.667$



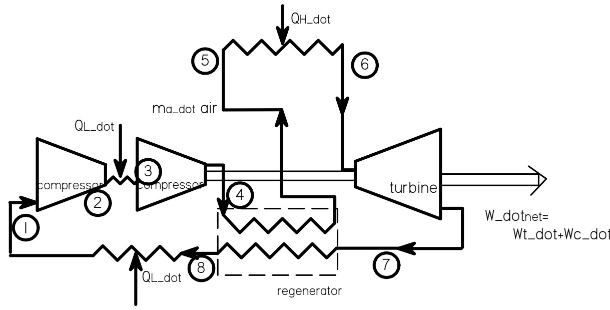


as we observed in class both T_H and T_L are lowered by intercooling. Intercooling (by itself) slightly reduces ideal efficiency. Increased number of stages doesn't reduce efficiency significantly further.

▶ reset variables

Intercooled Regenerative Brayton cycle

T3 = low temperature from first intercooler, T4 second compressor. additional stages replicated at T3 and T4 which = T1 and T2 respectively. T5 is turbine inlet



$\gamma := 1.667$ for these calculations $T_1 := 300$ $T_5 := 1200$ maximum

$\eta_{th\_ic} = 1 + \frac{Q_L}{Q_H}$ as ... assume ... $T_4 := T_2$ observe .. $T_3 := T_1$

taking advantage of constant c_{po} for all intercooled stages $pr := 1.01 .. 5.01$ start with 1+ as $\eta = 1$ mathematically

intercooled only from above

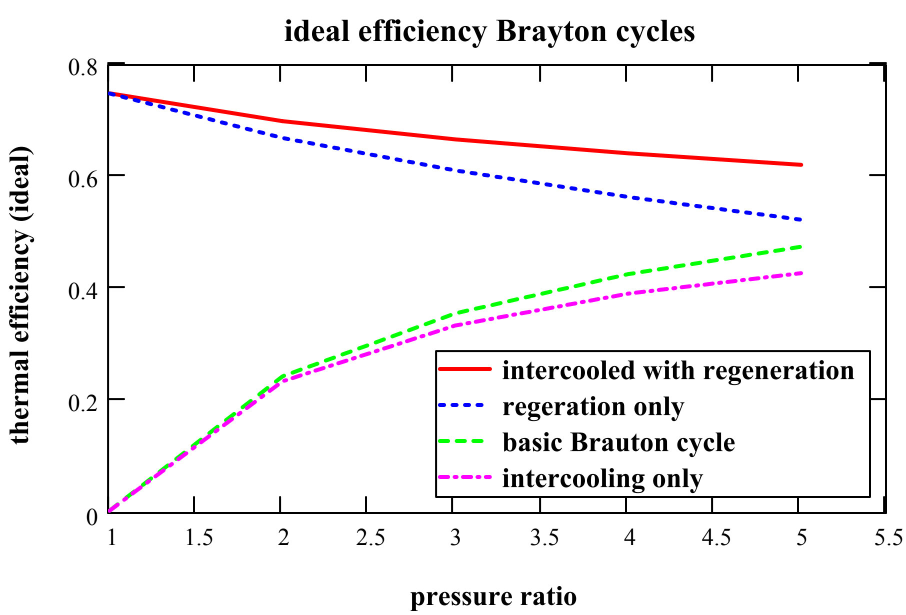
$\eta_{th\_ic} = 1 - \frac{T_6 - T_1 + N \cdot (T_2 - T_1)}{T_5 - T_2}$ with regeneration $Q_H = (T_5 - T_6)$ and ...initial stage of q_L is ... $T_2 - T_1$

so thermal efficiency becomes $\eta_{th\_ic\_reg} = 1 - \frac{T_2 - T_1 + N \cdot (T_2 - T_1)}{T_5 - T_6} = 1 - \frac{(N+1) \cdot (T_2 - T_1)}{T_5 - T_6}$

$power := \frac{\gamma - 1}{\gamma}$ $N := 2$ $T_6(pr) := T_5 \cdot \left(\frac{1}{pr}\right)^{power}$ $r_c(pr, N) := pr^{\frac{1}{N+1}}$

