comparison of rankine with single regeneration

$kPa := 10^3 \cdot Pa$ $MPa := 10^6 Pa$

basic reference problem ...
Rankine_class_example.mcd

$kN := 10^3 \cdot N$ $kJ := 10^3 \cdot J$ $bar := 0.1MPa$

state 1: condenser outlet

same as reference

Table 1 or Table A.1.1 $T_1 := 40$ $p_1 := 7.384kPa$ $v_{f\_1} := 0.0010078 \frac{m^3}{kg}$ $v_1 := v_{f\_1}$

$s_{f\_1} := 0.5725 \frac{kJ}{kg \cdot K}$ $s_1 := s_{f\_1}$ $s_{fg\_1} := 7.6845 \frac{kJ}{kg \cdot K}$ $h_{f\_1} := 167.57 \frac{kJ}{kg}$ $h_{fg\_1} := 2406.7 \frac{kJ}{kg}$ $h_1 := h_{f\_1}$

state 2: pump outlet

assume $v_f = v_1$ constant, isentropic, ds = 0 =>T*ds= 0 => h2 = h1+v1*dp from relationships Tds = dh + v*dp integrated with constant v and Tds = 0

$p_2 := 30bar$ $s_2 := s_1$ $h_2 := h_1 + v_1 \cdot (p_2 - p_1)$ $h_2 = 170.586 \frac{kJ}{kg}$

$w_p := h_1 - h_2$ using C_p $Cp := 4.184 \frac{kJ}{kg} \frac{kJ}{kg \cdot K}$ actual units

$w_p = -3.016 \frac{kJ}{kg}$ and ... eqn 5.18 $h_2 - h_1 = Cp \cdot (T_2 - T_1)$ $T_2 := T_1 + \frac{h_2 - h_1}{Cp}$ $T_2 = 40.721$

state 3: boiler outlet

$p_3 := p_2$ $T_3 := 460$ $p_3 = 3\,MPa$

▸ interpolation

from interpolation Table A.1.3 P=3MPa page 622 $h_3 = 3366.5 \frac{kJ}{kg}$ $s_3 = 7.113 \frac{kJ}{kg \cdot K}$

state 4: turbine outlet

isentropic expansion to 40 deg C
determine h_4 from x

$s_4 := s_3$

$s_4 = s_{f\_1} + x \cdot s_{fg\_1}$ => $x := \frac{s_4 - s_{f\_1}}{s_{fg\_1}}$ $x = 0.851$

$h_4 := h_{f\_1} + h_{fg\_1} \cdot x$ $h_4 = 2216 \frac{kJ}{kg}$ $w_t := h_3 - h_4$ $w_t = 1150 \frac{kJ}{kg}$

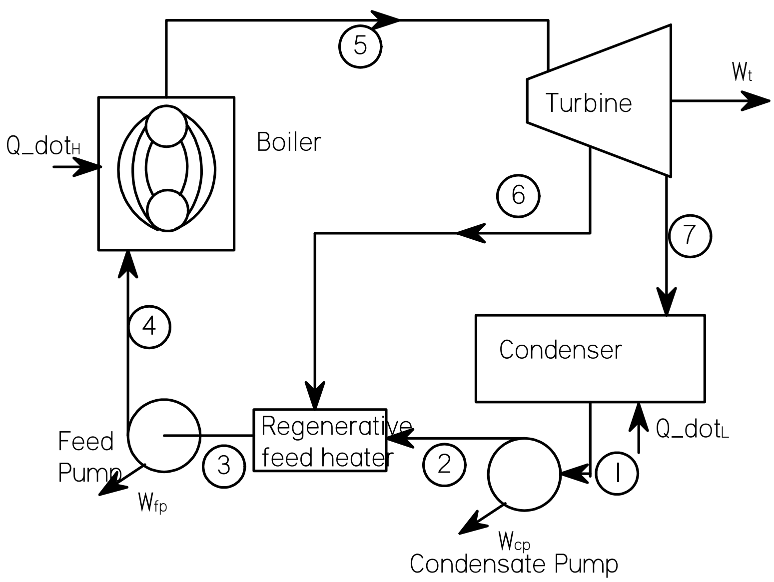
thermal efficiency

$$\eta_{th} = \frac{work\_net}{Q_H} = \frac{Q_H + Q_L}{Q_H} = \frac{w_t + w_p}{Q_H} = \frac{(h_3 - h_4) + (h_1 - h_2)}{h_3 - h_2}$$

