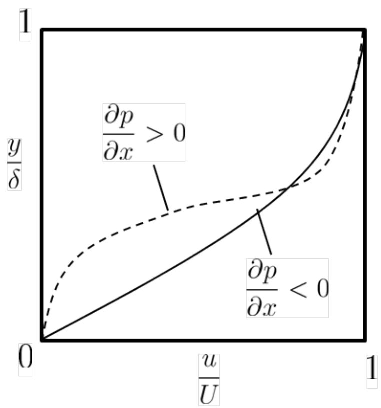
MIT Department of Mechanical Engineering
2.25 Advanced Fluid Mechanics

Problem 9.03

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



Consider a laminar boundary layer or the laminar sublayer of a turbulent boundary in two-dimensional flow. The fluid is incompressible and has constant viscosity.

Show that, at the wall, the velocity profile is concave upwards in flow with a favorable pressure gradient ($\partial p/\partial x$ ¡0). Whereas it is concave downwards for flow with an unfavorable pressure gradient ($\partial p/\partial x > 0$).

Solution:

Let's consider laminar boundary layer equation, i.e.,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{9.03a}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{9.03b}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9.03c}$$

The boundary conditions on the wall surface are given as

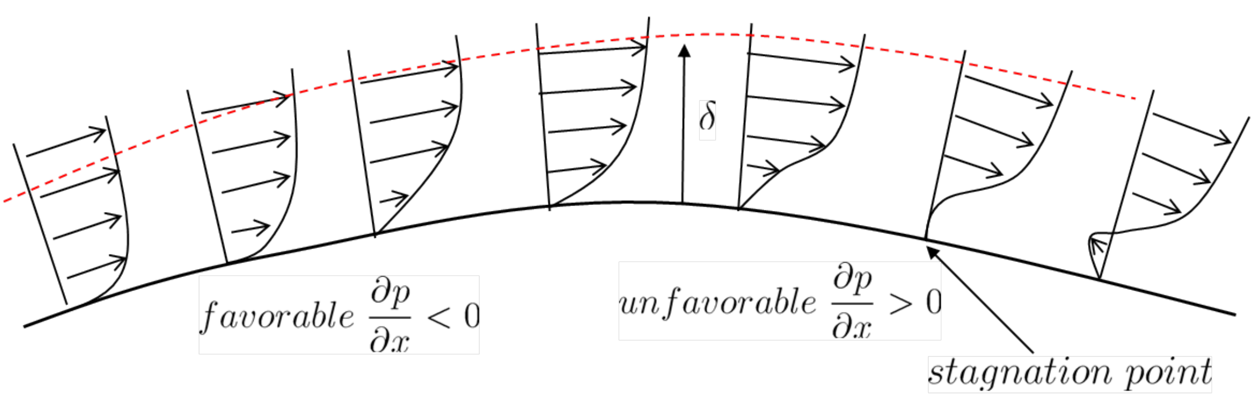
$$u = 0 \tag{9.03d}$$
$$v = 0 \tag{9.03e}$$

hence the x momentum equation becomes

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}} \tag{9.03f}$$

where the sign of the Eq. (9.03f) determines the sign of pressure gradient which also determines the sign of curvature. The separation point is defined as a point where shear stress becomes zero, i.e.,

$$\tau_o = \mu\frac{\partial u}{\partial y} = 0 \tag{9.03g}$$



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Problem Solution by J.Kim, Fall 2009

