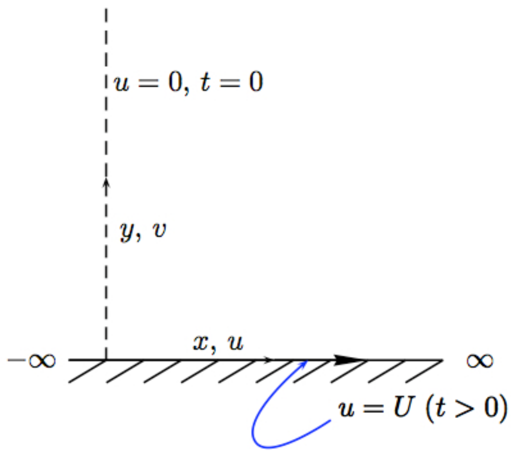
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Stokes First Problem ATP



Consider Stokes' First Problem: impulsive start of a flat plate beneath a semi-infinite layer of initially quiescent incompressible fluid. The governing equations (presuming parallel flow — no instabilities) for $u(y,t)$ are:

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}, \qquad 0 < y < \infty \ , \tag{1}$$

$$u(y=0,t) = U \ , \tag{2}$$

$$u(y \to \infty, t) \to 0 \ , \tag{3}$$

$$u(y, t=0) = 0 \ . \tag{4}$$

The shear stress at the wall is then given by

$$\tau_W(t) = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \ . \tag{5}$$

Here ρ is the density and μ is the dynamic viscosity. The shear stress at the wall will be of the form

$$\tau_W = C U^{\alpha_1} \rho^{\alpha_2} \mu^{\alpha_3} t^{\alpha_4} \ , \tag{6}$$

where C is a non-dimensional constant. Find the exponents α_1, α_2, α_3, and α_4 by dimensional analysis.

Hint (one approach): Write the equations in terms of u/U; apply Buckingham Pi with as few variables as possible; apply the chain rule.

