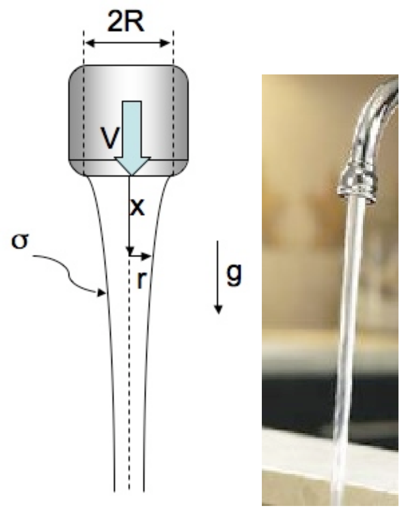
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 4.11

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



An incompressible, inviscid liquid flows with speed V vertically downward from the nozzle of the radius R. The liquid density ρ is high compared with that of the ambient air. The surface tension between the liquid and the air is σ.

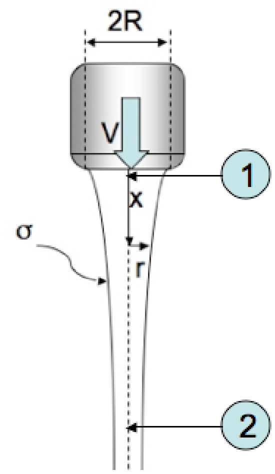
(a) Obtain an expression which relates the local radius r of the liquid stream to the distance x from the nozzle.

(b) Show that for sufficiently large x,

$$r \approx R\left(\frac{V^2}{2gx}\right)^{\frac{1}{4}}, \tag{4.11a}$$

(c) Write down all the criteria which must be satisfied for this expression to be a good approximation. State each criterion as 'x must be very large compared with y', where y is some combination of the given quantities V, R, g, and σ.

Solution:



- (*a*) Using Bernoulli between 1 and 2,

$$\frac{1}{2}\rho V^2 + P_a = \frac{1}{2}\rho V_1^2 - \rho g x + P_a, \tag{4.11b}$$

simplifying,

$$\frac{2}{\rho}(\frac{1}{2}\rho V^2 + \rho g x) = \frac{1}{2}\rho V_2^2 \frac{2}{\rho}, \tag{4.11c}$$

then,

$$V^2 + 2gx = V_2^2, \Rightarrow V_2 = \sqrt{V^2 + 2gx}. \tag{4.11d}$$

Also, from mass conservation

$$\pi R^2 V = \pi r^2 V_2, \Rightarrow \left(\frac{r}{R}\right)^2 = \frac{V}{V_2}. \tag{4.11e}$$

Finally adding the information from Bernoulli,

$$\left(\frac{r}{R}\right)^2 = \frac{V}{\sqrt{V^2 + 2gx}}. \tag{4.11f}$$

- (*b*) For $\frac{2gx}{V^2} \gg 1 \Rightarrow \left(\frac{r}{R}\right)^2 \approx \left(\frac{V^2}{2gx}\right)^{\frac{1}{2}}, \Rightarrow$

$$\boxed{\frac{r}{R} = \left(\frac{V^2}{2gx}\right)^{\frac{1}{4}}.} \tag{4.11g}$$

- (*c*) For the solution to apply,

$$\frac{V^2}{2gx} \ll 1, \qquad OR \qquad \frac{\rho V^2}{2\rho g x} \ll 1, \tag{4.11h}$$

then,

$$\boxed{x \gg \frac{V^2}{2g}.} \tag{4.11i}$$

Also, since we neglected surface tension,

$$\frac{\Delta P_\sigma}{\Delta P_x} = \frac{\frac{\sigma}{r}}{\rho g x} \ll 1, \Rightarrow \frac{\sigma}{r \rho g x} \ll 1. \tag{4.11j}$$

Now, let's get an estimate of the order of magnitude of $r(x)$ from (b),

$$r \approx R\left(\frac{V^2}{2gx}\right)^{\frac{1}{4}}, \Rightarrow r \sim \frac{RV^{\frac{1}{2}}}{(gx)^{\frac{1}{4}}}, \tag{4.11k}$$

now, substituting into the x requirement,

$$\frac{\sigma (gx)^{\frac{1}{4}}}{RV^{\frac{1}{2}} \rho g x} \ll 1, \Rightarrow x^{\frac{3}{4}} \qquad \frac{\sigma}{RV^{\frac{1}{2}} \rho g^{\frac{3}{4}}}, \tag{4.11l}$$

finally,

$$\boxed{x \gg \frac{\sigma^{\frac{4}{3}}}{R^{\frac{4}{3}} g \rho^{\frac{4}{3}} V^{\frac{2}{3}}}.} \tag{4.11m}$$

□

Problem Solution by MC, Fall 2008

