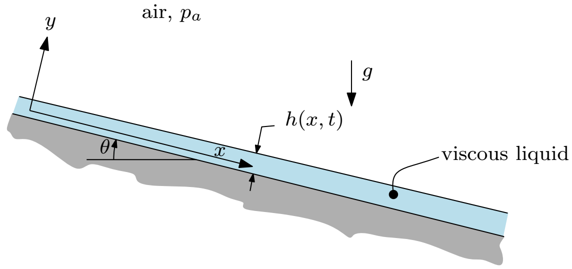
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 6.16

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A rigid plane surface is inclined at an angle θ relative to the horizontal and wetted by a thin layer of highly viscous liquid which begins to flow down the incline.

(a) Show that if the flow is two-dimensional and in the inertia-free limit, and if the angle of the inclination is not too small, the local thickness $h(x,t)$ of the liquid layer obeys the equation

$$\frac{\partial h}{\partial t} + c\frac{\partial h}{\partial x} = 0$$

where

$$c = \frac{\rho g h^2}{\mu}\sin\theta$$

(b) Demonstrate that the result of (a) implies that in a region where h decreases in the flow direction, the angle of the free surface relative to the inclined plane will steepen as the fluid flows down the incline, while in a region where h increases in the flow direction, the reverse is true. Does this explain something about what happens to slow-drying paint when it is applied to an inclined surface?

(c) Considering the result of (b) above, do you think that the steady-state solutions of the previous problems would ever apply in practice? Discuss.