$T_2(pr, N) := r_c(pr, N)^{power} \cdot T_1$

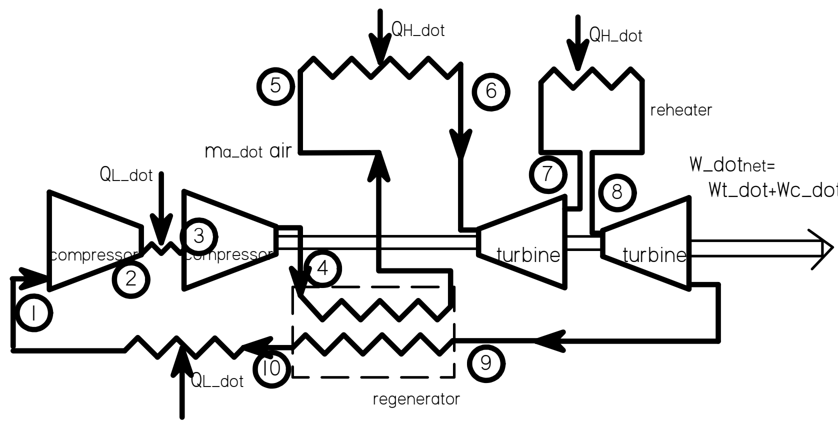
$\eta_{th\_ic\_reg}(pr, N) := 1 - \frac{(N+1) \cdot (T_2(pr, \ ) - T_1)}{T_5 - T_6(pr)}$



regeration was derived above leaving T1/T3 now renumbered to T1/T5 explicit. so variable T1/T5 inserted in arguments

▸ reset variables

intercooling, reheating and regenerative



example plot of intercooled Brayton cycle with reheat (and regeneration)

parameters for plot. to retain states 2, 3 & 4 as previously defined two points 1a and 1b are inserted rather than renumbering. for intercooling, T_1 => T_{1a} => T_{1b} =>T_2

p_1 => p_{1a} => p_{1b} =>p_2

s_1 => s_{1a} => s_{1b} =>s_2

for reheat return to $T_{3;}$ T_3 => T_{3a} => T_{3b} =>T_4

p_3 => p_{3a} => $p3_b$ =>p_4

s_3 => s_{3a} => s_{3b} =>s_4

starting conditions $p_{1\_plot} := 1$ $T_{1\_plot} := 25 + 273.15$ $s_{1\_plot} := 1$

after first stage compression $p_{1a\_plot} := \sqrt{10}$

intercooler final temperature $T_{1b\_plot} := T_{1\_plot}$

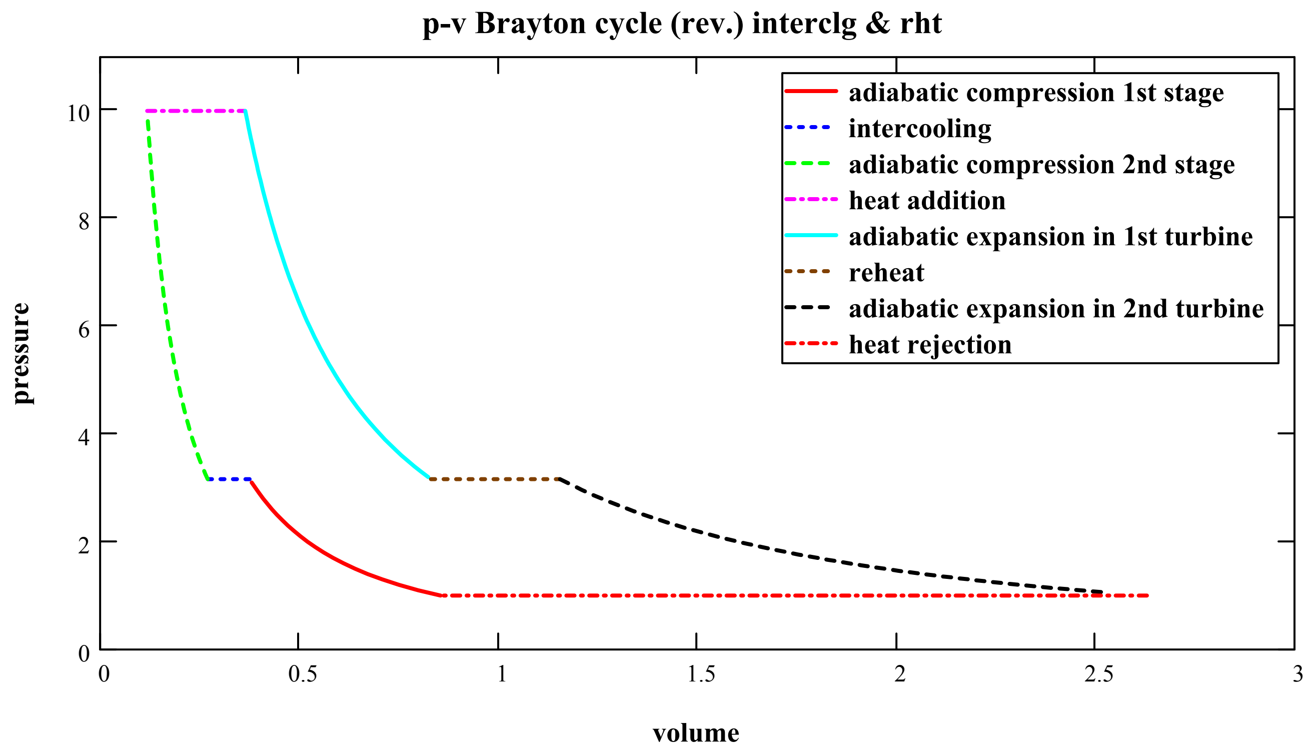
after second stage compression $p_{2\_plot} := 10$

