MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 1.00

This problem is from "2.25 Advanced Fluid Mechanics" by Ain Sonin

Rate of change of properties measured by a probe moving through the earth's atmosphere — plus some things about the earth and its atmosphere.

The pressure distribution in a static, constant-temperature planetary atmosphere modeled as an ideal gas is given by

$$p = p_0 e^{-z/H} \tag{1.00a}$$

where z is the altitude above a reference altitude $z = 0$, p_0 is the absolute pressure at $z = 0$, and

$$H = \frac{RT}{Mg} \tag{1.00b}$$

is a length scale that characterizes the atmosphere. Its value is determined by the strength of the gravitational acceleration and the parameters that appear in the ideal-gas equation of state,

$$p = \rho \frac{RT}{M} \; ; \tag{1.00c}$$

$R = 8.32$ JK^{-1} mol^{-1} is the universal gas constant, T is the absolute temperature (taken as constant in this model of the atmosphere), M is the molar mass of the gas (0.029 kg/mol if the gas is air), and g is the acceleration of gravity at or near the surface of the planet. For the "Standard" isothermal model of the earth's atmosphere, $T = 288$ K, $p_0 = 1.02 \times 10^5$ N/m^2 if $z = 0$ at sea level, and consequently $H = 8.43$ km. Note that the distribution given above is based on the assumption that $H \ll a$, where a is the planet's radius.

Suppose a sounding rocket or balloon equipped with a static-pressure sensor is traveling through the atmosphere with given velocity (v_x, v_y, v_z).

1. In terms of the given quantities and z, derive an expression for the rate of change of pressure recorded by the rocket's sensor.
2. Evaluate this time of change at an altitude $z = 20{,}000$ m for a rocket traveling upward through the earth's atmosphere with a direction of 30° from the vertical and a speed of 465 m/s. (Answer: -0.273 bar/min.)
3. Suppose a rocket carries instruments that measure both the instantaneous atmospheric pressure p and the rate of change of that pressure, dp/dt. Given the value of these two quantities at a particular time and p_0 and H, derive expressions for the rocket's instantaneous altitude and vertical (upward) velocity.

Additional things to think about, if you are so inclined:

4. Suppose the Earth's atmosphere is isothermal and radially symmetric around a perfectly spherical earth with radius $a = 6400$ km. What is the total mass of the Earth's atmosphere? What fraction is this of the solid and liquid parts of the planet's mass? (Answers: 5.35×10^{18} kg and 8.96×10^{-7}.)

5. Show that 99% of the Earth's atmosphere's mass resides below an altitude of 39 km.
6. If the atmosphere heats up by 10°C, by how much will the absolute pressure at sea level change? (Answer: it will not change at all.)

