

Massachusetts Institute of Technology
DEPARTMENT OF MECHANICAL ENGINEERING

2.611/612 SHIP POWER AND PROPULSION

Problem Set 6 Solutions 2006

1. Describe the advantages and disadvantages of using a Gas Turbine vs. a Diesel Engine.

Advantages in general (not req'd for answer):

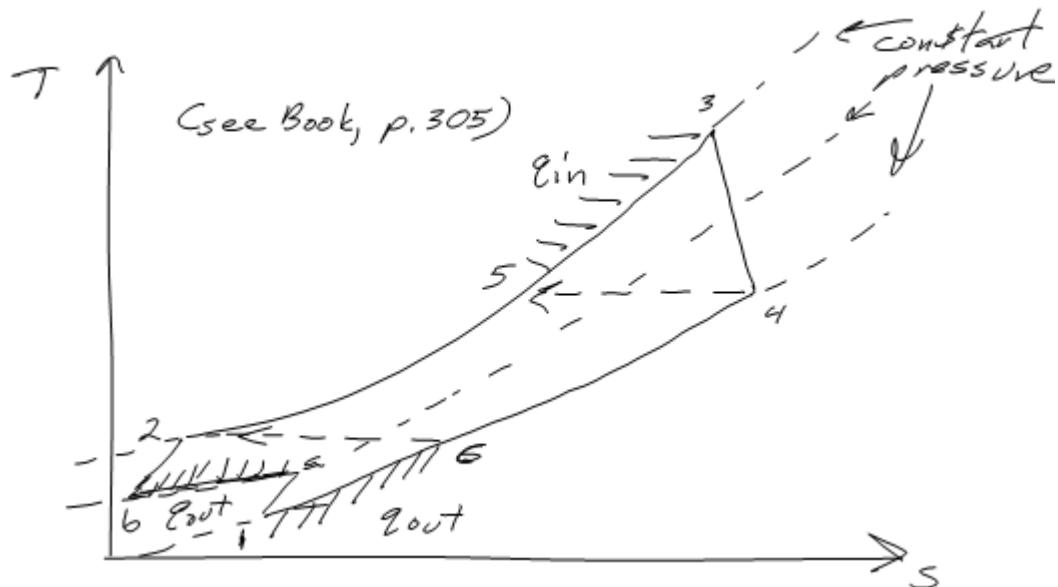
- 1) Fast Start-Up
- 2) Modular Construction
- 3) Easy Automation
- 4) High Reliability and Maintainability

Vs. Diesel

- low efficiency compared to diesel / higher fuel consumption than diesel
- requires higher fuel quality
- harder to repair underway
- higher power density than diesel so frees up space and weight

Ref: Woud, p. 137-138

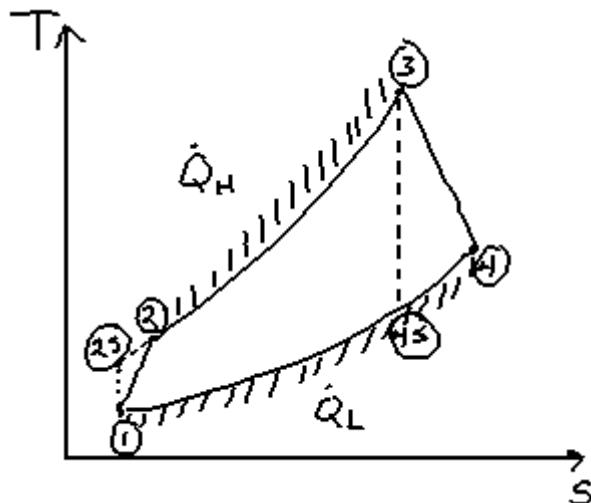
2. Draw a T-S diagram for an intercooled regenerative Brayton Cycle. Label the points and explain, in words, each portion of the cycle. Mark on your diagram the area where heat is transferred into the system and where it leaves the system.



- 1 to a: Compressor raises pressure - work in
 a - b: intercooler HX removes heat - q_{out}
 b-2: 2nd compressor raises pressure - work in
 2-5: Fluid pre-heated in regenerator (internal flow)
 5-3: Combustion - q_{in}
 3-4: Turbine - work out
 4-6: Exhaust fluid enters regenerator. Loses heat to the fluid in stage 2-5. (internal)
 6-1: In real cycle, heat is lost by exhausting the fluid. In closed cycle, heat is lost to a heat exchanger. - q_{out}

3. Simple closed-cycle Brayton engine

Given: $T_1 := 298.15K$
 $p_2_over_p_1 := 11$
 $\eta_c := 0.85$
 $T_3 := 1698K$
 $\eta_t := 0.92$
 $\Delta p_over_p := 0.06$
 $c_{p_air} := 1.00 \frac{kJ}{kg \cdot K}$
 $\gamma_{air} := 1.4$
 $m_dot := 60 \frac{kg}{s}$



