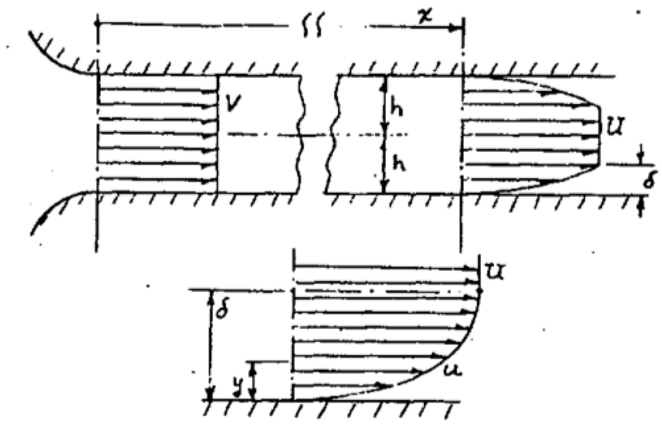
MIT Department of Mechanical Engineering

2.25 Advanced Fluid Mechanics

Problem 9.09

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin



Consider the two-dimensional, incompressible, steady flow of a fluid of constant viscosity in the entrance region of a slit of width $2h$. The flow enters the tube ($x = 0$) with the uniform velocity V. At distance x from the entrance, the boundary layer thickness is δ, and the core flow has the speed U. The boundary layer is laminar.

The ultimate objective is to analyze the entrance region, using momentum integral method, to determine how the boundary layer thickness, the skin friction stress, and the pressure gradient all vary with distance x. Since it is known that the fully-developed flow has parabolic velocity profile, it is agreed to assume, with some approximation, that the boundary layer velocity profile in the entrance region is also parabolic, following the equation

$$\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \tag{9.09a}$$

Carry your analysis only to the point of obtaining a differential equation relating the dimensionless boundary layer thickness δ/h as dependent variable to the dimensionless length x/h as the independent variable. Any constants of the problem may of course appear in the differential equation. Do not attempt to integrate the latter.

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