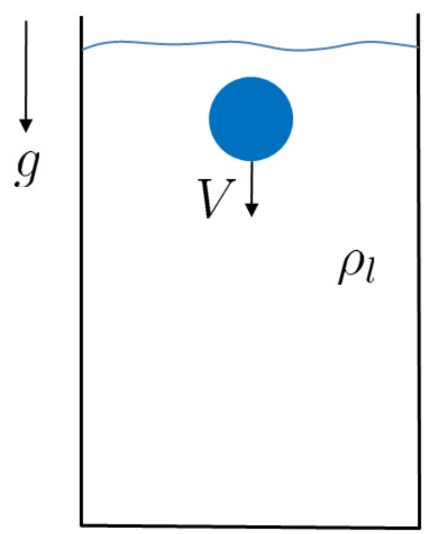
MIT Department of Mechanical Engineering

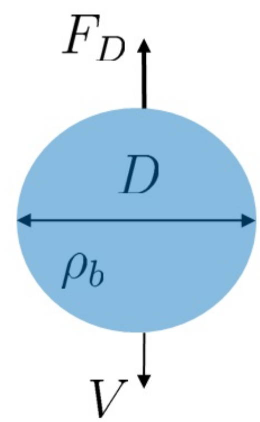
# 2.25 Advanced Fluid Mechanics

## Problem 7.03

*This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin*

A metal ball falls at steady speed in a large tank containing a viscous liquid. The ball falls so slowly that it is known that the inertia forces may be ignored in the equation of motion compared with the viscous forces.




(a) Perform a dimensional analysis of this problem, with the aim of relating the speed of fall $V$, to the diameter of the ball $D$, the mass density of the ball $\rho_b$, the mass density of the liquid $\rho_l$, and any other variables which play a role. Note that the "effective weight" of the ball is proportional to $(\rho_b - \rho_l)g$.

(b) Suppose that an iron ball (sp. gr.=7.9, $D$=0.3 cm) falls through a certain viscous liquid (sp. gr. = 1.5) at a certain steady-state speed. What would be the diameter of an aluminum ball (sp. gr. = 2.7) which would fall through the same liquid at the same speed assuming inertial forces are negligible in both flows?

## Solution:

(a) Non-dimensional Groups

In steady state, the body force (weight, $W$) must be balanced with buoyancy ($F_B$) and drag ($F_D$) forces.

$$W_{eff} = (\rho_b - \rho_l)g\left(\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\right) = F_D \tag{7.03a}$$

| $\rho_l$ | $\mu$ | $V$ | $D$ | $F_D$ |
|---|---|---|---|---|
| $[ML^{-3}]$ | $[ML^{-1}T^{-1}]$ | $[LT^{-1}]$ | $[L^1]$ | $[ML^1T^{-2}]$ |

Thus we have

$$\begin{aligned} n &= 5 \text{ variables} \\ k &= 3 \text{ primary variables} \\ \Rightarrow j &= 2 \text{ dimensionless group} \end{aligned}$$

For our primary variables, we choose (1) a fluid property: $\rho_l$, (2) a flow parameter: $V$, and (3) a geometric parameter: $D$. Therefore, the first dimensionless group is

$$\mu = f_1(\rho_l, V, D) \qquad or \qquad \underline{\Pi_1 = K_1 \mu \rho_l^a V^b D^c}$$

where $K_1$ is a constant. Thus,

$$\begin{aligned} M &: 0 = 1 + a \\ L &: 0 = -1 - 3a + b + c \\ T &: 0 = -1 - b \\ &\Rightarrow a = b = c = -1 \end{aligned}$$

$$\boxed{\Pi_1 = K_1 \frac{\mu}{\rho_l V D} = \frac{K_1}{Re}} \tag{7.03b}$$

Similarly, we can obtain the second non-dimensional parameter.

$$\Pi_2 = K_2 F_D \rho_l^a V^b D^c$$

$$\Rightarrow \boxed{\Pi_2 = K_2 \frac{F_D}{\rho_l V^2 D^2}} \tag{7.03c}$$

When $K_2 = 2$, this becomes the drag coefficient $C_D$, i.e.,

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

where $A$ is a characteristic cross-section area.

(b) Example of Similarity

In part (a), we obtained two non-dimensional variables. In highly viscous flows or fast speed flows, the drag force is a function of the Reynolds number. However, if the speed of the ball is very small ($Re << 1$), then the drag force is no longer a function of Reynolds number.

When the non-dimensional parameters are consistent in two situations, the flow fields are also similar. Let's make the drag coefficients are the same in the two cases.

$$C_D = \frac{(\rho_i - \rho_l) g \left(\frac{4}{3}\pi \left(\frac{D_i}{2}\right)^3\right)}{\rho V^2 D_i^2} = \frac{(\rho_a - \rho_l) g \left(\frac{4}{3}\pi \left(\frac{D_a}{2}\right)^3\right)}{\rho V^2 D_a^2}$$

where the subscripts $i$ and $a$ denote the iron and aluminum.

$$\Rightarrow \frac{(7.9 - 1.5) \times (0.3)^3}{(0.3)^2} = \frac{(2.7 - 1.5) \times D_a^3}{D_a^2}$$

Therefore, the diameter of an aluminum ball which satisfies the similarity is

$$\boxed{D_a = 1.6 \quad cm} \tag{7.03d}$$

□

*Problem Solution by Jaehyung Kim, Fall 2009*