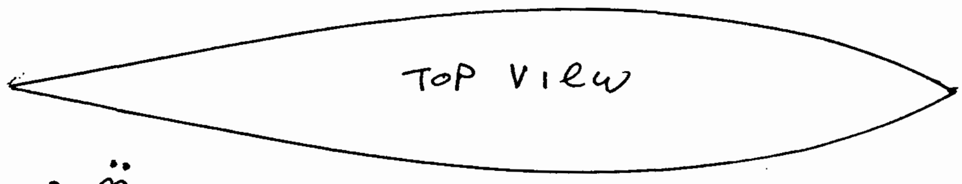
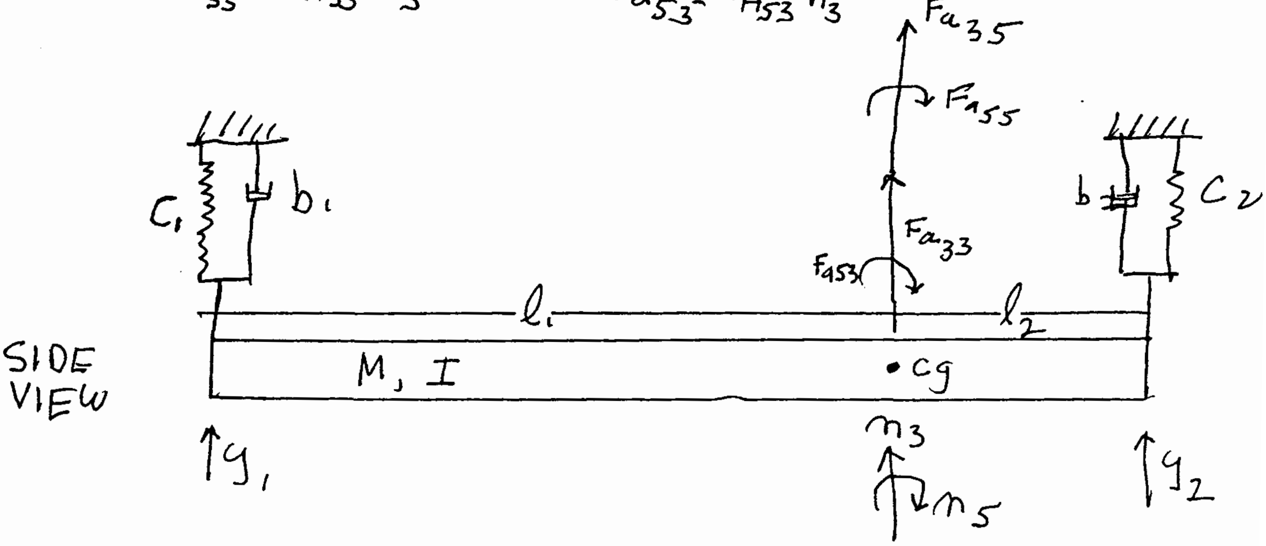
Oscillating Rigid Objects



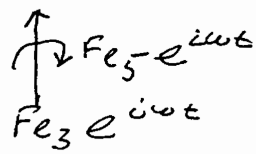
$$F_{a_{33}} = -A_{33}\ddot{\eta}_3 \qquad F_{a_{55}} = -A_{55}\ddot{\eta}_5$$

$$F_{a_{35}} = -A_{35}\ddot{\eta}_5 \qquad F_{a_{53}} = -A_{53}\ddot{\eta}_3$$



$$y_1 = \eta_3 + \ell_1\eta_5$$

$$y_2 = \eta_3 - \ell_2\eta_5$$



$$M\ddot{\eta}_3 = -A_{33}\ddot{\eta}_3 - A_{35}\ddot{\eta}_5 - c_1\eta_3 - c_1\ell_1\eta_5 - b_1\dot{\eta}_3 - b_1\ell_1\dot{\eta}_5 - c_2\eta_3 + c_2\ell_2\eta_5 - b_2\dot{\eta}_3 + b_2\ell_2\dot{\eta}_5 + F_{e_3}e^{i\omega t}$$

$$(M+A_{33})\ddot{\eta}_3 + \overbrace{(b_1+b_2)}^{B_{33}}\dot{\eta}_3 + \overbrace{(c_1+c_2)}^{C_{33}}\eta_3 + A_{35}\ddot{\eta}_5 + \underbrace{(b_1\ell_1 - b_2\ell_2)}_{B_{35}}\dot{\eta}_5 + \underbrace{(c_1\ell_1 - c_2\ell_2)}_{C_{35}}\eta_5 = F_{e_3}e^{i\omega t}$$

