DEPARTMENT OF OCEAN ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.20 Marine Hydrodynamics

SAMPLE FINAL EXAM

THREE HOURS CLOSED BOOKS

INSTRUCTIONS:

For the problems in section A, fill in the required answers where indicated by ______________, (or in the provided space). When a list of options, [. . .] [. . .] [. . .], is provided select (by circling) all (none, one or more of) the options which apply. For problems in section B, write your solutions in the exam book provided.

Unless otherwise indicated, use gravitational acceleration $g = 10\ \mathrm{m/s^2}$, water density $\rho = 10^3\ \mathrm{kg/m^3}$, and kinematic viscosity $\nu = 10^{-6}\ \mathrm{m^2/s}$; and air density $\rho = 1\ \mathrm{kg/m^3}$, and kinematic viscosity $\nu = 10^{-5}\ \mathrm{m^2/s}$. Give all your numerical results in SI (kg, m, s) units. For numerical answers, you MUST give also the proper units (e.g., $5\ \mathrm{m^2/s}$ not 5).

There are a large number of problems and each individual answer can only be worth that many points, do NOT spend a disproportionate amount of time on any one problem.

YOUR NAME:______________________________

If you want us to post your course grade so that you can know it next week, please provide a nontrivial alphanumeric code here (don't forget your code):

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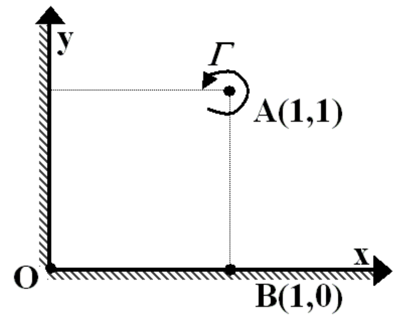
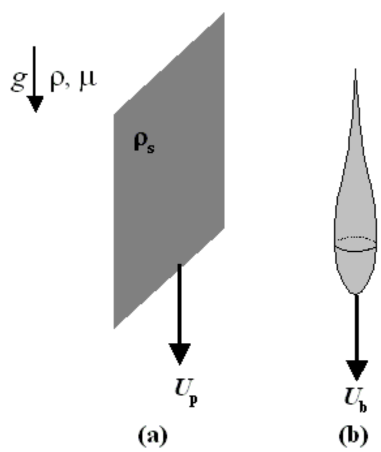
SECTION A [35%]

A1. A 2D point vortex of circulation Γ is placed at a point A (1, 1) in a corner created by the two walls x = 0 and y = 0. The horizontal force on the vortex (in terms of ρ and Γ) is F = ____________________ Neglecting gravity, the pressure difference between the two points O(0,0) and B(1,0) is: $(P_O - P_B)/\rho$ = ______________________



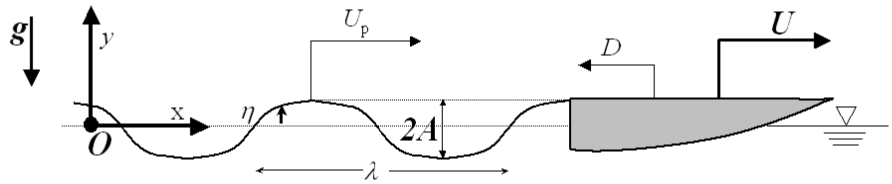
A2. A thin square plate (density $\rho_s = 2\,\rho$) of dimension 1cm×1cm and thickness 1mm is released in water from rest. If the plate falls parallel to the square side (figure (a) below), its terminal velocity is U_p = ___________________________________ (use friction coefficient $C_f = 1.328\, R_L^{-1/2}$ for laminar, and $C_f = 0.0725\, R_L^{-1/5}$ for turbulent boundary layer).

A teardrop-shaped streamlined body has the same length (1cm) and total wetted area (S=2cm^2) as the flat plate above, and has a weight in water (weight minus buoyancy) W=1.5·10^{-3}N. When dropped from rest vertically (figure (b) below), the terminal velocity of this body is found to be U_b = 0.25m/s. Based on this information, the form drag (total drag minus friction drag) of the teardrop-shaped body can be estimated to be D = ______________________.



A3. A 2D circle has an unsteady horizontal velocity $U(t)$ in an infinite fluid. The fluid itself has a uniform horizontal acceleration (to the right) of A = 1m/s^2. At some instant t = τ, the total horizontal hydrodynamic force on the circle is zero. Assuming potential flow, at t = τ, $\dot{U}(\tau)$ = ___________m/s^2 to the [right] [left].

A4. A 2D ship travels in deep water from left to rigth with constant speed U, generating a 2D wavetrain behind it of amplitude A. The waves are steady (fixed) with respect to the ship.