$\eta_{th} := \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$ $\eta_{th} = 0.359$ $\eta_{th} := \frac{w_t + w_p}{h_3 - h_2}$ $\eta_{th} = 0.359$

$Q_H := h_3 - h_2$ $Q_L := h_1 - h_4$ $\eta_{th\_1} := \frac{Q_H + Q_L}{Q_H}$ $\eta_{th\_1} = 0.359$

same cycle with regeneration ***extract steam at 400kPa (4 bar) from turbine to mix with condensate to become saturated liquid at 400kPa***



1 - vacuum; $(1\text{-}m_1)$ saturated liquid T=40 C

2 - sub cooled liquid at feed heater pressure P=400 kPa

3 - saturated liquid at 400 kPa

4 - sub cooled liquid at boiler pressure P=3 MPa

5 - superheated vapor T=460 C

6 - (m_1) superheated vapor @ 400 kPa or ... vapor + liquid @ saturation temperature and pressure tbd

7 - $(1\text{-}m_1)$ vapor + liquid @ saturation temperature and pressure

state 1: condenser outlet $(1\text{-}m_1)$

the state values are identical to the reference, however the fraction of total mass flow is less = $1\text{-}m_1$

Table 1 or Table A.1.1

$T_1 := 40$ $\quad p_1 := 7.384\text{kPa}$ $\quad v_{f\_1} := 0.0010078\,\frac{m^3}{kg}$

$s_{f\_1} := 0.5725\,\frac{kJ}{kg\cdot K}$ $\quad s_1 := s_{f\_1}$ $\quad s_{fg\_1} := 7.6845\,\frac{kJ}{kg\cdot K}$ $\quad h_{f\_1} := 167.57\,\frac{kJ}{kg}$ $\quad h_{fg\_1} := 2406.7\,\frac{kJ}{kg}$ $\quad h_1 := h_{f\_1}$

state 2: condensate pump outlet $(1\text{-}m_1)$

assume v_f constant, isentropic, ds = 0 =>T*ds= 0 => $h_2 = h_1+v_1*dp$ from relationships Tds = dh + v*dp integrated with constant v and Tds = 0

$p_2 := 4\text{bar}$ $\quad s_2 := s_1$ $\quad h_2 := h_{f\_1} + v_{f\_1}\cdot(p_2 - p_1)$ $\quad h_2 = 167.966\,\frac{kJ}{kg}$

$w_{cp} := h_{f\_1} - h_2$ $\quad w_{cp} = -0.396\,\frac{kJ}{kg}$ $\quad T_2 := T_1 + \frac{h_2 - h_1}{Cp}$ $\quad T_2 = 40.095$

state 3: regeneration out

mass rate = 1; saturated liquid at p_feed = 4bar

$p_3 := p_2$ $\quad T_3 := 143.63$ $\quad h_3 := 604.74\,\frac{kJ}{kg}$ $\quad s_3 := 1.7766\,\frac{kJ}{kg\cdot K}$

$p_2 = 400\,\text{kPa}$

$v_3 := 1.0836\cdot 10^{-3}\,\frac{m^3}{kg}$ $\quad h_{fg\_3} := 2133.9\,\frac{kJ}{kg}$ $\quad s_{fg\_3} := 5.1193\,\frac{kJ}{kg\cdot K}$

state 4: feed pump out

$p_4 := 3MPa$ $h_4 := h_3 + v_3 \cdot (p_4 - p_3)$ $h_4 = 607.557 \frac{kJ}{kg}$ $s_4 := s_3$

$w_{fp} := h_3 - h_4$ $w_{fp} = -2.817 \frac{kJ}{kg}$ $T_4 := T_3 + \frac{h_4 - h_3}{Cp}$ $T_4 = 144.303$

state 5: boiler outlet

$T_5 := 460$ Table 2 or interpolate as above

$p_5 := p_4$ $p_5 = 3\,MPa$ $h_5 := 3366.6 \frac{kJ}{kg}$ $s_5 := 7.1144 \frac{kJ}{kg \cdot K}$

state 6: turbine outlet #1, partial flow (m_1)

expansion to p3 with same Δs as regeneration heating determine h4 from x

$p_6 := p_3$ $s_6 := s_5$

$s_6 = s_3 + x_6 \cdot s_{fg\_3}$ => $x_6 := \frac{s_6 - s_3}{s_{fg\_3}}$ $x_6 = 1.043$