$$I\ddot{\eta}_5 = -A_{55}\ddot{\eta}_5 - A_{53}\ddot{\eta}_3 - c_1 l_1 \eta_3 - c_1 l_1^2 \eta_5 - b_1 l_1 \dot{\eta}_3 - b_1 l_1^2 \dot{\eta}_5$$
$$+ c_2 l_2 \eta_3 - c_2 l_2^2 \eta_5 + b_2 l_2 \dot{\eta}_3 - b_2 l_2^2 \dot{\eta}_5 + F_{e_5} e^{i\omega t}$$

$$A_{53}\ddot{\eta}_3 + \underbrace{(b_1 l_1 - b_2 l_2)}_{B_{53}}\dot{\eta}_3 + \underbrace{(c_1 l_1 + c_2 l_2)}_{C_{53}}\eta_3 + (I_5 + A_{55})\ddot{\eta}_5$$
$$+ \underbrace{(b_1 l_1^2 + b_2 l_2^2)}_{B_{55}}\dot{\eta}_5 + \underbrace{(c_1 l_1^2 + c_2 l_2^2)}_{C_{55}}\eta_5 = F_{e_5} e^{i\omega t}$$

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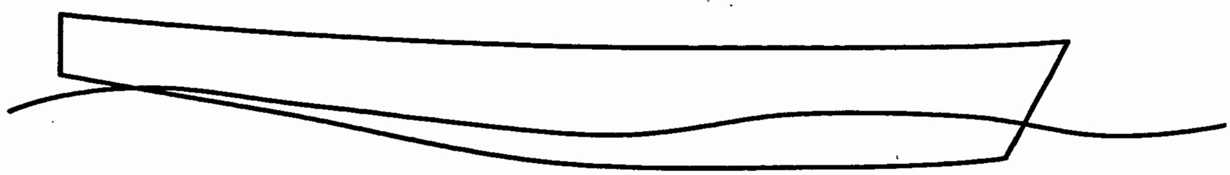
Potentials and Boundary Conditions



$$\phi_T = \phi_I + \phi_D + \sum_1^6 \zeta_j \phi_j$$

On the Free Surface:

$$\left[-\omega^2 - Ui\omega\frac{\partial}{\partial x} + u^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}\right]\phi_D = 0$$

and: $$\left[-\omega^2 - Ui\omega\frac{\partial}{\partial x} + u^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}\right]\phi_j = 0$$

On the Hull:

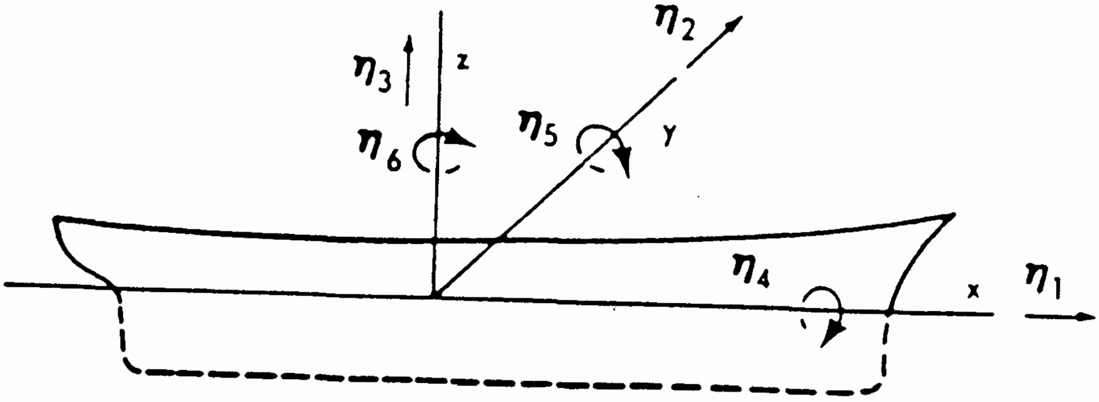
$$\frac{\partial \phi_I}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0$$

Pressure on the hull:

$$p = -\rho\left(i\omega - U\frac{\partial}{\partial x}\right)\phi_T e^{i\omega t} - \rho g(\zeta_3 + \zeta_4 y - \zeta_5 x)e_{-\omega t}$$

STI.Tex
→ STI(a), STI(b) STI (a)

Strip Theory



η_1 = surge $\quad$ η_3 = heave $\quad$ η_5 = pitch
η_2 = sway $\quad$ η_4 = roll $\quad$ η_6 = yaw

Sign convention for translatory and angular displacements

For inviscid, irrotational theory,

$$\phi_T = \phi_I + \phi_D + \sum_{k=1}^{6} \eta_j \phi_j + \text{interaction terms}$$

In linear theory the interaction terms are neglected. Here we focus on the ϕ_j's and the forces and moments associated with them.

