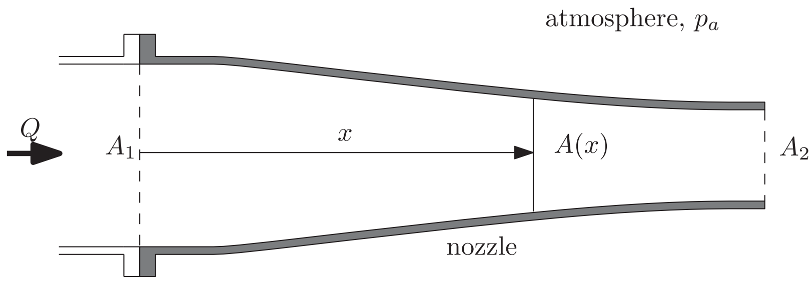
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 4.04

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



A nozzle with exit area A_2 is mounted at the end of a pipe of area A_1, as shown. The nozzle converges gradually, and we assum that the flow in it is (i) approximately uniform over any particular station x, (ii) incompressible, and (iii) inviscid. Gravitational effects are, furthermore, taken as negligible. The volume flow rate in the nozzle is given as Q and the ambient pressure is p_a.

(a) Derive an expression for the gage pressure at a station where the area is $A(x)$.

(b) Show, by integrating the x-component of the pressure force on the nozzle's interior walls, that the net x-component of force on the nozzle due to the flow is independent of the specific nozzle contour and is given by

$$F = \rho Q^2 \frac{(A_1 - A_2)^2}{2 A_1 {A_2}^2}$$

(c) The expression in (b) predicts that F is in the positive x-direction regardless of whether the nozzle is converging ($A_2 < A_1$) or diverging ($A_2 > A_1$). Explain.

Solution:

Given: Q, A_1, A_2 are constants.

(a) By mass conservation,

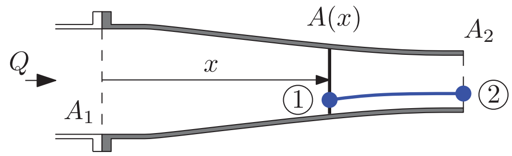
$$(\text{mass in}) = \rho v_1 A_1 = \rho v(x) A(x) = (\text{mass out})$$

Since there is no change in the mass inside the CV:

$$v_1 A_1 = Q = v(x)A(x)$$

$$\Rightarrow v(x) = \frac{Q}{A(x)}$$

Apply Bernoulli's equation along a stream line from station 1 to 2:



Note that all the assumption required for Bernoulli have been satisfied:

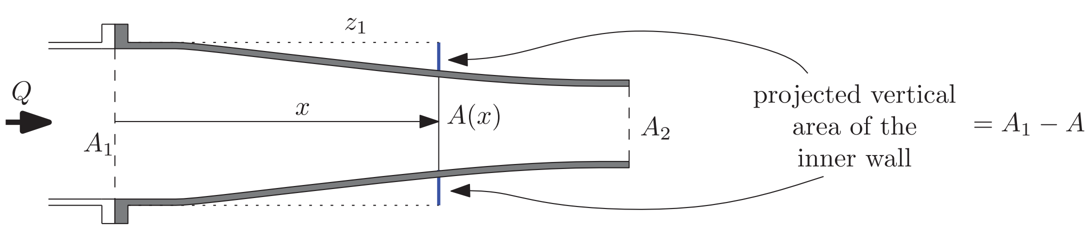
(a) inviscid
(b) along a streamline
(c) steady
(d) constant density
(e) no work/energy input or loss

$$\underbrace{p(x) + \frac{1}{2}\rho v(x)^2}_{\text{station 1}} = \underbrace{p_a + \frac{1}{2}\rho {v_2}^2}_{\text{station 2}}$$

Therefore,

$$p_g(x) = p(x) - p_a = \frac{1}{2}\rho\left({v_2}^2 - v^2\right) \qquad \Rightarrow \boxed{p_g(x) = \frac{1}{2}\rho Q^2\left(\frac{1}{{A_2}^2} - \frac{1}{A(x)^2}\right)} \qquad (4.04a)$$

(b) Integrate the pressure along the nozzle to obtain the x-component of pressure force



$$F_x = \int_1^2 dF_x = \int_1^2 p_g(x) d(\text{projected vertical area})$$

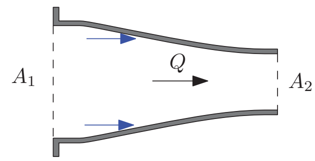
$$= \int_{A_1}^{A_2} p_g d\left(A_1 - A\right) = -\int_{A_1}^{A_2} p_g dA$$

We can reverse the integration limits to get rid of the minus sign in front and substitute Eq. (4.04a):

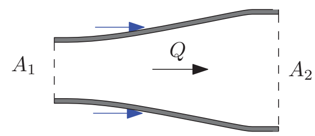
$$F_x = \int_{A_2}^{A_1} p_g dA$$
$$= \rho Q^2 \int_{A_2}^{A_1} \left(\frac{1}{{A_2}^2} - \frac{1}{A^2} \right) dA \qquad \Rightarrow \boxed{F_x = \frac{\rho Q^2}{2} \frac{(A_1 - A_2)^2}{A_1 {A_2}^2}} \qquad (4.04b)$$

(c)

For ($A_2 < A_1$), $p_g(x)$ is positive. Since the pressure is greater on the inside than the outside, the net pressure force acts on the inner wall.



For ($A_2 > A_1$), $p_g(x)$ is negative. Thus, the net pressure force acts on the outer wall, still pointing to the right.



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Problem Solution by Sungyon Lee, Fall 2005

