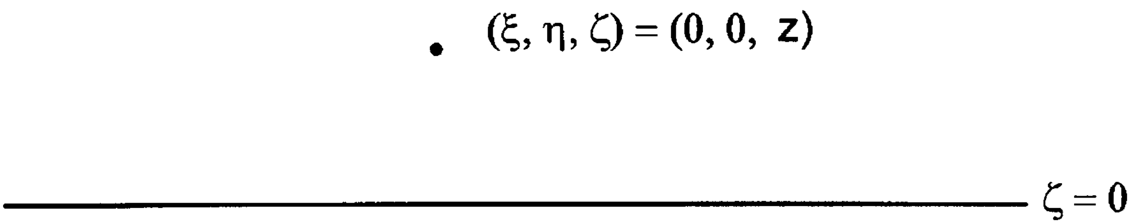
13.024 Problem Set 4:

Review of Fluid Dynamics

Due: March 11, 2003

1.



If you do not know it already, you will learn that a potential flow field due to an object with relative motion to a fluid can be represented by distributions of sinks (negative sources) and dipoles on all the boundaries of a fluid. Green's Theorem provides one way to choose the sink and dipole strengths. A frequently used Green Function is the Rankine sink potential G_r:

$$G_r = \frac{1}{R} = \frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}}$$

Consider a very deep ocean with a (ξ, $\acute{\eta}$, ς) coordinate system having its origin on the bottom, with ξ to the right and ς upwards. Suppose the sink is at (0,0,z) and z=1. Determine the velocity field associated with this sink. Since it is axisymmetric it is sufficient to look at the flow in the (ξ, 0, ς) plane. Plot the streamlines in MATLAB using the MATLAB *streamline* function. Look up the description of the use of the *streamline* function in the MATLAB help menu. It is suggested that you print out the description it provides. An example page of the use of this MATLAB function is at the end of this Problem Set description where the variables in the example should be obvious (us and ws are matrices of the horizontal and vertical velocities at various (xi, ze) positions). Each call to *streamline* draws one streamline. You will probably want to call the *streamline* function in a "for" loop with vectors of the streamline starting positions made in advance to have less typing than is in the example. Note that the streamlines are not tangent to the bottom so the Green function does not satisfy the bottom boundary condition. With this Green function and a moving object in the water above the bottom, calculating the flow caused by the moving object would require integrations over both the object and the bottom.

In all cases you have to figure out and compute the flow field. A reasonable range of ξ is -10 to 10 and ς should go from 0 to something between 0.5 and 0.9 as you choose. We are interested in seeing the streamlines near the bottom.

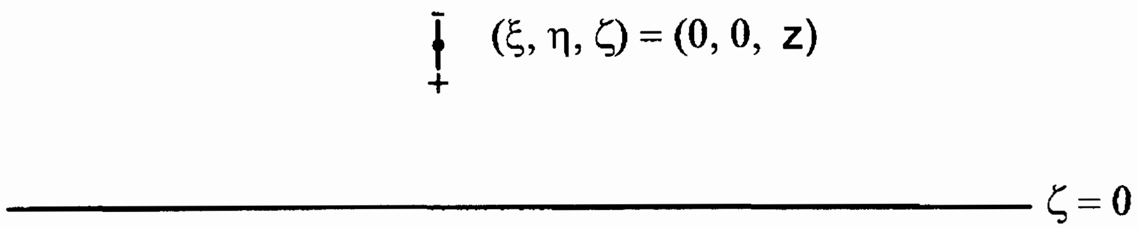
2.
An alternative Green function is $G = 1/R + G_a$, where G_a is a function that satisfies Laplace's equation throughout the fluid domain. Determine such a Green function with the constraint that it be a velocity potential whose associated flow is tangent to the bottom. In other words, the resulting velocity would have no component normal to the bottom. That would eliminate

the need for one of the integral in Green's Theorem on the bottom surface when solving for the flow about a moving object, or for a stationary object in a streaming flow.

Hint: Consider a method of images.