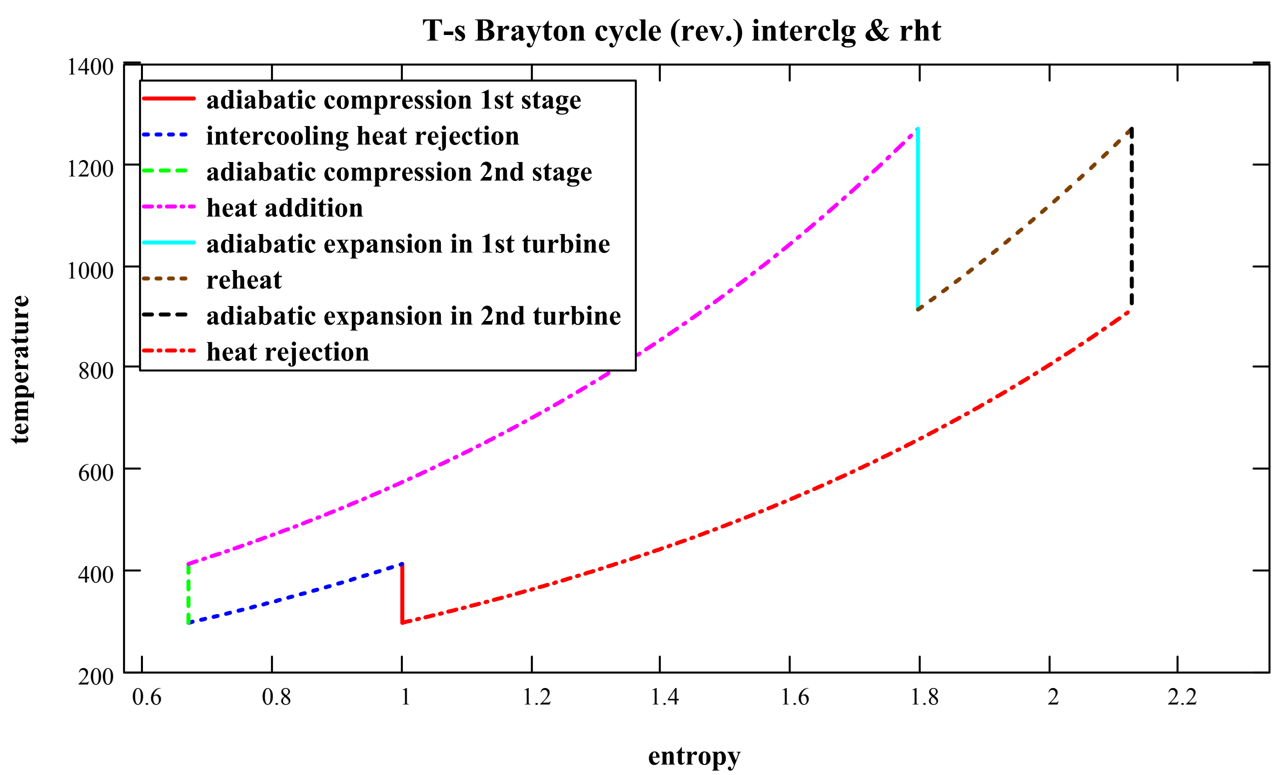
max temperature after heat addition $T_{3\_plot} := 1000 + 273.15$