Compressor:

$$\frac{T_{2S}}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = p_2_over_p_1^{\frac{\gamma_{air}-1}{\gamma_{air}}} = 1.984$$

and for the efficiency

$$\eta_c = \frac{T_1 \left(\frac{T_{2S}}{T_1} - 1 \right)}{T_2 - T_1}$$

so rearrange to get:

$$T_2 = \frac{T_1 \left(\frac{T_{2S}}{T_1} - 1 \right)}{\eta_c} + T_1 \quad \text{and using the result above}$$

$$T_2 := \frac{T_1 \left(p_2_over_p_1^{\frac{\gamma_{air}-1}{\gamma_{air}}} - 1 \right)}{\eta_c} + T_1$$

$$T_2 = 643.301K$$

Turbine:

$$\frac{p_3}{p_4} = \frac{p_2}{p_1} \cdot \left(1 - \frac{\Delta p}{p}\right) = p_{2_over_p1} \cdot (1 - \Delta p_{over_p}) = 10.34$$

and by gas properties:

$$\frac{T_{4S}}{T_3} = \left(\frac{p_4}{p_3} \right)^{\frac{\gamma}{\gamma-1}} = \left[\frac{1}{p_{2_over_p1} \cdot (1 - \Delta p_{over_p})} \right]^{\frac{\gamma_{air}}{\gamma_{air}-1}} = 0.513$$

$$T_{4S_over_T3} := \left[\frac{1}{p_{2_over_p1} \cdot (1 - \Delta p_{over_p})} \right]^{\frac{\gamma_{air}}{\gamma_{air}-1}}$$

and

$$\eta_t = \frac{T_3 - T_4}{T_3 \cdot \left(1 - \frac{T_{4S}}{T_3}\right)}$$

so rearrange and use the result above for p_3/p_4 .

a) $T_4 := T_3 - \eta_t \cdot T_3 \cdot (1 - T_{4S_over_T3})$ $T_4 = 937.264\text{K}$

Now that we have the temperatures, we can do the rest of the analyses.

$$\dot{m}_{dot} := 60 \frac{\text{kg}}{\text{s}}$$

b). ratio of $W_{dot_compressor}/W_{dot_turbine}$

$$W_{dot_compressor} = \dot{m} \cdot c_{p_air} \cdot (h_1 - h_2) = \dot{m} \cdot c_{p_air} \cdot (T_2 - T)$$

$$W_{dot_turbine} = \dot{m} \cdot c_{p_air} \cdot (h_3 - h_4) = \dot{m} \cdot c_{p_air} \cdot (T_3 - T_4)$$

so:

$$\frac{W_{dot_compressor}}{W_{dot_turbine}} = \frac{T_2 - T}{T_3 - T_4} = 0.454$$

This is also called the "Back work" ratio.

$$\text{Net_power} = \dot{W}_{\text{compressor}} + \dot{W}_{\text{turbine}}$$

c. Net power

$$= \boxed{\dot{m} \cdot c_{p,\text{air}} \cdot (T_1 - T_2) + \dot{m} \cdot c_{p,\text{air}} \cdot (T_3 - T_4) = 2.494 \times 10^4 \text{ kW}}$$

d. Heater heat transfer rate

$$\dot{Q}_{\text{H}} = \boxed{\dot{m} \cdot c_{p,\text{air}} \cdot (T_3 - T_2) = 6.328 \times 10^4 \text{ kW}}$$

e. Thermal efficiency

$$\eta_{\text{th}} = \frac{\text{Net_power}}{\dot{Q}_{\text{H}}} = \boxed{\frac{(T_1 - T_2) + (T_3 - T_4)}{T_3 - T_2} = 0.394}$$

4. Regenerative closed-cycle Brayton engine

Given: All values from problem 3 + ...

$$\varepsilon_{\text{reg}} := 0.90 \quad T_5 := T_2 + 0.9 \cdot (T_4 - T_2) \quad T_5 = 907.868 \text{ K}$$

$$q_{\text{in}} := (T_3 - T_5) \cdot c_{p,\text{air}} \quad q_{\text{in}} = 790.132 \frac{\text{kJ}}{\text{kg}} \quad Q_{\text{in}} := q_{\text{in}} \cdot \dot{m}$$

