The geographer, speaking specially of the sandhill, says:—"The hill of sounding sand stretches 80 li east and west and 40 li north and south. It reaches a height of 500 ft. The whole mass is entirely constituted of pure sand. In the height of summer the sand gives out sounds of itself, and if trodden by men or horses, the noise is heard 10 li away. At festivals people clamber up and rush down again in a body, which causes the sand to give a loud rumbling sound like thunder. Yet when you look at it next morning the hill is just as steep as before."

Mr. Lionel Giles, from whose translation of the Tun-Huang-Lu these extracts are made, mentions that this sounding sandhill is referred to in another old Chinese book, the Wu Tai Shih.

JOSEPH OFFORD.

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The Green Flash.

I CAN confirm Dr. Schuster's observation of the green flash at sunrise, as in September last I saw a green segment herald the sun as it rose from the sea into a sky which was free from atmospheric glare (see the *Observatory*, December, 1914). Observations had previously been made at sunset, in one of which the eye was unquestionably fatigued, and the green flash was seen upon turning *away* from the sun at the instant after sunset. In a later sunset experiment precautions were taken to prevent retinal fatigue, and again the flash was seen.

My opinion is confirmed by Prof. Porter's experiment that "the reason why doubt has been cast upon records of the green flash is that the colour may arise in two different ways (complementary colour due to retinal fatigue, or dispersion by the atmosphere), and that the observer has not always been careful to avoid retinal fatigue, as was the case in my first (sunset) observation."

My observation, No. 2 (*loc. cit.*), is also in agreement with Dr. Schuster's experience, that with a very red sun no flash is to be seen.

W. GEOFFREY DUFFIELD.

University College, Reading, March 6.

Measurements of Medieval English Femora.

As the Editor of NATURE has insisted upon the great pressure at present upon his space I propose to reply to Dr. Parsons's letter, in the issue of March 11, adequately elsewhere. KARL PEARSON.

Galton Laboratory, March 15.

THE PRINCIPLE OF SIMILITUDE.

I HAVE often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of "laws" are put forward as novelties on the basis of elaborate experiments, which might have been predicted *a priori* after a few minutes' consideration. However useful verification may be, whether to solve doubts or to exercise students, this seems to be an inversion of the natural order. One reason for the neglect of the principle may be that, at any rate in its applications to particular cases, it does not much interest mathematicians. On the other hand, engineers, who might make much more use of it than they have done, employ a notation which tends to obscure it. I refer to the manner in which gravity is treated. When the question under consideration depends essentially upon gravity, the symbol of gravity (g) makes no appearance, but when gravity does not enter into the question at all, g obtrudes itself conspicuously.

I have thought that a few examples, chosen almost at random from various fields, may help to direct the attention of workers and teachers to the great importance of the principle. The statement made is brief and in some cases inadequate, but may perhaps suffice for the purpose. Some foreign considerations of a more or less obvious character have been invoked in aid. In using the method practically, two cautions should be borne in mind. First, there is no prospect of determining a numerical coefficient from the principle of similarity alone; it must be found if at all, by further calculation, or experimentally. Secondly, it is necessary as a preliminary step to specify clearly *all* the quantities on which the desired result may reasonably be supposed to depend, after which it may be possible to drop one or more if further consideration shows that in the circumstances they cannot enter. The following, then, are some conclusions, which may be arrived at by this method:—

Geometrical similarity being presupposed here as always, how does the strength of a bridge depend upon the linear dimension and the force of gravity? In order to entail the same strains, the force of gravity must be inversely as the linear dimension. Under a given gravity the larger structure is the weaker.

The velocity of propagation of periodic waves on the surface of deep water is as the square root of the wave-length.

The periodic time of liquid vibration under gravity in a deep cylindrical vessel of any section is as the square root of the linear dimension.

The periodic time of a tuning-fork, or of a Helmholtz resonator, is directly as the linear dimension.

The intensity of light scattered in an otherwise uniform medium from a small particle of different refractive index is inversely as the fourth power of the wave-length.

The resolving power of an object-glass, measured by the reciprocal of the angle with which it can deal, is directly as the diameter and inversely as the wave-length of the light.

The frequency of vibration of a globe of liquid, vibrating in any of its modes under its own gravitation, is independent of the diameter and directly as the square root of the density.

The frequency of vibration of a drop of liquid, vibrating under capillary force, is directly as the square root of the capillary tension and inversely as the square root of the density and as the $1\frac{1}{2}$ power of the diameter.

The time-constant (*i.e.*, the time in which a current falls in the ratio $e:1$) of a linear conducting electric circuit is directly as the inductance and inversely as the resistance, measured in electro-magnetic measure.

The time-constant of circumferential electric currents in an infinite conducting cylinder is as the square of the diameter.

In a gaseous medium, of which the particles repel one another with a force inversely as the nth power of the distance, the viscosity is as the $(n+3)/(2n-2)$ power of the absolute temperature. Thus, if $n=5$, the viscosity is proportional to temperature.