In general, a force in the j'th direction will lead to motions in the six degrees of freedom, η_k, $k = 1, 2, 3, 4, 5, 6$.

Sinusoidal forces which generate sinusoidal motions are considered. The equations of motion for sinusoidal excitation are:

$$\sum_{k=1}^{6} \{(M_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k\} = F_j e^{i\omega t}, \qquad j = 1, 2, 3, 4, 5, 6$$

$$\eta_k(t) = \zeta_k e^{i\omega t}$$

$$\sum_{k=1}^{6} \left\{-\omega^2 (M_{jk} + A_{jk})\zeta_k + i\omega B_{jk}\zeta_k + C_{jk}\zeta_k\right\} = F_j, \qquad j = 1, 2, 3, 4, 5, 6$$

$$\sum_{k=1}^{6} \left\{\left[-\omega^2 (M_{jk} + A_{jk}) + i\omega B_{jk} + C_{jk}\right] \zeta_k\right\} = F_j, \qquad j = 1, 2, 3, 4, 5, 6$$

$-M_{jk}\ddot{\eta}_k$ is an inertial force in the j'th direction due to motion in the k'th direction.

$-A_{jk}\ddot{\eta}_k$, $-B_{jk}\dot{\eta}_k$, and $-C_{jk}\eta_k$ are hydrodynamic and hydrostatic forces in the j'th direction due to motion in the k'th direction.

For a ship having port/starboard symmetry:

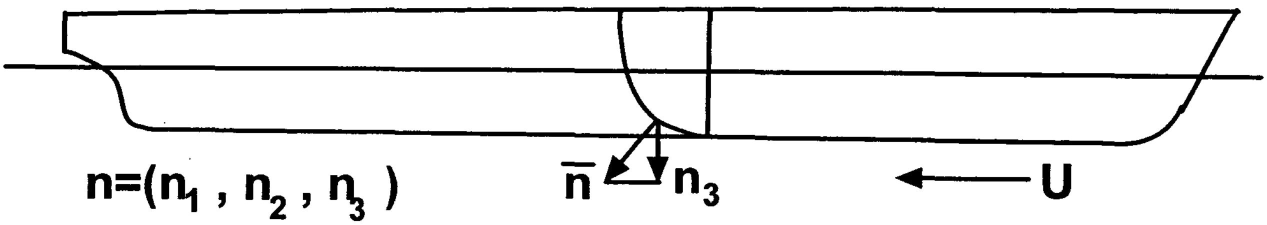
$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_c & 0 \\ 0 & M & 0 & -Mz_c & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_c & 0 & I_4 & 0 & -I_{46} \\ Mz_c & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{64} & 0 & I_6 \end{bmatrix}$$

$$A_{jk} = \begin{bmatrix} A_{11} & 0 & a_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix}$$

$$B_{jk} = \begin{bmatrix} B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\ 0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\ B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\ 0 & A_{B2} & 0 & B_{44} & 0 & B_{46} \\ B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\ 0 & B_{62} & 0 & B_{64} & 0 & B_{66} \end{bmatrix}$$

$$C_{jk} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & C_{35} & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & C_{53} & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Boundary Conditions on Hull