after first turbine expansion $p_{3a\_plot} := \sqrt{10}$

max temperature after reheat addition $T_{3b\_plot} := 1000 + 273.15$

▸ calculations


p-v Brayton cycle (rev.) interclg & rht
adiabatic compression 1st stage
intercooling
adiabatic compression 2nd stage
heat addition
adiabatic expansion in 1st turbine
reheat
adiabatic expansion in 2nd turbine
heat rejection
pressure
volume
0
2
4
6
8
10
0
0.5
1
1.5
2
2.5
3


T-s Brayton cycle (rev.) interclg & rht
adiabatic compression 1st stage
intercooling heat rejection
adiabatic compression 2nd stage
heat addition
adiabatic expansion in 1st turbine
reheat
adiabatic expansion in 2nd turbine
heat rejection
temperature
entropy
200
400
600
800
1000
1200
1400
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2

$$\eta_{th\_ic\_reh\_reg} = 1 + \frac{Q_L}{Q_H}$$

$\gamma := 1.667$ for these calculations

$T_1 := 300$ $T_5 := 1200$ maximum

figure later
T5 inlet to turbine, stages of turbine are at T5 - T6 for all, for ease of calculations number of reheat and intercooling are the same so pressure ratios are identical

taking advantage of constant c_{po} as ...

assume ... $T_4 := T_2$

observe .. $T_3 := T_1$
for all intercooled stages

and upper and lower temperature for reheat are at T5 and T6

$$\eta_{th\_ic\_reh\_reg} = 1 - \frac{(N+1)\cdot(T_2 - T_1)}{(N+1)\cdot(T_5 - T_6)}$$

$N := 2$

$pr := 1.01 .. 5.01$

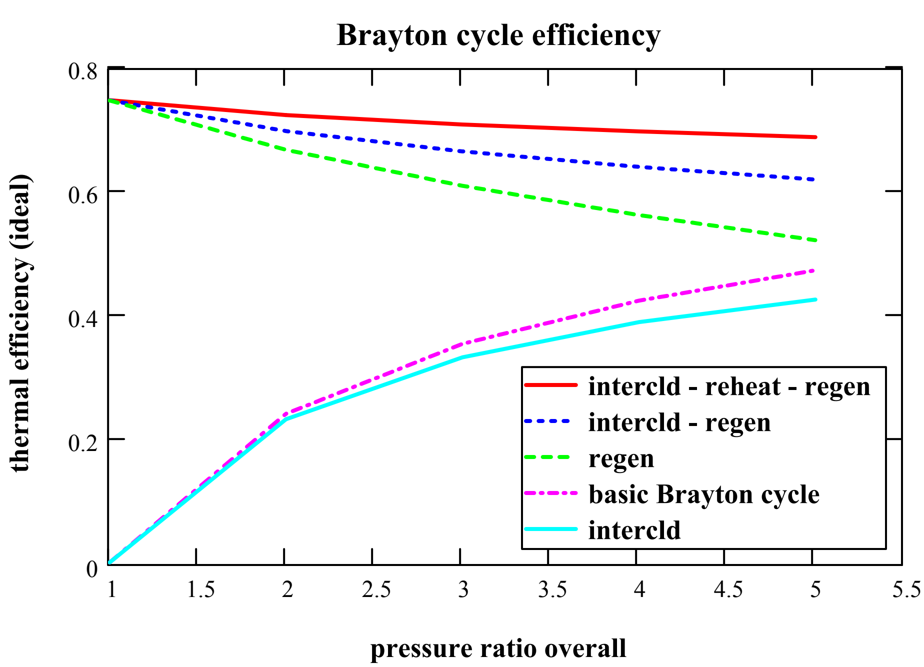
$power := \frac{\gamma - 1}{\gamma}$

$r_c(pr, N) := pr^{\frac{1}{N+1}}$

$T_2(pr, N) := r_c(pr, N)^{power} \cdot T_1$

$T_6(pr, N) := T_5 \cdot \left(\frac{1}{r_c(pr, \ )}\right)^{power}$

$$\eta_{th\_ic\_reh\_reg}(pr, N) := 1 - \frac{(N+1)\cdot(T_2(pr, N) - T_1)}{(N+1)\cdot(T_5 - T_6(pr, N))}$$



