2.25 Advanced Fluid Mechanics

October 15th 2008

Couette & Poiseuille Flows . [1] Some of the fundamental solutions for fully developed viscous flow are shown next. The flow can be pressure or viscosity driven, or a combination of both. We consider a fluid, with viscosity μ and density ρ. (Note: W is the depth into the page.)

- a) PLANE Wall-Driven Flow (Couette Flow)

  Parallel flow: $\underline{u}(y) = u(y)\hat{x}$, flow between parallel plates at $y = 0$ and $y = H$, wall-driven, and resisted by fluid viscosity.

$$U_{max} = \max(U_{top}, U_{bottom})$$

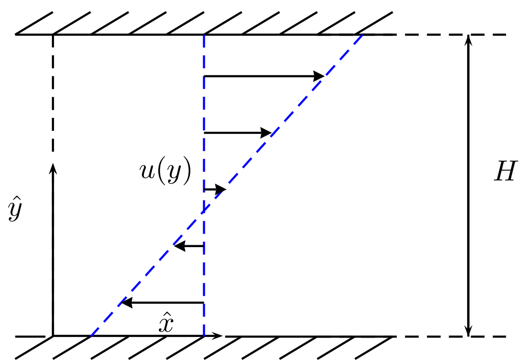
$$U_{min} = \min(U_{top}, U_{bottom})$$

$$U_{avg} = 0.5 * (U_{top} + U_{bottom})$$

$$Q' = Q/W$$

$$Q' = \frac{H}{2}(U_{top} - U_{bottom}) \text{ (signed)}$$

$$\tau_{w,top} = -\mu\frac{(U_{top} - U_{bottom})}{H}$$



$$u(y) = (U_{top} - U_{bottom})(y/H) + U_{bottom}$$

- b) PLANE Pressure-Driven Flow (Poiseuille Flow) (Stationary walls)

  Parallel flow: $\underline{u}(y) = u(y)\hat{x}$, flow between parallel plates at $y = 0$ and $y = H$, pressure driven.

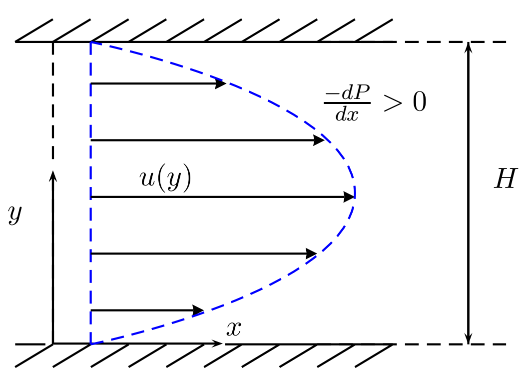
$$U_{max} = -\frac{dP}{dx}\frac{H^2}{8\mu}$$

$$U_{avg} = \frac{2}{3}U_{max}$$

$$Q' = -\frac{dP}{dx}\frac{H^3}{12\mu}$$

$$\tau_w = \frac{H}{2}(-\frac{dP}{dx})$$

(τ_w on either wall in the x direction)



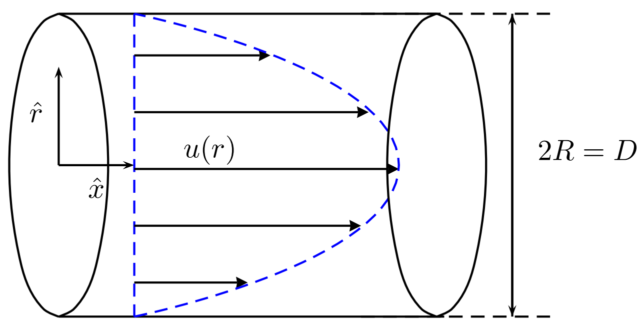
$$u(y) = \left(\frac{H^2}{2\mu}\right)(-\frac{dP}{dx})(\frac{y}{H}(1 - \frac{y}{H}))$$

[1] 2.25 Advanced Fluid Mechanics Problem

- c) TUBE Pressure Driven Flow (Poiseuille Flow) (Stationary walls)

  Parallel flow: $\underline{u}(r) = u(r)\hat{x}$, flow along a tube.

$\frac{-dP}{dx} > 0$



$$U_{max} = \left(\frac{R^2}{4\mu}\right)\left(-\frac{dP}{dx}\right)$$

$$U_{avg} = \left(\frac{R^2}{8\mu}\right)\left(-\frac{dP}{dx}\right)$$

$$Q = -\frac{dP}{dx}\frac{\pi R^4}{8\mu}$$

$$\tau_w(2\pi RL) = \pi R^2 L\left(-\frac{dP}{dx}\right)$$

(τ_w in the x direction)

$$u(r) = \left(\frac{R^2}{4\mu}\right)\left(-\frac{dP}{dx}\right)\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$f = \frac{\Delta P}{\frac{1}{2}\rho U^2 \frac{L}{D}} = \frac{16}{Re_D}$$

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