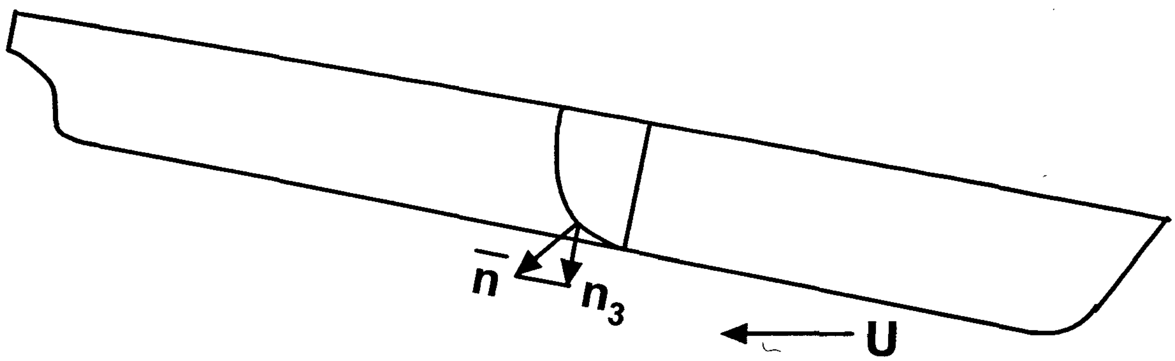


Definitions: $(n_1, n_2, n_3) = \vec{n}$ $(n_4, n_5, n_6) = \vec{r} \times \vec{n}$

Except for j = 5 and j = 6:

$$\frac{\partial}{\partial n}\zeta_j \phi_j e^{i\omega t} = \frac{\partial}{\partial t}\zeta_j e^{i\omega t} n_j \qquad \zeta_j \frac{\partial \phi_j}{\partial n} e^{i\omega t} = \zeta_j i\omega n_j e^{i\omega t} \qquad \frac{\partial \phi_j}{\partial n} = i\omega n_j$$

For j = 5 (pitch) and for j = 6 (yaw) there is a change in the normal velocity associated with U:



Here, for positive pitch, there is an upward (positive z) component of velocity equal to $-U\zeta_5 e^{i\omega t}$. This has a component normal to the hull surface of $-U\zeta_5 e^{i\omega t} n_3$.

For n = 5: $$\frac{\partial \phi_5}{\partial n} = i\omega n_5 + U n_3$$

For n = 6: $$\frac{\partial \phi_6}{\partial n} = i\omega n_6 - U n_2$$

For slender ships surge forces F_1 are much smaller than the other forces. Furthermore, surge motions have little effect except for the special case of towing. We neglect surge here.

Under these conditions, the pitch and heave equations are decoupled from the sway, roll and yaw equations.

PITCH AND HEAVE EQUATIONS

$$\left[-\omega^2(M + A_{33}) + i\omega B_{33} + C_{33}\right]\zeta_3 + \left[-\omega^2 A_{35} + i\omega B_{35} + C_{35}\right]\zeta_5 = F_3$$

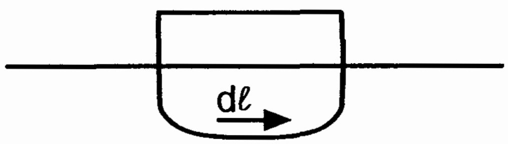
$$\left[-\omega^2 A_{53} + i\omega B_{53} + C_{53}\right]\zeta_3 + \left[-\omega^2(I_5 + A_{55}) + i\omega B_{55} + C_{55}\right]\zeta_5 = F_5$$

Hydrodynamic and Hydrostatic Coefficients

The hydrodynamic forces on the ship due to oscillatory motions of the ship are called radiation forces. For a motion of the form $\eta_k(t) = \zeta_k e^{i\omega t}$, the linearized force in the j direction is called $T_{jk}e^{i\omega t}$. T_{jk} will be a complex number having a real part and an imaginary part. It is conventionally written in the following form:

$$T_{jk} = (\omega^2 A_{jk} - i\omega B_{jk} - C_{jk})\zeta_k$$

A_{jk} is the added mass for forces in the j direction due to motion in the k direction.
B_{jk} is the damping coefficient for forces in the j direction due to motion in the k direction.
C_{jk} is the hydrostatic "spring constant".



$$A_{33} = \int_L a_{33}d\xi - \frac{U}{\omega^2}b_{33}^A \qquad B_{33} = \int_L b_{33}d\xi + Ua_{33}^A$$

$$A_{35} = -\int_L \xi a_{33}d\xi - \frac{U}{\omega^2}B_{33}^o + \frac{U}{\omega^2}x_A b_{33}^A - \frac{U^2}{\omega^2}a_{33}^A$$

$$B_{35} = -\int_L \xi b_{33}d\xi + UA_{33}^o - Ux_A a_{33}^A - \frac{U^2}{\omega^2}b_{33}^A$$

A_{33}^o and B_{33}^o are the speed independent parts of the respective coefficients.