Eiffel found that the resistance to a sphere moving through air changes its character somewhat suddenly at a certain velocity. The consideration of viscosity shows that the critical velocity is inversely proportional to the diameter of the sphere.

If viscosity may be neglected, the mass (M) of a drop of liquid, delivered slowly from a tube of diameter (a), depends further upon (T) the capillary tension, the density (σ), and the acceleration of gravity (g). If these data suffice, it follows from similarity that

$$M=\frac{Ta}{g}F\left(\frac{T}{g\sigma a^2}\right),$$

where F denotes an arbitrary function. Experiment shows that F varies but little and that within somewhat wide limits may be taken to be 3·8. Within these limits Tate's law that M varies as a holds good.

In the Æolian harp, if we may put out of account the compressibility and the viscosity of the air, the pitch (n) is a function of the velocity of the wind (v) and the diameter (d) of the wire. It then follows from similarity that the pitch is directly as v and inversely as d, as was found experimentally by Strouhal. If we include viscosity (ν), the form is

$$n=v/d.f(\nu/vd),$$

where f is arbitrary.

As a last example let us consider, somewhat in detail, Boussinesq's problem of the steady passage of heat from a good conductor immersed in a stream of fluid moving (at a distance from the solid) with velocity v. The fluid is treated as incompressible and for the present as inviscid, while the solid has always the same *shape* and presentation to the stream. In these circumstances the total heat (h) passing in unit time is a function of the linear dimension of the solid (a), the temperature-difference (θ), the stream-velocity (v), the capacity for heat of the fluid per unit volume (c), and the conductivity (κ). The density of the fluid clearly does not enter into the question. We have now to consider the "dimensions" of the various symbols.

Those of a are	$(\text{Length})^1$,
. . . . . . v . .	$(\text{Length})^1 (\text{Time})^{-1}$,
. . . . . . θ . .	$(\text{Temperature})^1$,
. . . . . . c . .	$(\text{Heat})^1 (\text{Length})^{-3} (\text{Temp.})^{-1}$,
. . . . . . κ . .	$(\text{Heat})^1 (\text{Length})^{-1} (\text{Temp.})^{-1} (\text{Time})^{-1}$,
. . . . . . h . .	$(\text{Heat})^1 (\text{Time})^{-1}$.

Hence if we assume

$$h=a^x\theta^y v^z c^u \kappa^v,$$

we have

by heat	$1=u+v$
by temperature	$0=y-u-v$,
by length	$0=x+2-3u-v$,
by time	$-1=-z-v$;

so that

$$h=\kappa a\theta\left(\frac{avc}{\kappa}\right)^z.$$

Since z is undetermined, any number of terms of this form may be combined, and all that we can conclude is that

$$h=\kappa a\theta.F(avc/\kappa),$$

where F is an arbitrary function of the one variable avc/κ. An important particular case arises when the solid takes the form of a cylindrical wire of any section, the length of which is perpendicular to the stream. In strictness similarity requires that the length l be proportional to the linear dimension of the section b; but when l is relatively very great h must become proportional to l and a under the functional symbol may be replaced by b. Thus

$$h=\kappa l\theta.F(bvc/\kappa).$$

We see that in all cases h is proportional to θ, and that for a given fluid F is constant provided v be taken inversely as a or b.

In an important class of cases Boussinesq has shown that it is possible to go further and actually to determine the form of F. When the layer of fluid which receives heat during its passage is very thin, the flow of heat is practically in one dimension and the circumstances are the same as when the plane boundary of a uniform conductor is suddenly raised in temperature and so maintained. From these considerations it follows that F varies as $v^{\frac{1}{2}}$, so that in the case of the wire

$$h\propto l\theta.\sqrt{(bvc/\kappa)},$$

the remaining constant factor being dependent upon the shape and purely numerical. But this development scarcely belongs to my present subject.

It will be remarked that since viscosity is neglected, the fluid is regarded as flowing past the surface of the solid with finite velocity, a serious departure from what happens in practice. If we include viscosity in our discussion, the question is of course complicated, but perhaps not so much as might be expected. We have merely to include another factor, ν^w, where ν is the kinematic viscosity of dimensions $(\text{Length})^2 (\text{Time})^{-1}$, and we find by the same process as before

$$h=\kappa a\theta.\left(\frac{avc}{\kappa}\right)^z.\left(\frac{c\nu}{\kappa}\right)^w.$$

Here z and w are both undetermined, and the conclusion is that

$$h=\kappa a\theta.F\left\{\frac{avc}{\kappa},\ \frac{c\nu}{\kappa}\right\},$$

where F is an arbitrary function of the *two* variables avc/κ and $c\nu/\kappa$. The latter of these, being the ratio of the two diffusivities (for momentum and for temperature), is of no dimensions; it appears to be constant for a given kind of gas, and to vary only moderately from one gas to another. If we may assume the accuracy and universality of this law, $c\nu/\kappa$ is a merely numerical constant, the same for all gases, and may be omitted, so that h reduces to the forms already given when viscosity is neglected altogether, F being again a function of a single variable, avc/κ or bvc/κ. In any case F is constant for a given fluid, provided v be taken inversely as a or b.