$T_6 := T_3$

new!! ... mass fraction has to be > 1 for heat balance to work. this says extraction steam is superheated. x relationship doesn't apply. need to interpolate in superheated region

$p = 4bar = 400kPa$ $s_6 = 7.114 \frac{1}{K} \frac{kJ}{kg}$ find T and h by interpolation

▸ interpolation

$h_6 := 2834.491 \frac{kJ}{kg}$ $T_6 := 187.702$

state 7: turbine outlet #2, partial flow ($1-m_1$)

expansion to p3 with same Δs as regeneration heating; determine h7 from x

$s_7 := s_5$ $T_7 := T_1$

$s_7 = s_1 + x \cdot s_{fg\_1}$ => $x_7 := \frac{s_7 - s_1}{s_{fg\_1}}$ $x_7 = 0.851$

$h_7 := h_1 + h_{fg\_1} \cdot x_7$ $h_7 = 2216.42 \frac{kJ}{kg}$

now we can do the turbine

flow through 6 = m_1 treat like x; combination of m_1 at h6 and ($1-m_1$) at $h_2 = h_3$ for heat balance out of feed heater

$m_1 \cdot h_6 + (1 - m_1) \cdot h_2 = h_3$ $m_1 = \frac{h_3 - h_2}{h_6 - h_2}$ $m_1 := \frac{h_3 - h_2}{h_6 - h_2}$ $m_1 = 0.164$

$w_t := h_5 - h_6 + (1 - m_1) \cdot (h_6 - h_7)$ $w_t = 1048.94 \frac{kJ}{kg}$

thermal efficiency

$\eta_{th} := \frac{w_t + w_{cp} \cdot (1 - m_1) + w_{fp}}{h_5 - h_4}$ $\eta_{th} = 0.379$

$w_{fp} = -2817.36\,Sv$ $\eta_{th\_1} = 0.359$

▸ data for plots

Thermal efficiency using entropy average temperature approach

$T_5 = 460$

$h_5 = 3366.6 \frac{kJ}{kg}$ $\quad h_4 = 607.557 \frac{kJ}{kg}$ $\quad T_1 = 40$

$s_6 = 7.114 \frac{kJ}{kg \cdot K}$ $\quad s_4 = 1.777 \frac{kJ}{kg \cdot K}$ $\quad s_1 = 0.572 \frac{1}{K} \frac{kJ}{kg}$

we need to redefine efficiency for t average calculations

$\eta_{th} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$ $\quad Q_H = m_H \cdot \int T\,ds$ $\quad T\_bar_H = \frac{Q_H}{m_H \cdot (\Delta s_H)} = \frac{m_H \cdot (\Delta h_H)}{m_H \cdot (\Delta s_H)} = \frac{\Delta h_H}{\Delta s_H}$

and ... $\quad T\_bar_H \cdot m_H \cdot (\Delta s_H) = Q_H$

$T\_bar_H := \frac{h_5 - h_4}{s_5 - s_4}$ $\quad T\_bar_H = 516.888\,K$ $\quad m_H := 1$

$Q_L = m_L \cdot \int T\,ds$ $\quad$ T constant $\quad T\_bar_L := (T_1 + 273.15) \cdot K$ $\quad T\_bar_L = 313.15\,K$

$T\_bar_L \cdot m_L \cdot (\Delta s_L) = Q_L$

but ... $\quad (\Delta s_H = s_5 - s_4) \neq (\Delta s_L = s_7 - s_1)$ $\quad$ as ... $\quad s_5 = s_6 = s_7$ $\quad$ but ... $\quad s_4 \neq s_1$

and when inserting into Q η relationship have to put in m_1 values ... $\quad \eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T\_bar_L \cdot m_L \cdot (\Delta s_L)}{T\_bar_H \cdot m_H \cdot (\Delta s_H)}$

$\eta_{t\_avg\_1} := 1 - \frac{T\_bar_L \cdot (1 - m_1) \cdot (s_7 - s_1)}{T\_bar_H \cdot (s_6 - s_3)}$ $\quad \eta_{t\_avg\_1} = 0.379$ $\quad$ matches as expected