$$A_{53} = -\int_L \xi a_{33} d\xi + \frac{U}{\omega^2} B_{33}^o + \frac{U}{\omega^2} x_A b_{33}^A$$

$$B_{53} = -\int_L \xi b_{33} d\xi - U A_{33}^o - U x_A b_{33}^A$$

$$A_{55} = -\int_L \xi^2 a_{33} d\xi + \frac{U^2}{\omega^2} A_{33}^o - \frac{U}{\omega^2} x_A^2 b_{33}^A + \frac{U^2}{\omega^2} x_A a_{33}^A$$

$$B_{55} = \int_L \xi^2 b_{33} d\xi + \frac{U^2}{\omega^2} B_{33}^o - U x_A^2 a_{33}^A + \frac{U^2}{\omega^2} x_A b_{33}^A$$

$$C_{33} = \rho g \int_L b d\xi = \rho g A_{WP}$$

b is the beam of each section and A_{WP} is the waterplane area.

$$C_{35} = C_{53} = -\rho g \int_L \xi b d\xi = -\rho g M_{WP}$$

$$C_{55} = \rho g \int_L \xi^2 b d\xi = \rho g I_{WP}$$

M_{WP} is the moment of area of the waterplane and I_{wp} is the moment of inertia of the waterplane.

$$F_3 = \rho \alpha \int_L (f_3 + h_3) d\xi + \rho \alpha \frac{U}{i\omega} x_A h_3^A$$

$$F_5 = -\rho \alpha \int_L \left[\xi (f_3 + h_3) + \frac{U}{i\omega} h_3 \right] d\xi - \rho \alpha \frac{U}{i\omega} x_A h_3^A$$

α is the wave amplitude. $f_3 = F_3 e^{i\omega t}$ is the Froude-Krilov force.

$$F_3(x) = ge^{-ikx\cos\beta} \int_{C_x} N_3 e^{iky\sin\beta} e^{kz} d\ell$$

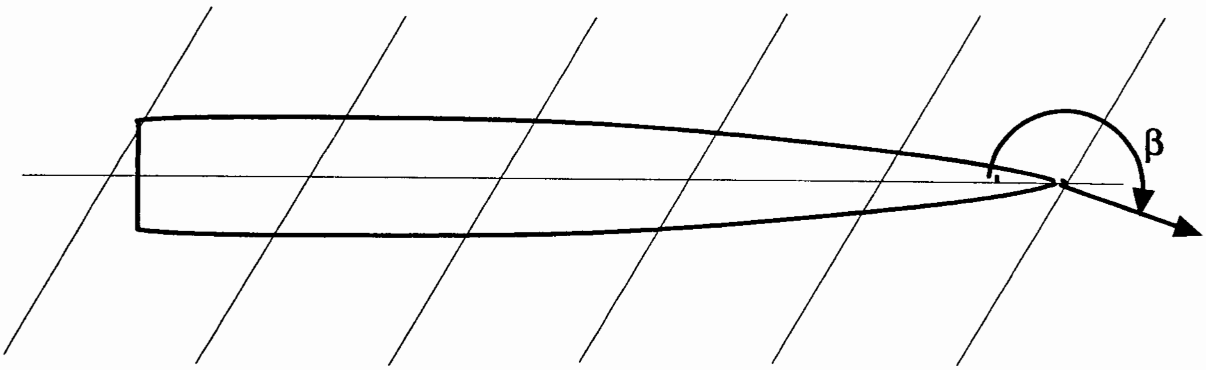
N_3 is the vertical component of the 2D normal to the section. N_2 is the horizontal component of the 2D normal. β is the wave propagation angle. $d\ell$ is the element of arc length around the section.

h_3 is the sectional diffraction force.

$$h_3(x) = \omega_o e^{-ikx\cos\beta} \int_{C_x} (iN_3 + N_2 \sin\beta) e^{iky\sin\beta} e^{kz} \psi_3 d\ell$$

$$\omega_o = \sqrt{gk} \qquad \omega_o = \omega + kU\cos\beta$$

ψ_3 is the velocity potential for a 2D cylinder of shape C_x oscillating in heave. ψ_3 is the solution to $\nabla^2 \psi_3 = 0$ subject to the boundary condition for heave motion. It can be obtain by several ways including panel methods.



Thus, to do the the longitudinal integrals ($d\xi$), one must know the 2D hydrostatic terms and the 2D added mass, damping, and velocity potential for heave.