example plot of multiple intercooled Brayton cycle with multiple reheat (and regeneration

parameters for plot. to retain states 2, 3 & 4 as previously defined two points 1a and 1b are inserted rather than renumbering. for intercooling, T_1 => T_{1a} => T_{1b} =>T_2

p_1 => p_{1a} => p_{1b} =>p_2

s_1 => s_{1a} => s_{1b} =>s_2

for reheat return to $T_{3;}$ T_3 => T_{3a} => T_{3b} =>T_4

p_3 => p_{3a} => $p3_b$ =>p_4

s_3 => s_{3a} => s_{3b} =>s_4

starting conditions $p_{1\_plot} := 1$ $T_{1\_plot} := 25 + 273.15$ $s_{1\_plot} := 1$

pressure ratio $pr\_plot := 20$

number of compression stages ... $n\_comp := 4$

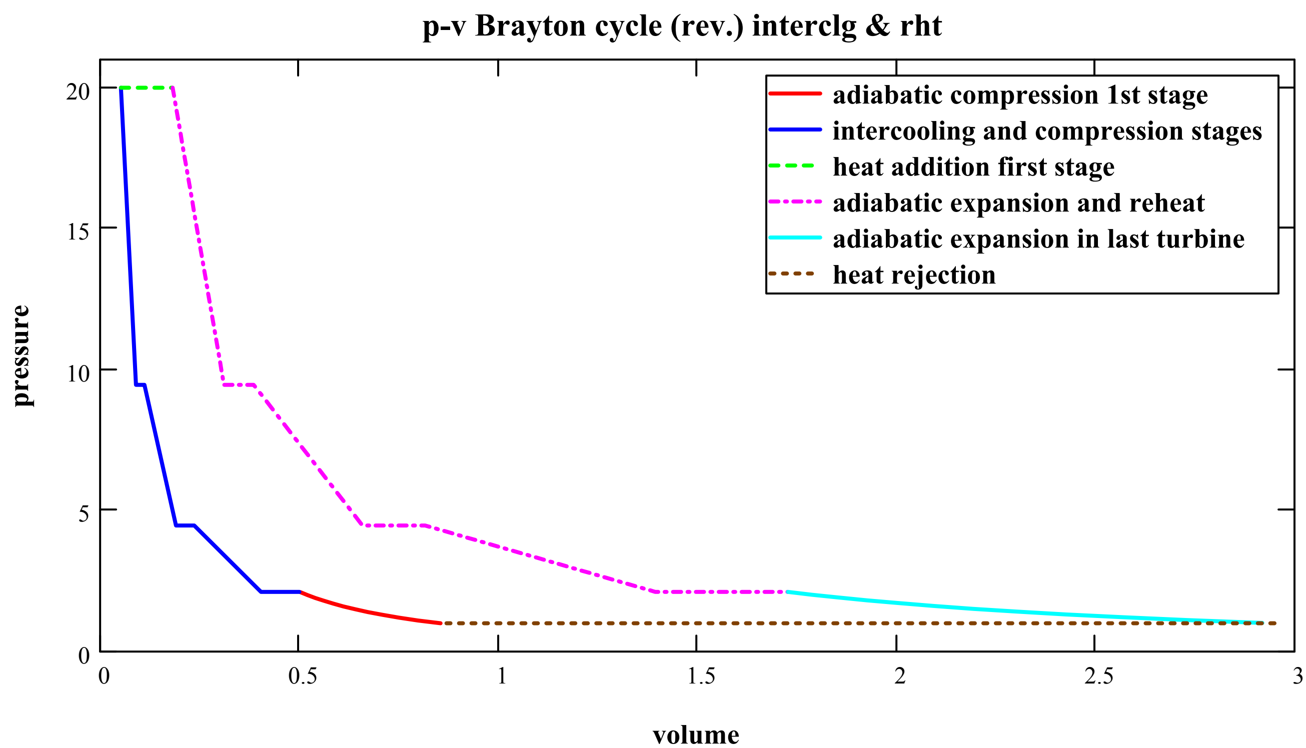
intercooler final temperature T_{1_plot}