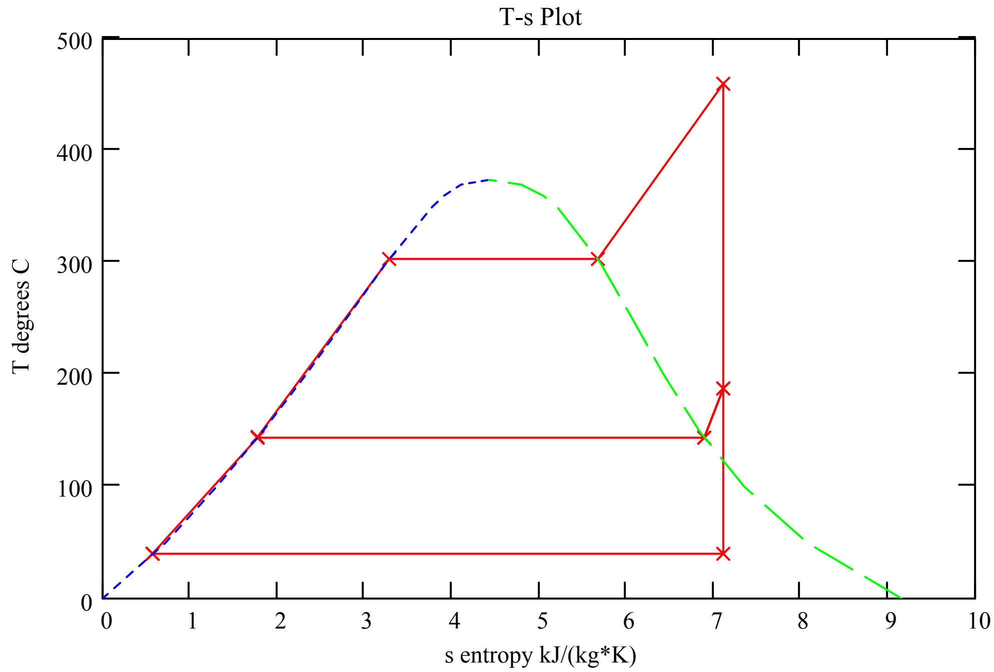
suppose we didn't allow for the difference in Δs and mass flow and just said

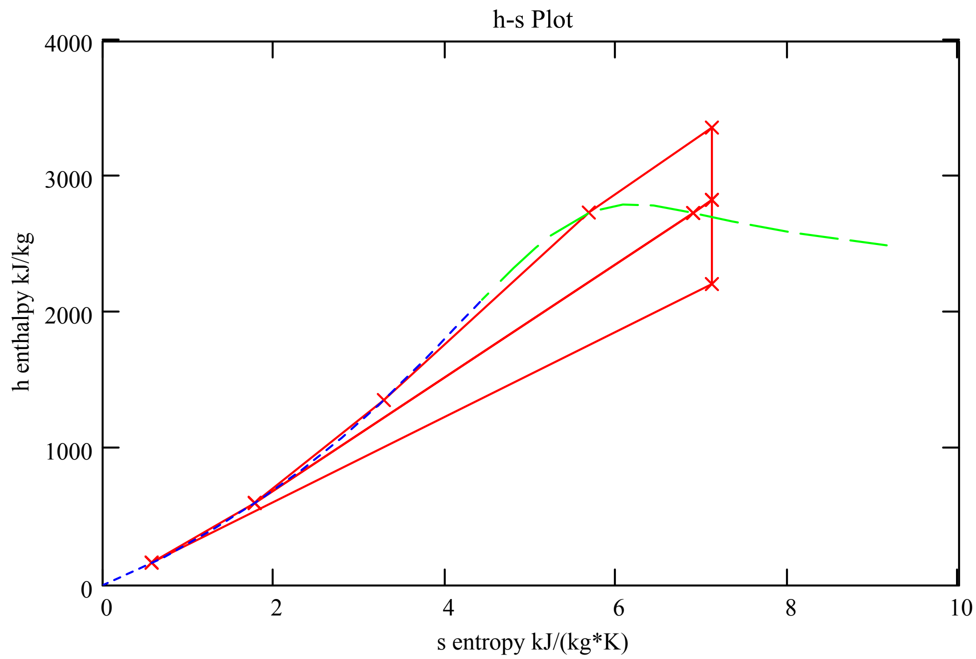
$\eta_{t\_avg\_2} := 1 - \frac{T\_bar_L}{T\_bar_H}$ $\quad \eta_{t\_avg\_2} = 0.394$

precise T avg	estimated T avg	actual regenerative	reference
$\eta_{t\_avg\_1} = 0.379$	$\eta_{t\_avg\_2} = 0.394$	$\eta_{th} = 0.379$	$\eta_{th\_1} = 0.359$