SWAY, ROLL AND YAW EQUATIONS

$$\left[-\omega^2(A_{22}+M)+i\omega B_{22}\right]\zeta_2+\left[-\omega^2(A_{24}-Mz_c)+i\omega B_{24}\right]\zeta_4+\left[-\omega^2 A_{26}+i\omega B_{26}\right]\zeta_6=F_2$$

$$\begin{aligned}\left[-\omega^2(A_{42}-Mz_c)+i\omega B_{42}\right]\zeta_2+\left[-\omega^2(A_{44}+I_4)+i\omega B_{44}+C_{44}\right]\zeta_4&\\ +\left[-\omega^2(A_{46}-I_{46})+i\omega B_{46}\right]\zeta_6 &= F_4\end{aligned}$$

$$\left[-\omega^2 A_{62}+i\omega B_{62}\right]\zeta_2+\left[-\omega^2(A_{64}-I_{46})+i\omega B_{64}\right]\zeta_4+\left[-\omega^2(A_{66}+I_6)+i\omega B_{66}\right]\zeta_6=F_6$$

All the coefficients can be determined from the 2D sectional sway and roll added mass and damping, the 2D sectional potentials for sway and roll and the hydrostatic roll restoring force.

For all the five motions considered, the response at the resonant frequency is largely controlled by the wave generation damping (B coefficients) except for roll where the damping at resonance is dominated by the viscous damping. Therefore, for strip theory to give accurate results for roll, an estimate for the viscous damping coefficient must be added to B_{55}.

SIMULATIONS OF SHIP MOTIONS IN RANDOM SEAS

The complete problem includes effects of waves coming from all directions. Here, for simplicity and clarity we will consider long-crested random waves coming from one direction.

The "system functions", $\xi_j(\omega)$, are complex numbers, dependent on frequency. For each one, the magnitude is the ratio of the sinusoidal motion amplitude to the wave amplitude. The phase is the phase lead of the motion with respect to the wave elevation at the origin of the coordinate system.

Suppose we have a wave spectrum, $S_e(\omega)$. The wave elevation at the origen of the ship, or offshore structure, at the origin of the coordinate system, can be simulated as:

$$\zeta_w(t) = \sum_{n=-N}^{N} Z_n e^{i(n\delta\omega)t}$$

$$\text{where:} \quad Z_n = e^{i\alpha_n}\sqrt{\frac{1}{2}S_e(n\delta\omega)\delta\omega}$$

Within the restrictions of linear theory, each ship motion can be simulated in the specified random wave field as:

$$\eta_j(t) = \sum_{n=-N}^{N} \xi_j(n\delta\omega) Z_n e^{i(n\delta\omega)t}$$

ST4b

Added Resistance and Drift Forces

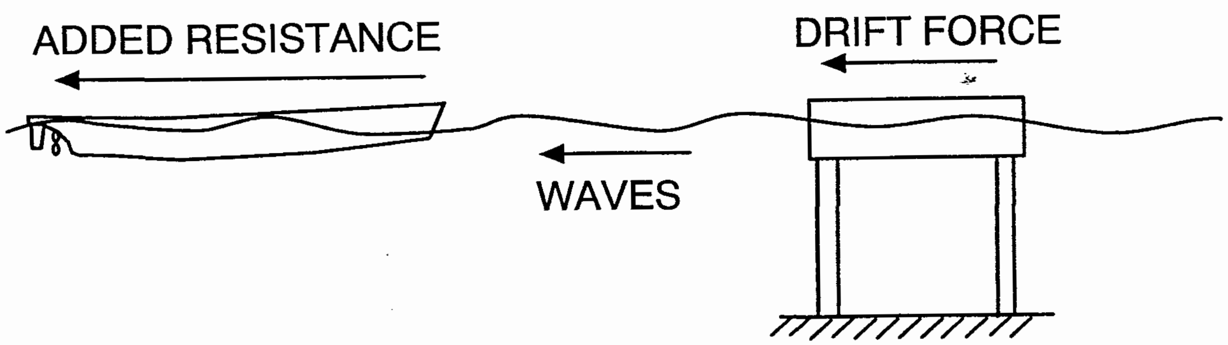
An important "second order" effect is the average force an oscillating wave field can imoise on an object. These forces are typically small in comparison of oscillating forces and spring-like resorting forces so the horizontal mean forces are most important. For a ship, this force is called the added resistance and for an offshore structure it is called the drift force.

