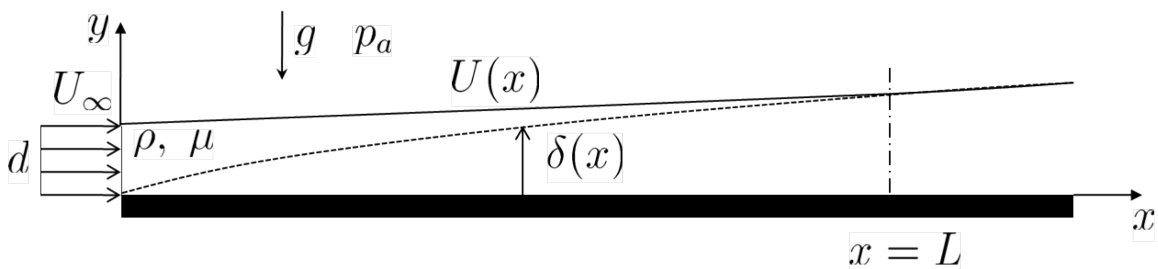
1. Boundary Layer on a Free Surface

A two-dimensional liquid fluid stream with height d, uniform velocity U_∞, density ρ and dynamic viscosity μ is incident on the top side of a flat plate as shown in the Figure. The gravitational acceleration is g and the atmospheric pressure is assumed equal to p_a. A laminar boundary layer develops over the flat plate and its displacement thickness grows and finally it reaches the free surface at a distance L from the leading edge of the plate.



Denote by δ(x) the displacement thickness of the boundary layer and by U(x) the horizontal fluid velocity at the edge of the displacement thickness. Assume that the horizontal velocity profile u(x, y) inside the boundary layer at a distance x downstream of the leading edge is given by the expression

$$\frac{u(x,y)}{U(x)} = a(x)\left(\frac{y}{\delta(x)}\right) + b(x)\left(\frac{y}{\delta(x)}\right)^3,\ \ 0 < y < \delta(x)$$

Assume that at the edge of the displacement thickness of the boundary layer the horizontal fluid velocity is equal to that of the inviscid flow above it.

a) (3 points) Determine $a(x)$ and $b(x)$ by providing appropriate physical justifications.

b) (3 points) Derive an expression relating $U(L)$ with U_∞, d and $\delta(L)$.

c) (3 points) Apply the mass conservation principle and derive an expression for the displacement thickness at x=L, i.e., $\delta(L)$.

d) (3 points) Determine the pressure variation at x =L : $p(L,y),\ \ 0 < y < \delta(L)$.

e) (4 points) Apply the momentum conservation principle to determine the horizontal force acting on the plate over its length 0<x<L.

f) (4 points) Assuming that the velocity profile $u(x,y)$ for x>L is given by the same expression as for x<L, obtain the shear stress (as a function of x) for x>L on the plate.