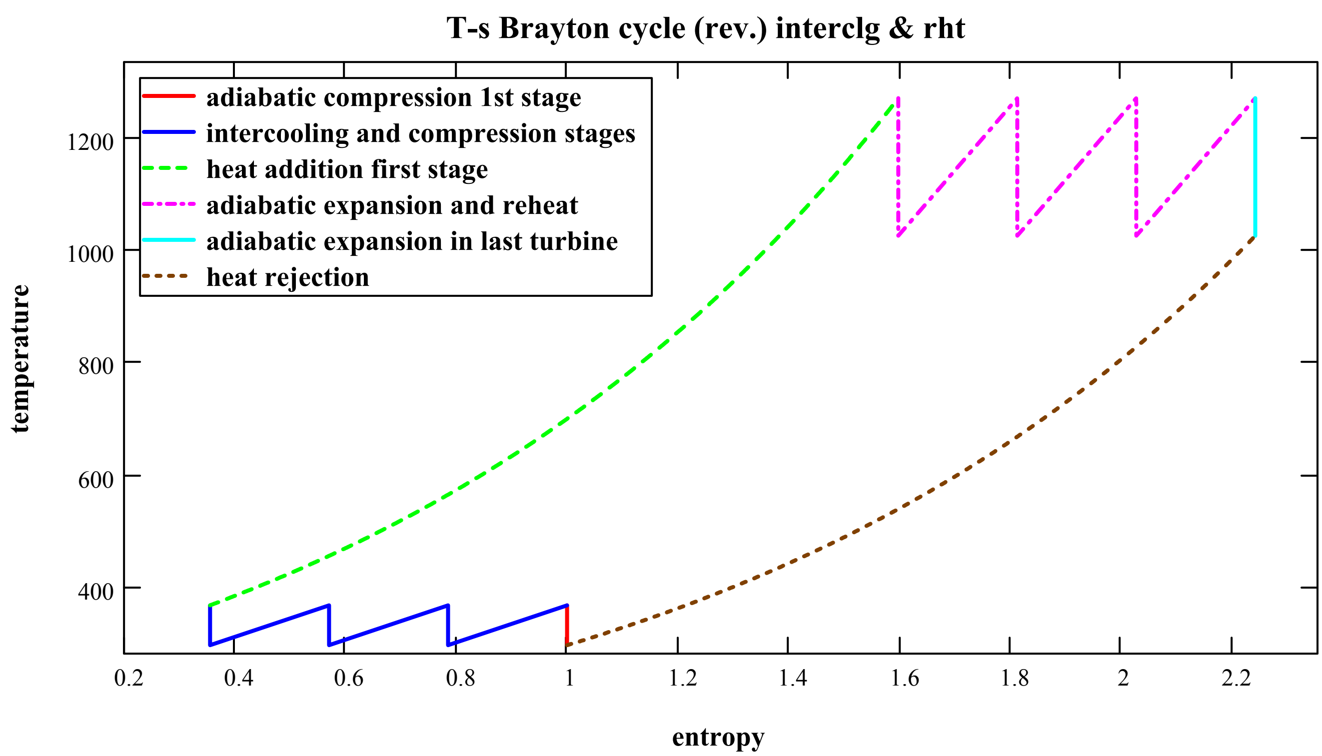
max temperature after heat addition $T_{3\_plot} := 1000 + 273.15$

number of expansion stages ... $n\_exp := 4$

max temperature after reheat addition T_{3_plot}

calculations


p-v Brayton cycle (rev.) interclg & rht
adiabatic compression 1st stage
intercooling and compression stages
heat addition first stage
adiabatic expansion and reheat
adiabatic expansion in last turbine
heat rejection
pressure
volume
0
5
10
15
20
0.5
1
1.5
2
2.5
3


T-s Brayton cycle (rev.) interclg & rht
adiabatic compression 1st stage
intercooling and compression stages
heat addition first stage
adiabatic expansion and reheat
adiabatic expansion in last turbine
heat rejection
temperature
entropy
400
600
800
1000
1200
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2

as number of reheat and intercooled stages increases, ideal efficiency should approach Carnot

$\eta_{th\_carnot} := 1 - \frac{T_1}{T_5}$ $N := 1 .. 20$ $pr := 5$

this calculation fixes pressure ratio overall = 5 and looks at variation with number of stages of intercooling and reheat (same)