One again, for simplicity and clarity we will consider waves propagating in a single direction. If a sinusoidal wave has amplitude A, the dominant mean force has the form $r_a(\omega)A^2$. $r_a(\omega)$ is called the added resistance operator. It is found by solving the second order hydrodynamic problem. However, Some first order effects contribute.

Consider the term in Euler's equation $(\vec{V} \cdot \nabla)\vec{V}$ When $\vec{V}$ is sinusoidal, this term will contribute zero frequency terms.

In the presence of a wave spectrum $S(\omega)$, the total added reesistance is:

$$R_{added} = \int_0^\infty 2r_a(\omega)S(\omega)d\omega$$



Gerritsma and Beukelman Theory for Added Resistance

The "exact" formulation for the added resistance operator, $r_a(\omega_e)$ requires solution of the complicated 3-D second order problem. However, Gerritsma and Beukelman [1] developed a semi-empirical formulation for $r_a(\omega_e)$ based on strip theory that is remarkably accurate. The basis of their theory was as follows:

1. Each section of the ship encounters a relative vertical velocity that depends on the wave, and the heave and pitch of the ship.
2. This relative vertical motion generates waves which carry energy away from the ship.
3. Equating this radiated energy, per unit time, to the added resistance times the ship speed provides a formula for the added resistance.

This formula is:

$$r_a(\omega_e) = \frac{k}{2\omega_e}\int_0^{\mathcal{L}}\left[N(x) - V\frac{dm(x)}{dx}\right]V_z^2(x)dx$$

where the ship extends from 0 to $\mathcal{L}$ and k = wavenumber of the wave,
$N(x)$ = heave damping coefficient per unit length of ship at position x,
V = forward speed of ship,
$m(x)$ = added mass per unit length of ship cross section at position x,
$V_z(x)$ = relative vertical water velocity amplitude at position x.

$$V_z = \dot{z} - x\dot{\theta} + V\theta - \zeta_a$$

where $\dot{z}$ is the heave velocity of the ship at $x = 0$, θ is the pitch angle of the ship, and ζ_a is the average of the velocity of the fluid motion in the wave over the width of the ship cross section at its local depth.

[1] Gerritsma, J., and Beukelman, W., "Analysis of the Increase in Resistance in Waves of a Fast Cargo Ship", Technical report 169 s, Netherlands Ship Research Centre TNO, April, 1972

Nonlinear Wave Force Calculations

Second order wave forces arise both from the second order potential, which increases the accuracy with which the nonlinear terms in the free surface boundary condition are met, and also from the influence of the first order solution on the nonlinear term in Euler's equation. For example, consider the term: $u\frac{\partial u}{\partial x}$. As an example, a field of two sinusoidal waves is :

$$u = A_1 \sin(k_1 x - \omega_1 t) + A_2 \sin(k_2 x - \omega_2 t)$$
$$\frac{\partial u}{\partial x} = A_1 k_1 \cos(k_1 x - \omega_1 t) + A_2 k_2 \cos(k_2 x - \omega_2 t)$$

At $x = 0$:

$$u = -A_1 \sin(\omega_1 t) - A_2 \sin(\omega_2 t)$$
$$\frac{\partial u}{\partial x} = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$$

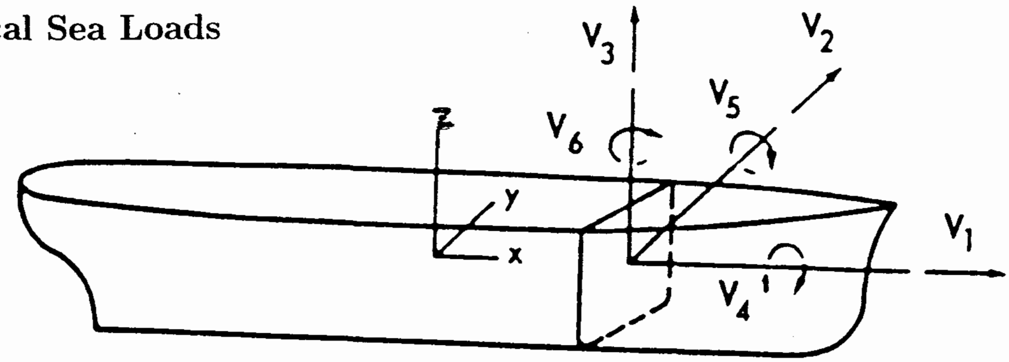
$$\begin{aligned} u\frac{\partial u}{\partial x} &= -A_1^2 k_1 \sin(\omega_1 t)\cos(\omega_1 t) - A_1 A_2 k_2 \sin(\omega_1 t)\cos(\omega_2 t) \\ &\quad -A_1 A_2 k_1 \sin(\omega_2 t)\cos(\omega_1 t) - A_2^2 k_2 \sin(\omega_2 t)\cos(\omega_2 t) \end{aligned}$$