RAYLEIGH.

PERISCOPES.

WHILE the periscope of the submarine is developing in the direction of greater optical perfection and elaboration, there has been a return to the simplest and earliest types of periscope for use in land warfare. Some of these trench periscopes recall the polemoscope, described by Helvelius in the seventeenth century for military purposes; this polemoscope in its simplest form consisted of two mirrors with their reflecting surfaces parallel to each other, and inclined at 45° to the direction of the incident light. These mirrors were mounted in a tube and separated a convenient distance (Fig. 1).

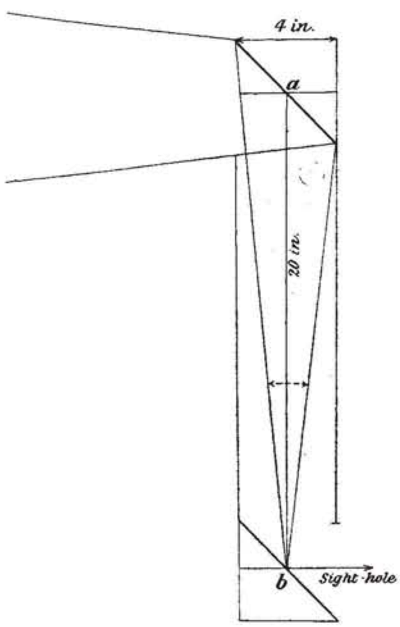

FIG. 1.

For modern trench warfare the convenient separation is about 18 to 24 in., and the mirrors are mounted in tubes, in boxes of square or oblong section, or attached to a long rod. In each case it is necessary that the mirrors should be fixed at the correct angle, and that there should be no doubling or distortion of the image.

The principal requirements of these trench periscopes are portability, lightness, small size and inconspicuous appearance, and large field of view. When there are no lenses the field of view is exactly the same as would be obtained by looking through a tube of the same length and diameter. Thus, with mirrors of 2 in. by 3 in. and a separation of about 22 in., a field of view of 5° would be obtained; and by moving the eye about, this field could be nearly doubled.

By using a box of oblong section the horizontal field of view can be increased without unduly increasing the size of the periscope. As the field of view is somewhat limited in any case, the principal objection to the use of a telescope or binocular, viz., the reduced field, no longer applies, and many periscopes are arranged to be used with a monocular or a binocular telescope.

Most periscopes can be used with a magnification of two or three, *i.e.*, with one tube of an ordinary opera glass; but when higher magnification is to be used the mirrors must be of better quality, both as regards flatness of surfaces and parallelism of the glass. When the mirrors are large enough—8 to 10 centimetres wide—both telescopes of the binocular may be used, but in this case the requirements for the mirrors are even more stringent, as the images formed by the two telescopes will not coincide unless the mirrors are plane. When suitable lenses are placed between the mirrors, the size of the mirrors can be reduced or the field of view increased; it is easy to provide a small magnification of the image or even to arrange for a variable magnification.

In such cases the lenses must be arranged to give an erect image, or mirrors or prisms employed to erect the image. An example of a periscope of this type is shown in Fig. 2, where the mirrors are replaced by reflecting prisms, and the prisms erect the image in much the same way as the prisms of a prism binocular.

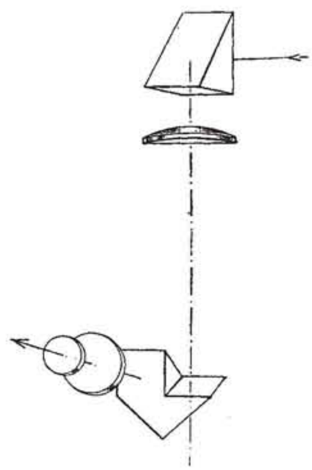

FIG. 2.

This arrangement is very suitable for a large magnification, but for larger fields the prism is unsuitable, unless it be silvered, and it is preferable to erect the image by means of lenses.

When longer tubes are used or larger fields are required, the design should approximate to that used in the submarine periscope.

This optical system has been steadily developed since its first introduction by Sir Howard Grubb in 1901.

The system consists of two telescopes, of which one is reversed, so that the image would be reduced in size, while the other magnifies this image, so that the final image is of the same size as the object, or is magnified one and a quarter or one and a half times. (As a very large angular field of view is required in these periscopes, the beam reflected into the tube must cover a large angle, and would soon fall on the sides of the tube; the reversed telescope, however, reduces the angle of the beam, and so enables it to pro-