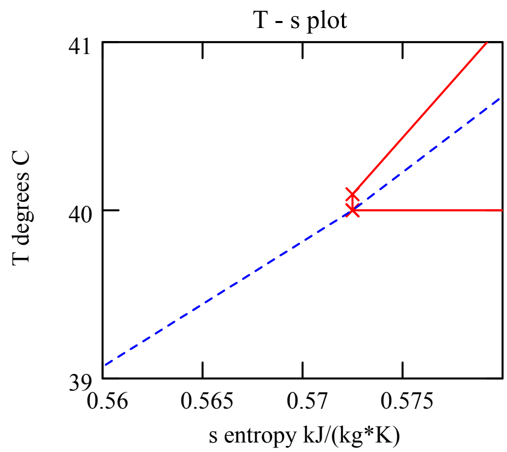
▶ "perfect regeneration"

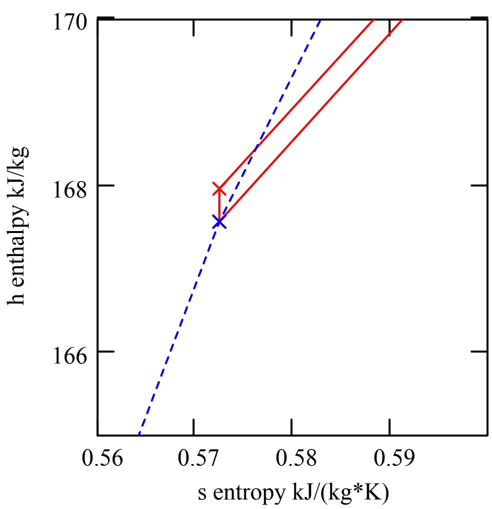
▶ data for saturation curve


T-s Plot
T degrees C
s entropy kJ/(kg*K)
0
100
200
300
400
500
0
1
2
3
4
5
6
7
8
9
10

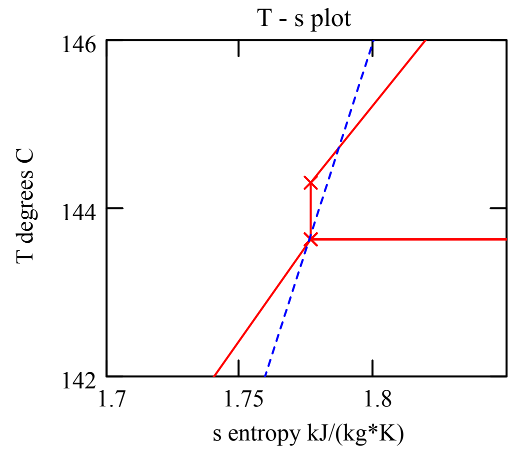

h-s Plot
h enthalpy kJ/kg
s entropy kJ/(kg*K)
0
1000
2000
3000
4000
0
2
4
6
8
10

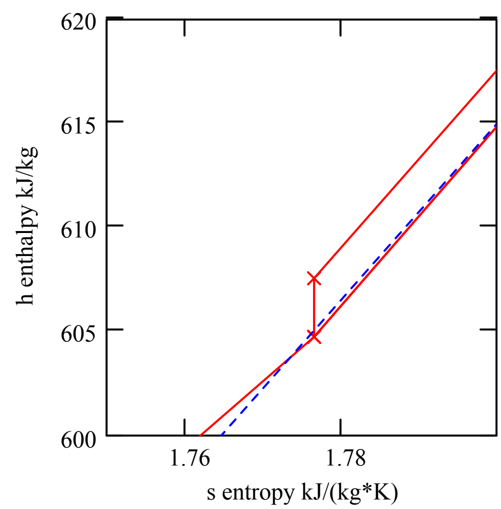
close up of points 1 and 2





close up of points 3 and 4





N.B. these scales are very exaggerated !!