Consider just one of the four terms:

$$\begin{aligned} \left(u\frac{\partial u}{\partial x}\right)_1 &= -A_1 A_2 k_2 \sin(\omega_1 t)\cos(\omega_2 t) \\ &= -\frac{A_1 A_2 k_2}{2}\sin(\omega_1 + \omega_2)t - \frac{A_1 A_2 k_2}{2}\sin(\omega_1 - \omega_2)t \end{aligned}$$

If ω_1 and ω_2 are only slightly different, $\omega_1 + \omega_2$ is a comparatively high frequency and $\omega_1 - \omega_2$ is a very low frequency.

Vertical Sea Loads



V_1 = compression force
V_2 = horizontal shear force
V_3 = vertical shear force
V_4 = torsional moment
V_5 = vertical bending moment
V_6 = horizontal bending moment

The complete strip theory sea loads are considered in the reference "Ship Motions and Sea Loads", by Salvesen, Tuck and Faltinsen. Here we consider the vertical loads which lead to the shear force, V_3' and the longitudinal vertical bending moment, V_5'. Sinusoidal forces and motions are considered. For example,

$V_j' = V_j e^{i\omega t}$ where the real part of all complex expressions is implied

$$\text{Likewise, } \eta_j = \zeta_j e^{i\omega t}$$

The fluid forces are separated into hydrostatic forces R_j, sea wave exciting forces E_j, and hydrodynamic forces resulting from unsteady ship motions D_j. I_j is the inertial component of the j^{th} structural force due to motions of the ship. Then, the structural loads can be expressed symbolically as:

$$V_j = I_j - R_j - E_j - D_j$$

We are concerned with terms having subscripts 3 and 5. All the longitudinal integrals in the following are over the portion of the ship forward of the section under consideration. We denote these integrals as:

$$\int_{L_f} ....d\xi$$

$$I_3 = \int_{L_f} -\omega^2 m(\xi)\ [\zeta_3 - \xi\zeta_5]\ d\xi$$

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$$I_5 = -\int_{L_f} -\omega^2 m(\xi)\,[\xi - x]\,[\zeta_3 - \xi\zeta_5]\,d\xi$$

$$R_3 = -\rho g \int_{L_f} b(\xi)\,[\zeta_3 - \xi\zeta_5]\,d\xi$$

$$R_5 = \rho g \int_{L_f} b(\xi)\,[\xi - x]\,[\zeta_3 - \xi\zeta_5]\,d\xi$$

$$E_3 = \rho\alpha \left\{ \int_{L_f} (f_3 + h_3) d\xi + \left(\frac{U}{i\omega} h_3 \right)_{\xi = x} \right\}$$

$$E_5 = \rho\alpha \int_{L_f} \left\{ (\xi - x)(f_3 + h_3) + \frac{U}{i\omega} h_3 \right\} d\xi$$

$$\begin{aligned} D_3 \;=\; & -\int_{L_f} \left\{ \omega^2 a_{33}(\zeta_3 - \xi\zeta_5) + i\omega b_{33}(\zeta_3 - \xi\zeta_5) + U b_{33}\zeta_5 + i\omega U a_{33}\zeta_5 \right\} d\xi \;- \\ & - \left\{ i\omega a_{33} U(\zeta_3 - \xi\zeta_5) + U b_{33}(\zeta_3 - \xi\zeta_5) + U^2 a_{33}\zeta_5 - \frac{iU^2}{\omega} b_{33}\zeta_5 \right\}_{\xi = x} \end{aligned}$$

$$\begin{aligned} D_5 \;=\; & \int_{L_f} a_{33}(\xi - x) \left\{ -\omega^2(\zeta_3 - \xi\zeta_5) + i\omega b_{33}(\zeta_3 - \xi\zeta_5) \right\} d\xi \;+ \\ & \int_{L_f} \left\{ i\omega U a_{33}(\zeta_3 - x\zeta_5) + U b_{33}(\zeta_3 - x\zeta_5) + U^2 a_{33}\zeta_5 - \frac{iU^2}{\omega} b_{33}\zeta_5 \right\} d\xi \end{aligned}$$