

Turbulent flow in a pipe (scaling)

The notion of being steady in time in laminar flow is shown in Figure 1. However this is slightly different when you deal with turbulent flows (Figure 2).

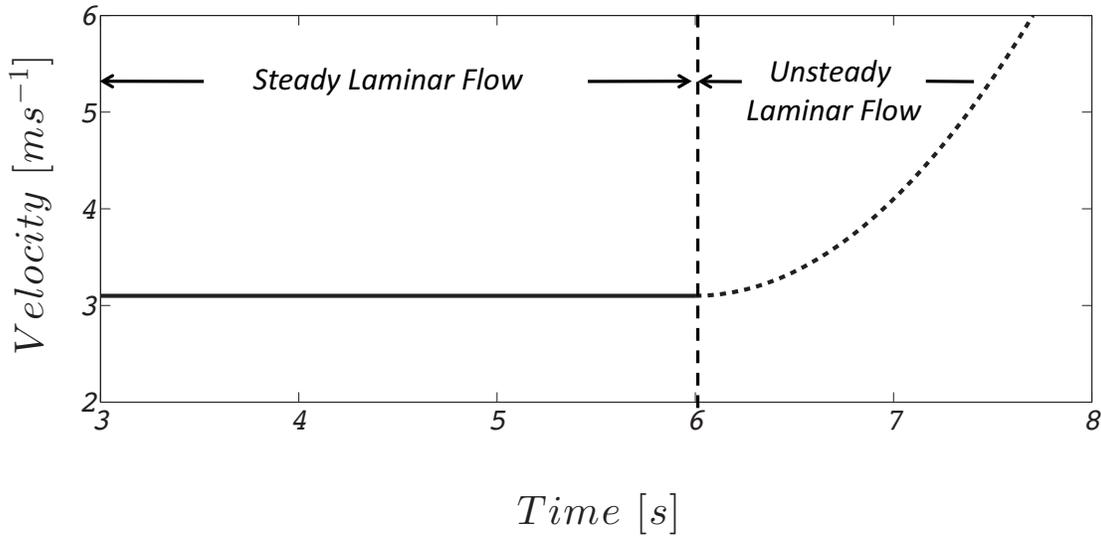


Figure 1: Laminar Flow (Steady vs. Unsteady)

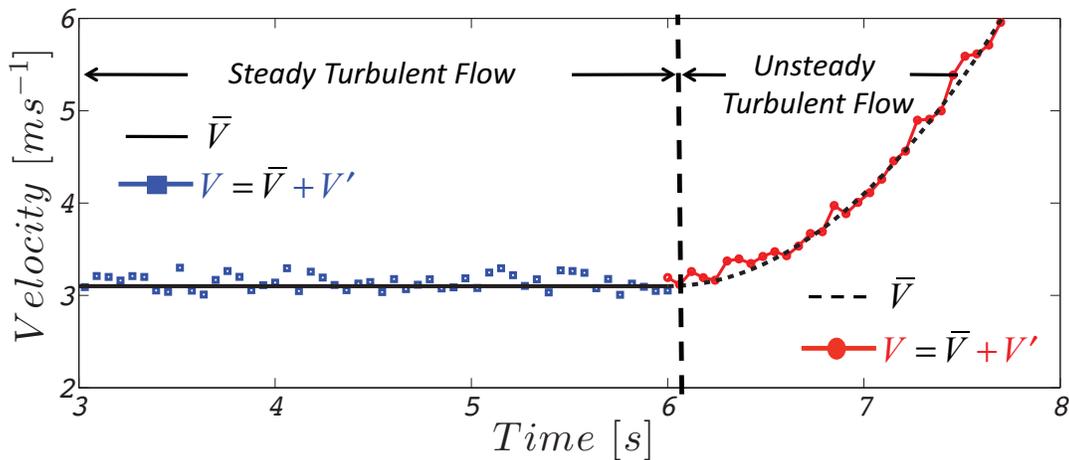


Figure 2: Turbulent Flow (Steady vs. Unsteady).

(a) Generally in turbulent flow your velocity is not steady and you have fluctuations. A common approach is to write every parameter in the following form:

$$V = \bar{V} + V' \quad (1)$$

Any variable can be resolved into a mean value \bar{V} plus a fluctuating value V' where by definition:

$$\bar{V} \equiv \frac{1}{T} \int_{t_0}^{t_0+T} V dt \quad (2)$$

where T is large compared to the relevant period of the fluctuations. The mean values \bar{V} can also change slowly with time, which is referred as unsteady turbulent flow. What in Shapiro 7.14 is mentioned as “steady and fully developed in the mean” describes the fact that:

$$\frac{\partial \bar{V}_z}{\partial t} = 0, \quad \frac{\partial \bar{V}_z}{\partial z} = 0 \quad (3)$$

The important fact is that when you try to deduce the governing equations of motion you should carry these fluctuation terms, V' , all along. Finally you will end up getting an equation very similar to Navier-Stokes which is called the Reynolds equation (there is still an ongoing debate on whether the idea of time-averaging in the turbulent flow is the best approach but those discussions are way beyond the scope of this course).

The Reynolds equation will have the following form:

$$\rho \frac{D\bar{V}}{Dt} + \rho \frac{\partial}{\partial x_j} \left(\overline{v'_i v'_j} \right) = -\nabla \bar{P} + \mu \nabla^2 \bar{V} + \rho \underline{g} \quad (4)$$

For the simple pipe flow, in the turbulent case (unlike the laminar case), even though the flow is “steady and fully developed in mean” i.e. $\rho \frac{D\bar{V}}{Dt} = 0$ the left hand side will not turn into zero. The equation (when simplified) turns into the following:

$$\rho \frac{\partial}{\partial x_j} \left(\overline{v'_i v'_j} \right) = -\nabla \bar{P} + \mu \nabla^2 \bar{V} \quad (5)$$

which shows that both ρ and μ are important parameters in the problem and we can not get rid of density as we did in the laminar case.

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