Make a streamline plot using MATLAB for the flow from this new Green Function. The example using *streamline* does this for a second figure where usi and wsi are the horizontal and vertical flow velocities.

3.



$\phi \dfrac{\partial G}{\partial n}$ is the velocity potential of a dipole pointing in the *n* direction. For a downward pointing Rankine dipole located at (ξ, ή, ς) = (0,0,z) the velocity potential is:

$$\phi_d = \frac{-(z-\zeta)}{[(z-\zeta)^2+\xi^2+\eta^2]^{3/2}}$$

The dipole was chosen as downward pointing because that gives an axisymmetric flow which can be completely defined by its behavior in the (ξ, 0, ς) plane. In an actual problem the dipoles point in directions normal to all surfaces. For this case make a streamline plot in the vicinity of the bottom showing the flow field of the dipole located at (0,0,1). The third figure in the following example shows the long-written way to do this in MATLAB. As before, you can define some MATLAB vectors and make all the streamlines in a "for" loop. You should see that the dipole has a flow component perpendicular to the bottom so using this form for $\partial G/\partial n$ will still require an integral on the bottom when Green's Theorem is used.

4. Determine an analytic function (a function that satisfies Laplace's equation in the fluid) which can be added to the form of $\partial G/\partial n$ above so the flow very near the dipole is still a dipole flow, but which has no flow perpendicular to the bottom. It should be the $\partial G/\partial n$ with the G you found in problem 2. Once again make a MATLAB streamline plot of this flow. The fourth figure in the following example is the "long writing" way to do the graphing with the horizontal and vertical components of the flow pre-calculated and called udi and wdi.

Note: Once again the method of images is the easiest route to the correct function.

Example MATLAB CODE for Four Streamline Plots.

```
[X,Z] = meshgrid(xi,ze);
h = streamline(X,Z,us,ws,-10,0.00);
h = streamline(X,Z,us,ws,-10,0.05);
h = streamline(X,Z,us,ws,-10,0.10);
h = streamline(X,Z,us,ws,-5,0.0);
h = streamline(X,Z,us,ws,0.0,0.0);
h = streamline(X,Z,us,ws,10,0.05);
h = streamline(X,Z,us,ws,10,0.10);
h = streamline(X,Z,us,ws,5,0.0);
h = streamline(X,Z,us,ws,10,0.00);
figure;
h = streamline(X,Z,usi,wsi,-10,0.05);
h = streamline(X,Z,usi,wsi,-10,0.0025);
h = streamline(X,Z,usi,wsi,-10,0.1);
h = streamline(X,Z,usi,wsi,10,0.05);
h = streamline(X,Z,usi,wsi,10,0.0025);
h = streamline(X,Z,usi,wsi,10,0.1);
figure
h = streamline(X,Z,ud,wd,-5,0.00);
h = streamline(X,Z,ud,wd,-4,0.00);
h = streamline(X,Z,ud,wd,-3,0.00);
h = streamline(X,Z,ud,wd,-2.0,0.0);
h = streamline(X,Z,ud,wd,-1.0,0.0);
h = streamline(X,Z,ud,wd,1.0, 0.0);
h = streamline(X,Z,ud,wd,2,0.0);
h = streamline(X,Z,ud,wd,3.0,0.0);
h = streamline(X,Z,ud,wd,4.0,0.0);
h = streamline(X,Z,ud,wd,5.0,0.00);
figure;
h = streamline(X,Z,udi,wdi,-5,0.05);
h = streamline(X,Z,udi,wdi,-4.0,0.0025);
h = streamline(X,Z,udi,wdi,-3.00,0.05);
h = streamline(X,Z,udi,wdi,-2.00,0.05);
h = streamline(X,Z,udi,wdi,-1.0,0.05);
h = streamline(X,Z,udi,wdi,1.0,0.05);
h = streamline(X,Z,udi,wdi,2.0,0.05);
h = streamline(X,Z,udi,wdi,3.0,0.05);
h = streamline(X,Z,udi,wdi,4.0,0.0025);
h = streamline(X,Z,udi,wdi,5.0,0.05);
```