Relative to a fixed point, ***O***, (not moving with the ship), the wave train propagates from left to right with phase velocity U_p = ____________________; wave number k = ______________________; and frequency ω = ________________________ (all in terms of U and g). The wave resistance of the ship is D = _________________________ (in terms of A, ρ and g).

A5. A helicopter, H, is chasing after a wave train with free surface elevation given by $\eta = A\cos(kx - \omega t)$. The helicopter has a (constant) speed given by U_H, so that its position is given by $x_H = U_H t$.

If the wave elevation below the helicopter is $\eta_H(t) = \eta(x = x_H, t)$, then $d\eta_H/dt$ = ____ (in terms of ω, k, U_H and A). The elevation $\eta_H(t)$ is sinusoidal in time and can be written as $\eta_H = A\cos(\omega_E t)$. The "encounter frequency" of the wave that the helicopter sees is ω_E = ____ (in terms of ω, k and U_H).

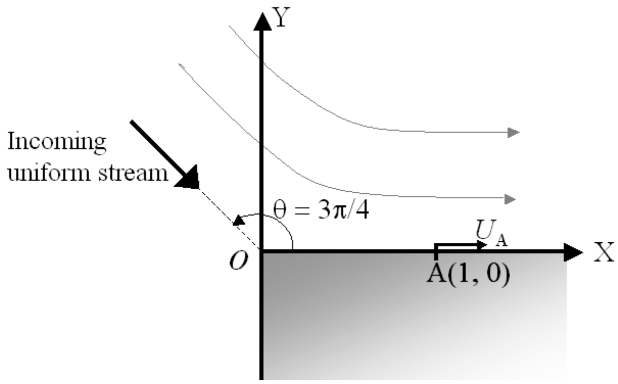
A6. A movie maker uses a small model ship of length L_m=1m to mimic the waves made by a large ship of length L_s=100m and speed U_s=10m/s. In order for the pictures in the movie to look "realistic," the speed of the model ship should be U_m =______________________. In addition, the time in the film (with the model) has to be [sped up] [slowed down] by a factor of _______________________.

A7. A vertical pipette (small hollow cylinder) of (inner) radius a is filled to a height h with a fluid of density ρ, and dynamic viscosity μ. The gravitational acceleration is g. At t = 0 the bottom of the pipette is removed. At this initial instant, the flow rate out of the pipette is given by Q =____________________ and the rate of change of h is given by dh/dt = ____.

SECTION B [65%]

[Note that in problems with multiple parts, later parts can often be solved without getting earlier parts (completely).]

B1. A steady uniform stream flows against the corner of a 2D rectangular building. The incoming uniform stream direction is symmetric with respect to the building so that the flow between $0 \le \theta \le 3\pi/4$ can be modeled as an interior corner flow (see "top view" figure below). Ignore gravity. At a point A=(X,Y)=(1,0) on the wall, the velocity is found to be U_A.



(a) Assuming potential flow, calculate the horizontal velocity $U_W(X)/U_A$ $(= U(X,0)/U_A)$ along the OX wall.

(b) Our final interest is to calculate the variation (with respect to x) of the boundary layer on the wall. To do this, we now assume a simple boundary layer profile given by:

$$\frac{u(y;x)}{U(x)} = \begin{cases} \sin\left(\dfrac{\pi}{2}\dfrac{y}{\delta(x)}\right) & 0 \le \dfrac{y}{\delta(x)} \le 1 \\ 1 & \dfrac{y}{\delta(x)} \ge 1 \end{cases}$$

For this velocity profile determine δ^*, θ, τ_o in terms of U, the fluid density ρ, the dynamic viscosity μ, and the single parameter describing the profile $\delta(x)$.

Hint: $\int \sin^2 ay\, dy = \frac{1}{4a}\left(2ay - \sin 2ay\right)$

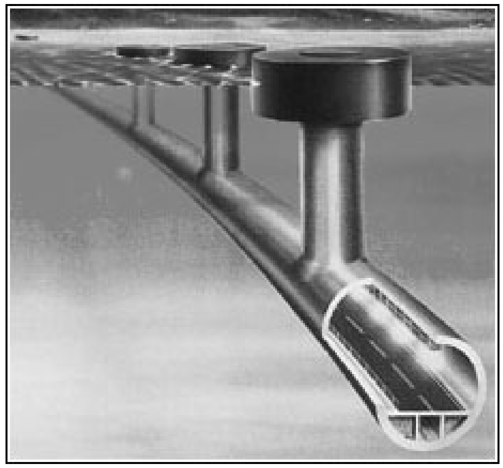
(c) The von Karman momentum integral equation for a boundary layer with a variable (potential flow) outer velocity $U(x)$ is given by:

$$\frac{\tau_o}{\rho} = \frac{d}{dx}\left(U^2\theta\right) + U\frac{dU}{dx}\delta^*$$

