MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 6.05

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin

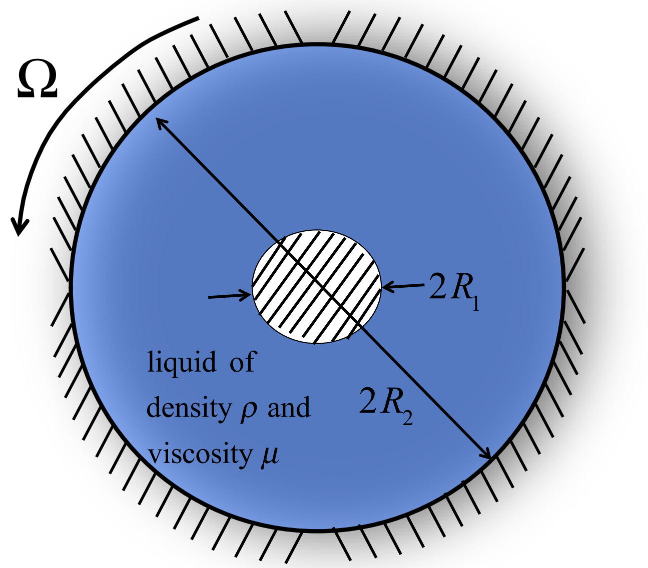

Figure 1: Geometry of the problem.

The general definition of the coefficient of viscosity, as applied to two-dimensional motions, is

$$\boxed{-\mu \equiv \frac{\tau}{d\gamma/dt}} \qquad (6.05a)$$

where $d\gamma/dt$ is the rate of change of the angle between two fluid lines which at time t are mutually perpendicular, the rate of change being measured by an observer sitting on the center of mass of the fluid particle.

- (a) Show that in terms of streamline coordinates,

$$\boxed{\tau = \mu\,(dV/dn - V/R)} \qquad (6.05b)$$

where V is the resultant velocity, R is the radius of curvature of the streamline, and n is the outward-going normal to the streamline.

- (b) A long, stationary tube of radius R_1 is located concentrically inside of a hollow tube of inside radius R_2, and the latter is rotated at constant angular speed ω. The annulus cottons fluid of viscosity μ. Assuming laminar flow, and neglecting end effects, demonstrate that

$$\frac{P}{\mu\omega^2 R_2^2} = \frac{4\pi}{(R2/R1)^2 - 1} \tag{6.05c}$$

  where P is the power required to turn unit length of the hollow tube.

- (c) Find the special form of (b) as $R_2/R_1 \to 1$, in terms of the gap width $h = R_2 - R_1$ and the radius R.

2.25 Advanced Fluid Mechanics
Fall 2013