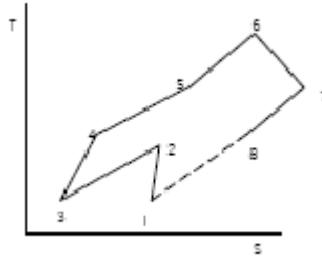
$$\text{Net_power} := [(T_1 - T_2) + (T_3 - T_4)] \cdot c_{p,\text{air}}$$

$$\boxed{Q_{\text{in}} = 4.741 \times 10^4 \text{ kW}}$$

$$\eta_{\text{th}} := \frac{\text{Net_power}}{q_{\text{in}}}$$

$$\boxed{\eta_{\text{th}} = 0.526}$$

5. Intercooled Recuperative Gas Turbine



$$T_{\text{a}} := 310 \text{ K}$$

$$\gamma_a := 1.4$$

$$T_3 := T_1$$

$$c_{\text{pa}} := 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$P2_over_P1 := 5$$

$$\gamma_p := 1.32$$

$$T_6 := 1350 \text{ K}$$

$$c_{\text{pp}} := 1.130 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\text{Pressure_loss} := .1$$

$$\text{LHV} := 43000 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{pc}} := .85$$

$$T_\phi := 298.15 \text{ K}$$

$$\eta_{\text{pt}} := .9$$

$$\eta_{\text{comb}} := .92$$

$$T_2 := T_1 \cdot P2_over_P1 \cdot \frac{\gamma_a - 1}{\gamma_a} \cdot \frac{1}{\eta_{\text{pc}}}$$

$$\varepsilon := .85$$

$$T_2 = 532.488 \text{ K}$$

$$T_4 := T_3 \cdot P4_over_P3 \cdot \frac{\gamma_a - 1}{\gamma_a} \cdot \frac{1}{\eta_{\text{pc}}}$$

$$T_4 = 532.488 \text{ K}$$

$$P6_over_P7 := P2_over_P1 \cdot P4_over_P3 \cdot (1 - \text{Pressure_loss})$$

$$P6_over_P7 = 22.5$$

$$T_7 := T_6 \cdot P6_over_P7 \cdot (-1) \cdot \left(\frac{\gamma_p - 1}{\gamma_p} \right) \cdot \eta_{\text{pt}}$$

$$T_7 = 673.566 \text{ K}$$

$$T_5 := T_4 + \varepsilon \cdot (T_7 - T_4)$$

$$T_5 = 652.404 \text{ K}$$

a

$$\text{fuel_air_ratio} := \frac{c_{\text{pp}} \cdot (T_6 - T_\phi) - c_{\text{pa}} \cdot (T_5 - T_\phi)}{\eta_{\text{comb}} \cdot \text{LHV} - c_{\text{pp}} \cdot (T_6 - T_\phi)}$$

$$\boxed{\text{fuel_air_ratio} = 0.022}$$

b

$$\text{specific_power} := (1 + \text{fuel_air_ratio}) \cdot c_{pp} \cdot (T_6 - T_7) - c_{pa} \cdot (T_2 - T_1) - c_{pa} \cdot (T_4 - T_3)$$

c.

$$sfc := \frac{\text{fuel_air_ratio}}{\text{specific_power}}$$

$$\boxed{\text{specific_power} = 333.755 \frac{\text{kW}}{\frac{\text{kg}}{\text{s}}}}$$

$$\boxed{P_{\text{turb}} := 20000 \text{ kW}}$$

$$m_{\text{dot}}_a := \frac{P_{\text{turb}}}{\text{specific_power}} \quad m_{\text{dot}}_a = 59.924 \frac{\text{kg}}{\text{s}}$$

d.

Assume ambient air conditions outside the ship

$$\text{Press}_{\text{air}} := 1 \cdot \text{bar} \quad T_{\text{amb}} := 298 \text{ K}$$

$$vel_{\text{duct}} := 25 \cdot \frac{\text{m}}{\text{s}}$$

molar weight of air (assume .8 N2 and .2 O2):

$$MW_{\text{air}} := .8 \cdot 28 \frac{\text{gm}}{\text{mol}} + .2 \cdot 32 \frac{\text{gm}}{\text{mol}}$$

$$MW_{\text{air}} = 0.029 \frac{\text{kg}}{\text{mol}}$$

Find air density using PV=nRT

$$\rho_{\text{ambair}} = \frac{n}{V} \cdot MW_{\text{air}} = \frac{P}{R \cdot T}$$

$$R_{\text{con}} := 0.08206 \frac{\text{L} \cdot \text{atm}}{\text{K} \cdot \text{mol}}$$

$$n_{\text{over}}_V := \frac{\text{Press}_{\text{air}}}{R_{\text{con}} \cdot T_{\text{amb}}}$$

$$n_{\text{over}}_V = 40.359 \frac{\text{mol}}{\text{m}^3}$$

$$\rho_{\text{ambair}} := n_{\text{over}}_V \cdot MW_{\text{air}}$$

$$\rho_{\text{ambair}} = 1.162 \frac{\text{kg}}{\text{m}^3}$$

$$V_{\text{flow}} := \frac{m_{\text{dot}}_a}{\rho_{\text{ambair}}}$$

$$V_{\text{flow}} = 51.555 \frac{\text{m}^3}{\text{s}}$$

$$A_{\text{duct}} := \frac{V_{\text{flow}}}{vel_{\text{duct}}}$$

$$\boxed{A_{\text{duct}} = 2.062 \text{ m}^2}$$

6.

