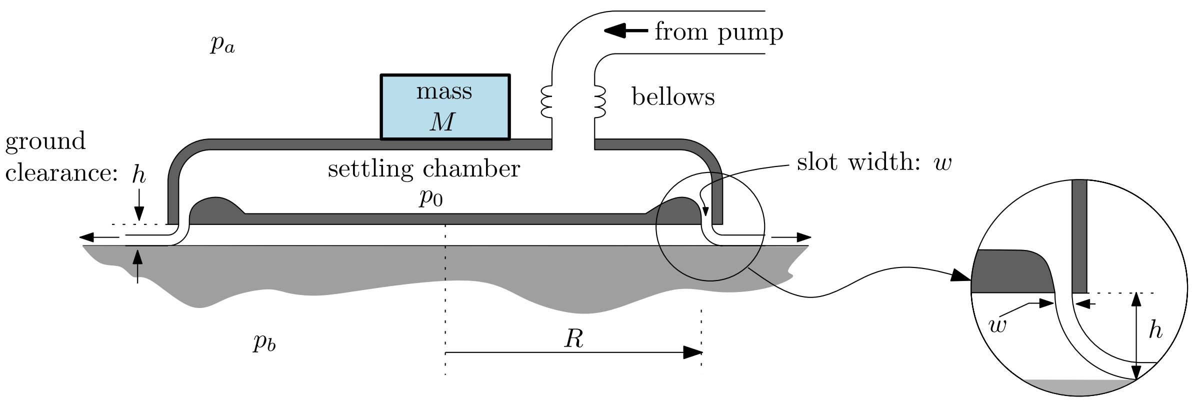
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 4.19

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A circular hovering platform of radius R is to support a mass M (its own mass plus a load). A thin, sheet-like jet (width w) is directed downward at the platform's periphery, as shown. The jet is fed from a settling chamber which is maintained at a pressure p_0 by an external pump. The system is to hover at an elevation h which is large compared to the width w of the sheet-like jet, but small compared with R.

When the jet is turned on, the pressure under the platform builds up and the platform rises until a steady state is reached. It is this steady state that we are concerned with.

(a) Describe the physical mechanism which allows the pressure p_b under the platform be higher than the atmospheric pressure p_a, in steady state, and thus to support a weight

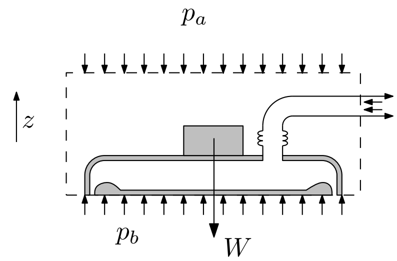
$$Mg = (p_b - p_a)\,\pi R^2$$

(b) Given the system weight Mg, the platform radius R, the jet width w, and the air density ρ, derive approximate expressions for (i) the volume flow rate Q of air required and (ii) the gage pressure p_0 required in the settling chamber, in order to maintain a ground clearance h. You may assume incompressible, inviscid flow in the peripheral jet, and make physical approximations consistent with the jet being thin compare with h ($w \ll h$) and the gage pressure $p_b - p_a$ below the platform being very small compared with the gage pressure $p_0 - p_a$ in the settling chamber.

Solution:

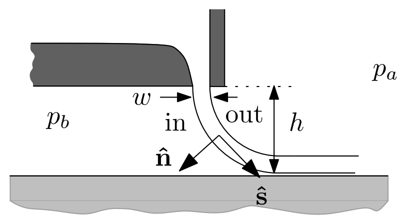
Given: Mg, R, w, ρ; **Unknown:** Q, p_0; **Assume:** incompressibility, inviscid, $R \gg h \gg w$

Consider a FBD that consists of the entire system:



$$F_z = 0 = -Mg - p_a(\pi R^2) + p_b(\pi R^2) \qquad \Rightarrow \boxed{p_b - p_a = \frac{Mg}{\pi R^2}} \tag{4.19a}$$

Consider streamline coordinates that describe the jet flowing outward:



The normal component of Euler's equation states that

$$-\frac{v^2}{r} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial n} - g\frac{\partial z}{\partial n} \qquad \text{(from streamline handout)} \qquad \Rightarrow \frac{\partial}{\partial n}(p + \rho g z) = \rho\frac{v^2}{r}$$

Integrate both sides with respect to dn:[1]

$$p + \underbrace{\rho g z\Big|_{\text{out}}^{\text{in}}}_{\approx 0} = \int_{\text{out}}^{\text{in}} \rho\frac{v^2}{r}dn \approx \frac{v^2}{h}w \qquad \Rightarrow \boxed{p_b - p_a = \rho\frac{v^2}{h}w} \tag{4.19b}$$

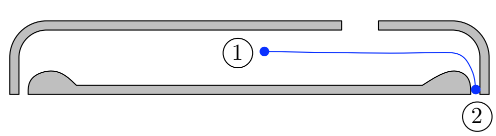
Combine Eqs. (4.19a) and (4.19b) to solve for v:

$$\rho\frac{v^2}{h}w = \frac{Mg}{\pi R^2} \qquad \Rightarrow \boxed{v = \sqrt{\frac{Mgh}{\rho\pi R^2 w}}} \tag{4.19c}$$

Since $Q = v \times \text{Area}$,

$$Q = \underbrace{(2\pi R)w}_{\approx\text{jet area}}\sqrt{\frac{Mgh}{\rho\pi R^2 w}} \qquad \Rightarrow \boxed{Q = 2\sqrt{\frac{Mg\pi wh}{\rho}}} \tag{4.19d}$$

Consider a streamline inside hovercraft:



[1]The hydrostatic term is negligible because the fluid sheet is thin.

Since $R \gg w$, assume quasisteady at station 1.

$$\Rightarrow p_1 + \cancel{\frac{1}{2}\rho {v_1}^2}^{\approx 0} + \cancel{\rho g h_1} = p_a + \frac{1}{2}\rho v^2 + \cancel{\rho g h_2}$$

$$\Rightarrow p_o = p_1 - p_a = \frac{1}{2}\rho v^2 \qquad \Rightarrow \boxed{p_o = \frac{1}{2}\left(\frac{Mgh}{\pi R^2 w}\right)} \tag{4.19e}$$

□

Problem Solution by Sungyon Lee, Fall 2005

