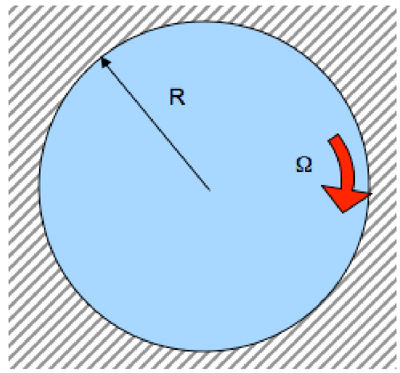
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 8.02

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



An infinitely long, cylindrical container of radius R rotates at the angular speed Ω. It contains water which is also in solid-body rotation with angular speed Ω. At time $t = 0$, the container suddenly stops rotating, and the contained water gradually comes to rest.

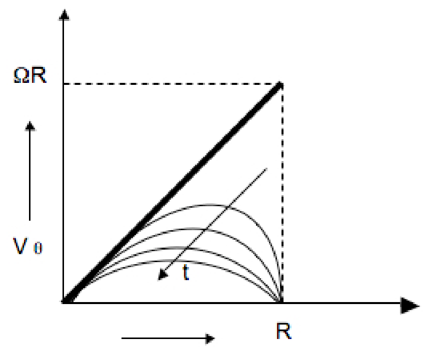
In all that follows, the possible effects of turbulence and other instabilities are to be considered absent.

- (a) Sketch curves of V_θ versus r, showing how the circumferential velocity varies with radius for several successive times, $t > 0$.
- (b) What is the order of magnitude of the time, t_R, up to which the Rayleigh's solution for impulsive start of a flat plate would describe the motion near the wall?
- (c) Suppose that $\Omega = 33 - 1/3$ $[rpm]$, $R = 10[cm]$, and that the fluid is water at 20 degrees celcius.

Make a very rough estimate of the time, in seconds required for most of the motion to disappear.

Solution:

- (a)



- (b) As soon as the cylinder is stopped, there is diffusion of momentum from the wall towards the center. The present problem in cylindrical geometry differs from Rayleigh's problem in that the velocity away from the wall (at $r = 0$) is zero at all times. In Rayleigh's problem the velocity boundary condition away from the wall is U_∞. By $t_R = \frac{R^2}{\nu}$, he diffusion scale, the impact of the wall has diffused towards the center, where the BC, as we discussed, is different from Rayleigh's problem. Hence, by $t_R = \frac{R^2}{\nu}$, there is already a "back diffusion" of this BC towards the wall and the near-wall region has started feeling this effect. For Rayleigh's solution to be valid near the wall, we have

$$t_R = \frac{R^2}{\nu}. \tag{8.02a}$$

- (c) For a very rough estimate, we again observe that the diffusion time scale, $t_L = \frac{L^2}{\nu}$. We know that the Rayleigh's solution involves the error function. Assuming a similar variation here for a very rough estimate, we can say

$$\frac{V_\theta}{r\Omega} \sim erf\left(\sqrt{\frac{t_R}{t}}\right). \tag{8.02b}$$

From the behavior of the error function, we know that all the motion will decay within $t \sim 10t_R$ (rough order of magnitude). Hence, motion-decay time is of the order of 10^5 seconds.

□

Problem Solution by MK/MC(Updated), Fall 2008

