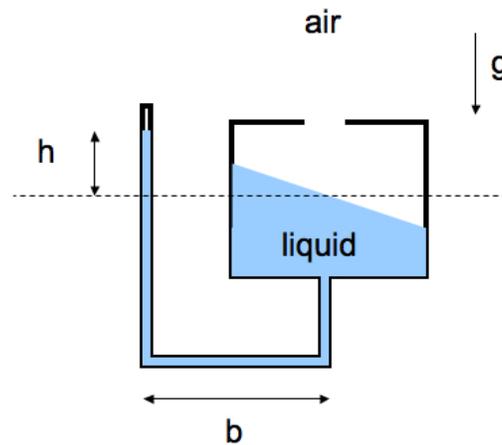


MIT Department of Mechanical Engineering  
2.25 Advanced Fluid Mechanics

**Problem 1.13**

*This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin*

*Accelerometer*



It is proposed to use the type of system shown in the sketch as an accelerometer for measuring the horizontal acceleration,  $a_x$ , and to obtain  $a_x$  from the formula

$$a_x = \frac{h}{b}g, \quad (1.13a)$$

where  $g$  is the gravitational acceleration.

- (a) Derive the formula used for  $a_x$  and state all assumptions clearly. Why doesn't the mass density of the liquid appear in the formula.
- (b) Under what circumstances would this be a good method of determining  $a_x$ , and under what conditions could it be not so good? Suggest improvements in the device.

**Solution:**

- (a) The fluid in the column behaves as if it were part of a larger container as long as it remains connected to the large reservoir (except for the neglected effects of surface tension). First, let's find the lines of constant pressure in the fluid. From hydrostatics,

$$\frac{\partial p}{\partial z} = -\rho g, \quad (1.13b)$$

$$\frac{\partial p}{\partial x} = -\rho a_x. \quad (1.13c)$$

$$(1.13d)$$

Then, integrating to obtain the pressure function,

$$p = -\rho g z + f_0(x), \quad (1.13e)$$

$$p = -\rho a_x x + f_1(z). \quad (1.13f)$$

$$(1.13g)$$

$$\Rightarrow p(x, z) = \Delta p + p_{atm} = -\rho(a_x x + g z) + p_{atm}. \quad (1.13h)$$

Now, for the line of constant pressure equal to  $p_{atm}$ ,

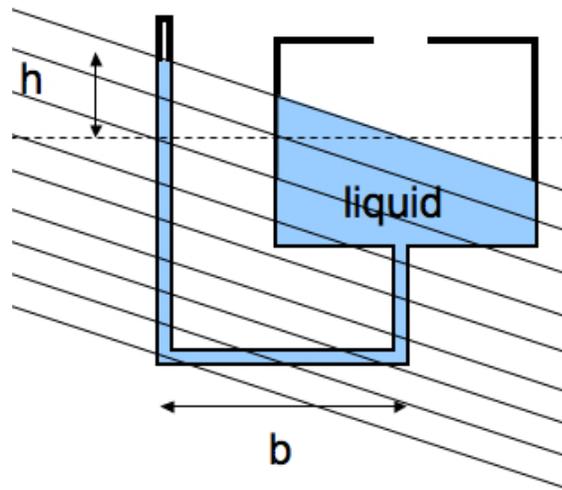
$$0 = a_x x + g z, \quad (1.13i)$$

at the water column,  $x = -b$ , and  $z = h$ , then

$$0 = -a_x b + g h, \Rightarrow a_x = \frac{h}{b} g. \quad (1.13j)$$

The density is not involved because even when the weight of the water column depends on it, the pressure driving the fluid up, also depends linearly on it, and the effect of the density cancels out.

- (b) If we draw lines of constant pressure, we'll notice that the perpendicular distance from the surface acts as an "effective depth distance". Then,  $b$  amplifies the magnitude of the read length,  $h$ , (that's why we have to divide  $h$  by it when getting the acceleration). Then, the longer  $b$  is, the better accuracy we'll have when reading the measurement. If  $b$  is too short, then the measurement error will grow, which sets a lower limits to the minimum required size of the device. Also, if too small surface tension and the effect of the meniscus will degrade the measurement.



Also, getting a number for rapid changing accelerations will prove to be challenging, since the height will change constantly. Besides that, any other form of acceleration will produce a considerable error (like rotation).

*Improvements:* There is always room for improvements

- (i) Measure the height difference between the front liquid level (not at the center as it is done now) and the meniscus in the tube.
- (ii) Reduce the volume of liquid in the container, most of it is not needed (not much because this volume stores the liquid that is driven into the tube when acceleration occurs, and acts as a hydraulic capacitor).

□

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