$$\begin{aligned}
 t_1 &:= 303\text{K} & p_2_{\text{over_p1}} &:= 5 & p_1 &:= .1\text{MPa} & \Delta p_{\text{over_p1}} &:= 0.0\epsilon \\
 \eta_{\text{ex}} &:= 0.85 & t_3 &:= 1373\text{K} & \eta_{\text{tr}} &:= 0.92 & c_{\text{p,mix}} &:= 1.00 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} & \eta &:= .85 \\
 \gamma_{\text{air}} &:= 1.4 & m_{\text{dot}} &:= 60 \frac{\text{kg}}{\text{s}} & & & & & \text{Power} &:= 18000\text{kW} \\
 n_C &:= 1.5 & n_T &:= 1.35 & \beta &:= 220 \frac{\text{kJ}}{\text{kg}} & c_p &:= \frac{m^2 \cdot 1000}{\text{K}\cdot\text{s}^2} & &
 \end{aligned}$$

a) The temp after polytropic compression

$$\frac{n_C^{-1}}{t_2 := p_2_{\text{over_p1}}^{n_C} \cdot t_1} \quad P_c := m_{\text{dot}} \cdot c_p \cdot (t_2 - t_1) \quad t_2 = 518.123\text{K}$$

b)

$$t_3 = 1.373 \times 10^3 \text{ K} \quad P_c = 1.291 \times 10^4 \text{ kW}$$

$$t_4 := t_3 \left(\frac{1}{p_2_{\text{over_p1}}} \right)^{.25} \quad t_4 = 918.18\text{K}$$

$$P_e := m_{\text{dot}} \cdot c_p \cdot (t_3 - t_4) \quad P_e = 2.729 \times 10^4 \text{ kW}$$

c)

$$t_5 := t_2 + \eta \cdot (t_4 - t_2) \quad t_5 = 858.172\text{K}$$

$$Q_{\text{rel}} := m_{\text{dt}} \cdot c_p \cdot (t_3 - t_5) \quad Q_{\text{rel}} = 4.212 \times 10^4 \text{ kW}$$

$$m_{\text{fuel}} := \frac{Q_{\text{rel}}}{43000 \frac{\text{kJ}}{\text{kg}}} \quad m_{\text{fuel}} = 0.98 \frac{\text{kg}}{\text{s}}$$

d)

$$P_B := P_e - P_c \quad P_B = 1.438 \times 10^7 \text{ W} \quad \frac{P_B}{Q_{\text{rel}}} = 0.341$$

7.

$$K_m := 5 \quad K_e := 30 \quad n := 3 \cdot \frac{1}{s} \\
 I_m := 10 \cdot A \quad R_1 := 2 \cdot \Omega$$

$$U_m := 400 \text{ V} \quad \Phi_m = 4.222 \text{ Wb}$$

$$E := U_m - I_m \cdot R_1 \quad E = 380 \text{ V} \quad \Phi_m := \frac{E}{K_e \cdot n} \quad \frac{E \cdot I_m}{n \cdot 2 \cdot \pi} = 201.596 \text{ N} \cdot \text{m}$$

8. AC motor

- a. $p := 6 \quad f := 60\text{-Hz} \quad n_r = 1164 \text{ rpm} \quad R_r := .1\Omega \quad X := .54\Omega$
 $n_s := 120 \frac{f}{p} \quad N_s = 1200 \text{ rpm}$
- b. $s_s := \frac{n_s - n_r}{n_s}$ $s_s = 0.03$
- c. $Z_r := \left(\frac{R_r}{s} \right) + .54i\Omega$ $Z_r = 3.377 \text{ ohms at an angle of } 9.2 \text{ degrees}$
- d. $I_r = E/Z_r$ $I_r = 44.4 \text{ amps at an angle of } -9.2 \text{ degrees}$
- e. Recalculate using same eqn. in step c above with new slip value
New rotor current = 18.7 amps, angle -3.9 deg.
- f. $n_f = 1185 \text{ rpm}$

Note: Think about the relationship between rotor current, rotor impedance and load.