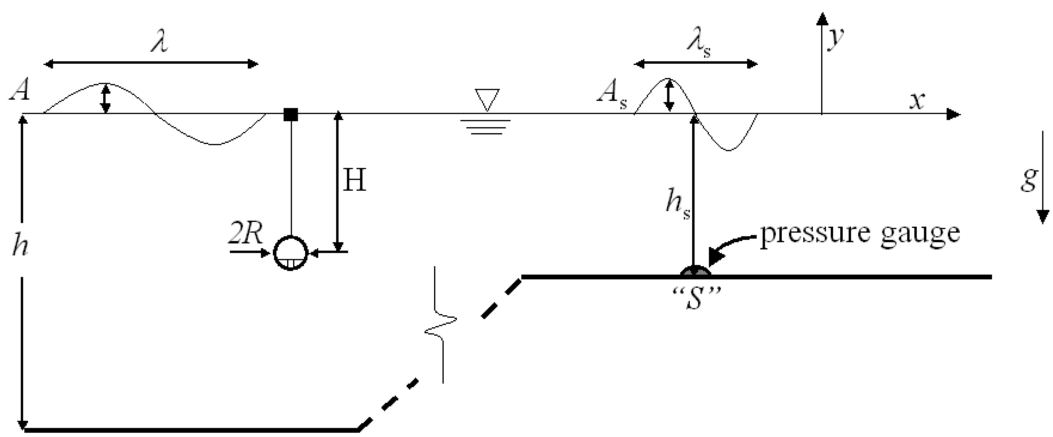
Substitute into this equation the results from (a) for $U(x)=U_w(X)$, and from (b) for δ^*, θ, τ_o to obtain an ODE for $\delta(x)$in terms of ρ, μ and U_A. Do not try to solve this ODE (yet).

(d) To see how the boundary layer thickness $\delta(x)$ increases with x, we assume that it has the form $\delta(x) \sim C\, x^\alpha$. Substitute this into the ODE you obtained in (c) to obtain the value of α (do not worry about the constant C).

B2. Recently, a novel design is proposed for the longest bridge in the world (spanning the Strait of Gibraltar connecting the continents of Europe and Africa). The bridge is in the form of a long submerged circular cylinder suspended from above by floating buoys (see below):



For an analysis of this design, we consider a 2D problem (below) consisting of a circular cylinder radius R=5m suspended at a depth H=30m. The average density of the bridge is $\rho_b=3\ \rho$. The bridge is to be deployed in water depth h=200m.



To estimate the incoming wave, a bottom mounted pressure gauge is placed at a station "S" which has a water depth of h_s=10m.

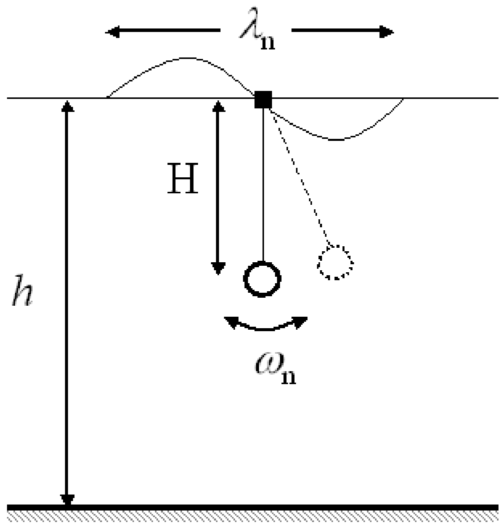
During a storm, the pressure gauge measures a dynamic pressure which is sinusoidal in time with period T = 10s and amplitude $(P_{max} - P_{min})/2 = 4\times10^4$Pa. Using linearized wave theory:

(a) Estimate the wavelength λ_s, the group velocity U_{gs} and the amplitude A_s of the waves at the station "S", h_S=10m.

(b) Estimate the wavelength λ, the group velocity U_g and the amplitude A of the waves at the bridge site h = 200m.

