Homework I Solution

Problem 1

1.1

Natural Uranium : X (g/yr)
Enriched Uranium :Y (g/yr)
Discarded Uranium: Z (g/yr)

Mass Conservation: X=Y+Z Eq.(1)
Mass Conservation of U-235:
0.71X=4.4Y+0.25Z Eq.(2)

Using Eqs. (1) and (2), we get
Y=0.1108X Eq.(3)

Hence, we lose almost 90% of the natural uranium in the enrichment process.

Since U-235 in the enriched Uranium is 4.4/100*Y, we can represent the used Uranium for fission from the natural Uranium as

4.4/100*0.111X=0.004877X. Eq.(4)

Hence, only 0.4877% of the natural Uranium is actually used for fission.

Then, power can be represented as

$0.004877X * \eta_{rankine} * \eta_{nuclear}$ *1MW-day/g=1000MW*365 day* $\eta_{capacity}$
where $\eta_{rankine}$ =0.35, $\eta_{nuclear}$ =0.95 and $\eta_{capacity}$ =0.9. Then, we get $X=2.026*10^8$ g/yr=202.6 ton/yr=0.56ton/day.

1.2

Average daily amount of coal used: X (kg/day)

$X * \eta_{steam}$ *27800BTU/kg*1J/$9.48*10^{-4}$BTU*1day/(24*3600sec) =$1000*10^6$ W* $\eta_{capacity}$
where η_{steam} =0.47[1]. Then, we get $X=6.04*10^6$kg/day=5640 ton/day.

[1] M.M. El-Wakil, Power Plant Technology, McGraw Hill, 2984, page 72

1.3

Area for the flat panel : X (cm^2)

500cal/(cm^2 day)/(0.239cal/J)* η_{conv} *X*1day/(24*3600sec)=1000*10^6W
where η_{conv} =0.12. Then, X=3.441*$10^{11}$$cm^2$. Also, total area required is 2X=68.8km^2.

Problem 2

2.1
P1=1atm, T1=300K $\rightarrow$ Isentropic compression I $\rightarrow$ P2,T2
P2,T2$\rightarrow$ Intercooling $\rightarrow$ P2'=P2,T2'=T1=300
P2',T2' $\rightarrow$ Isentropic compression II $\rightarrow$ P3=100atm,T3

In the compressions I and II, T and P have the following relationships

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} \text{ and } \frac{P_3}{P_2} = \left(\frac{T_3}{T_1}\right)^{k/(k-1)} \qquad \text{Eq. (5)}$$

Also, work can be represented as

$$w = c_p(T_3 - T_1) + c_p(T_2 - T_1) \qquad \text{Eq.(6)}$$

Using Eqs. (5) and (6), we get

$$w = c_p T_1 \left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} + \left(\frac{P_3}{P_2}\right)^{(k-1)/k} - 2 \right) \qquad \text{Eq.(7)}$$

By differentiating w by P_2, we get
$P_2 = \sqrt{P_1 P_3}$ =10atm.

2.2

From Eq. (5), we get T_2=T3=579K. Using Eq. (6) and cp~1kJ/kg K at T~450K, we get 558kJ/kg.

2.3

P1=1atm, T1=300K $\rightarrow$ Isentropic compression $\rightarrow$ P2=100atm,T2

Using Eq. (5) again, we get T2=1118K. Using $w = c_p(T_3 - T_1)$ and cp~1.1kJ/kg K at T~700K, we get $w = 900$kJ/kg.

It is clear that intercooling reduces the work required for compression significantly.

2.4
w=vdP=0.001m^3/kg*(10130kPa-101.3kPa)=10kJ/kg.

The compression work of liquids is typically 1-2% of that of gases.

2.5 Assuming η_c ~0.7-0.9, work in 2.2 and 2.3 should increase by 10-40%.

Problem 3

3.1
$\eta_{thermal}$ =0.5 : thermal efficiency
$\eta_{voltage}$ =0.7/1.23=0.569 : Voltage drop
η_{H2} =237/286=0.829 : H2 to electric power conversion efficiency

$\eta_{tot} = \eta_{thermal} * \eta_{voltage} * \eta_{H2}$ =0.236

3.2

η_{tot} =0.96* η_{ICE}

η_{ICE} =0.246

3.3

0.7* $\eta_{H2} * \eta_{voltage}$ =0.330

0.330=0.96 η_{ICE}

η_{ICE} =0.344

Without considering the refinery efficiency of fuels, it becomes

η_{ICE} =0.330