Flow inside a cylinder which is suddenly rotated

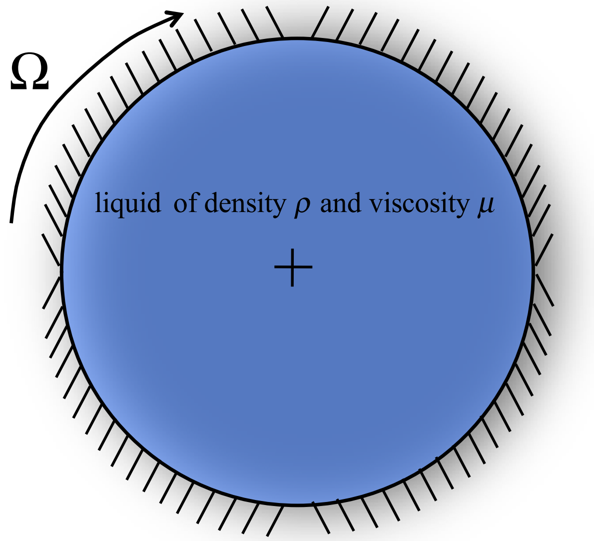

Figure 1: Geometry of the problem

A Newtonian liquid with density ρ and viscosity μ is initially at rest in a vertical, infinitely long cylinder of radius R. At time 0, the cylinder starts to rotate with a constant rotational speed, Ω about its axis. This problem is a transient flow like the Rayleigh plate problem (or Stokes' 1st problem). The governing equation is the following:

$$\boxed{\frac{\partial V_\theta}{\partial t} = \nu\left(\frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial V_\theta}{\partial r} - \frac{V_\theta^2}{r^2}\right)} \tag{1}$$

The boundary/initial conditions are the following:

$$\left\{\begin{array}{c} V_\theta = \Omega R \;\; at \;\; r = R,\, 0 < t \\ \\ V_\theta = finite \;\; at \;\; r = 0,\, 0 \le t \\ \\ V_\theta = 0 \;\; at \;\; t = 0,\, 0 \le r \le R \end{array}\right\} \tag{2}$$

This problem can be solved using separation of variables. The solution finally leads to Bessel equation and there is a bit of algebra in the process. The final solution will be:

$$\boxed{V_\theta(r,\theta) = \Omega r + 2\Omega R\sum_1^\infty \frac{J_1\left(\alpha_k r/R\right)}{\alpha_k J_0(\alpha_k)} exp\left(-\frac{\nu\alpha_k^2}{R^2}t\right)} \tag{3}$$

in which J_1 and J_0 are respectively the first and zeroth order Bessel functions of the first kind and α_k is the $k-th$ root of J_1.