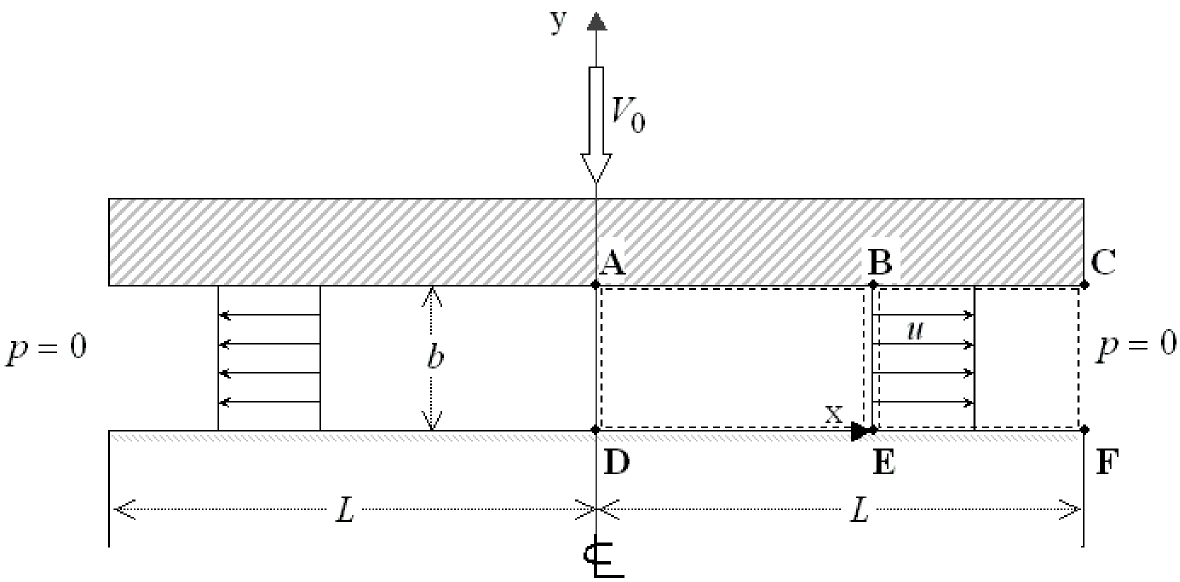
(c) Estimate the amplitude of the horizontal Froude-Krylov force F per unit length on the tube bridge.

(d) In general the seas at the bridge site contain wave components of different frequencies. What is the (i) frequency ω_n; and (ii) wavelength λ_n of the incident wave that will cause the bridge to be resonated in a swinging pendulum motion?



B3. The flow created by a flat plate falling onto a wall with constant velocity V_0 is modeled as a 2D ideal flow (ignore gravity). The width of the plate is $2L$, the height between the plate and the wall is $b(t)$. For $b << L$ the horizontal velocity in the gap can be assumed to be uniform in y: $u=u(x)$. By symmetry, on the centerline x = 0, $u(x=0) = 0$. Note that because of the velocity V_0 of the falling plate, the vertical velocity inside $v \neq 0$, and the flow is 2D.

Use the coordinate system in the figure below and give your answers in terms of L, $b(t)$, V_0, and the fluid density ρ.



(a) Applying conservation of mass in a control volume ABED from x = 0 to x = x, show that the horizontal velocity at any x is given by $u(x) = x\, V_0/b$.

(b) Using the result from (a), applying the differential form of the continuity equation and the proper boundary condition(s), show that the vertical velocity varies linearly with y and is independent of x. Write down $v(y)$ in terms of V_0 and b.

(c) Noting that $db/dt = -V_0$ obtain $\frac{\partial u}{\partial t}$ and $\frac{\partial v}{\partial t}$ as functions of b, V_0, x and y.

(d) Using the results from (c) and the inviscid Navier-Stokes (Euler) equation for v, show that the pressure inside the gap is a function of x only, i.e., $p = p(x)$.

(e) Using the results from (c) and the inviscid Navier-Stokes (Euler) equation for u, determine the pressure $p(x)$ at any point x inside the gap if the pressure at the exit x = L is zero.

(f) Calculate the vertical force F_y acting on the falling plate by the fluid for a given b. How does F_y behave for very small b?

{Extra Credit}

(g) It turns out that the result in (e) for p(x) can be obtained using conservation of horizontal momentum in a time varying control volume (without using the Euler equation). Choose a control volume BCEF with a fixed horizontal extent (from x = x to x = L) but with a height which changes in time (from y = 0 to y = b(t)).

(i) Using your results from (a) and (b), calculate the net horizontal momentum flux through the four surfaces of the control volume $\rho \oint_S u\left(\vec{U} \cdot \vec{n}\right)dS$ (hint: the horizontal momentum flux through BC is not zero because of V_0).

(ii) Noting that the control volume is changing, calculate the total change of horizontal momentum in this control volume given by the Eulerian change $\rho \int_\forall \frac{\partial u}{\partial t} d\forall$ plus the term $\rho \oint_S u\left(\vec{U} \cdot \vec{n}\right)dS$ obtained in (i).

(iii) Relating the momentum change in (ii) to the pressure acting on BE, obtain the expression for p(x).

*************************END OF FINAL EXAM*************************