Challenge Problem 1 **Name: ____________________**

1. So far, we have used a *fixed* control volume analysis to solve both steady and unsteady flow problems. Recall the transport theorem for a fixed control volume:

$$(1)\ \frac{dF_{system}}{dt} = \frac{d}{dt}\iiint_{M.V.(t)} f dV = \iiint_{C.V.} \frac{\partial f}{\partial t} dV + \iint_{C.S.} f\vec{V}\cdot\hat{n} dS$$

where

$F_{system} =$ value of a property F for a *system* of the same fluid particles (a system is the same as a material volume M.V. of fluid)
$f =$ property F per unit volume
$\vec{V} =$ velocity of fluid with respect to the chosen reference frame
M.V.(t) denotes a material volume (same as a fluid *system*) that is moving
C.V. denotes a *fixed* volume in space (a control volume)
C.S. denotes a *fixed* surface in space (a control surface)

Equation (1) can be extended to the case where we choose the control volume to be *deformable* by allowing the control surface to move in a convenient way for the problem we are studying (an example would be defining the space inside a cylinder with a moving piston as a control volume with the piston face as a moving control surface). If the control volume is *moving* in space and we examine it at an instant of time *t*, then we can consider it to *coincide* with a *fixed* volume at time *t*. We then perform the integrals over C.V.(*t*) and C.S.(*t*) just as we did in (1) for a fixed control volume. Thus the form of equation (1) is *not* changed for a moving control volume:

$$(2)\ \frac{dF_{system}}{dt} = \frac{d}{dt}\iiint_{M.V.(t)} f dV = \iiint_{C.V.(t)} \frac{\partial f}{\partial t} dV + \iint_{C.S.(t)} f\vec{V}\cdot\hat{n} dS$$

(A mathematical derivation of this fact, similar to the fixed control volume derivation in Recitation 2 notes, is possible.)

To gain insight in how to use (2) for a moving control volume, break the fluid velocity $\vec{V}$ at the moving surface C.S.(*t*) into two parts:

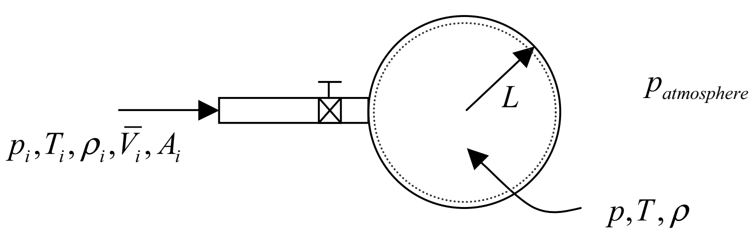
(3) $\vec{V} = \vec{U} + \vec{V}_r$ where

$\vec{U}$ = velocity of the control surface C.S.(t) in the frame of reference
$\vec{V}_r$ = velocity of the fluid relative to the control surface.

Now using (3), show that the following is an alternative form of (2) for a moving control volume:

$$(4)\quad \frac{dF_{system}}{dt} = \frac{d}{dt}\iiint_{C.V.(t)} f\,dV + \iint_{C.S.(t)} f\vec{V}_r \cdot \hat{n}\,dS$$

2. Consider the problem of an inflating balloon. Air enters the balloon from a supply pipe maintained at a constant pressure p_i and constant temperature T_i. The inlet air density is ρ_i, the inlet area is A_i, and the inlet average velocity is $\overline{V}_i$. Assume that the inlet density ρ_i is uniform over A_i and density ρ of the air in the balloon is uniform in space at any time t. Treat the air as an ideal gas where $p = \rho RT$. Approximate the balloon as a sphere of radius L, and assume it expands about a fixed center.



(a) Using conservation of mass, obtain an ordinary differential equation that describes the mass flow rate into the balloon $\rho_i A_i \overline{V}_i(t)$ in terms of the unknowns $L(t)$ and $\rho(t)$. (*Hint:* consider equation (4))

(b) As the pressure increases in the balloon, the temperature T is maintained directly proportional to the pressure p. Rewrite the equation in (a) for this case.

(c) Now replace the balloon with a rigid tank and assume that $\rho_i(t) = \rho(t)$. Solve for the log of the density ratio, $\ln(\rho/\rho_1)$, where ρ_1 is the density at a time t_1. Put your answer in terms of the tank volume $\forall$, A_i, and $\overline{V}_i$.

(d) Write a general equation similar in form to (4) for the conservation of momentum for a deformable control volume, using $\vec{V}$ to represent the unknown velocity field inside the balloon.

(e) In terms of $\vec{V}$ and the variables given in the problem statement, solve for the force on the balloon required to hold it on the pipe as it is being filled. Assume $p_i >> p_{atmosphere}$. Neglect any friction forces of the balloon on the air volume as it expands.
(You may not be able to evaluate all terms completely.)

(f) What is the force required to hold the balloon on the pipe once it has filled and $\overline{V}_i = 0$?