Ideal Symmetric Junction Elements

Having defined elements to model where energy can come from (sources) and where it can go to (dissipators) we next need to describe the ways these pieces can be assembled. To this end we will define ideal junction elements. Junction elements will describe the distribution of energy between other elements, so, in keeping with the philosophy of lumped-parameter modelling, we will assume that they don't do anything else. They don't supply, store or dissipate energy; they are *isenergic*[1].

The elements we have defined so far have a single *interaction port,* and one conjugate pair of variables was sufficient to describe their energetic transactions with their environments. The junction elements will be used to describe the interchange of energy between sets of elements, and therefore will have many ports (obviously, a one-port junction element would be pretty useless; it couldn't join anything to anything else). These *multi-port* elements obey the principle of conservation of energy which we will apply in differential form as a power balance condition, equation 3.4. For a junction element, stored energy and dissipated power are both zero, so summing across all ports results in the following condition.

$$\sum_{i=1}^{n} e_i f_i = 0 \tag{3.24}$$

where n is the number of ports. Now, if we add a symmetry[2] condition and require that each of the ports of a junction element be the same as each of the other ports, then it can be shown (see appendix) that *there are only two possible symmetric junction elements, and both are linear.* They are as follows.

<u>Common Effort Junction: Type Zero</u>

A common-effort or type zero junction has an unique effort associated with all connected bonds. A multi-port element with n ports is characterized by n constitutive equations. The n constitutive equations for an n-port zero junction are:

$$e_i = e \text{ for } i = 1 \text{ to } n \tag{3.25}$$

where e is the unique effort associated with the junction and the subscripts refer to labels on the n bonds or ports. The bond graph symbol for a four-port zero junction is shown in figure 3.26.

---

[1] Sometimes junction elements are called <u>non</u>-energic, but that is an odd and misleading term for one of the most important primitives of an energy-based modelling formalism. Junction elements are defined by the requirement that energy be constant; hence the term <u>iso</u>-energic or isenergic.

[2] The symmetry referred to is an invariance of properties under the operation of exchanging or permuting the interaction ports. See Appendix.

Identifying the unique (common) effort next to the zero symbol as in figure 3.26 is optional, but highly recommended.

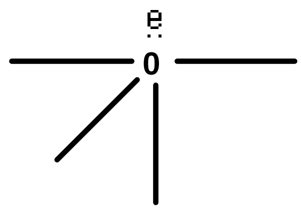

Figure 3.26: Bond graph symbol for a common effort or type zero junction.

Applying the power balance condition, equation 3.24, the net power flow into the junction is identically zero,

$$e \sum_{i=1}^{n} f_i + 0 \tag{3.26}$$

where the common effort, e, has been brought outside the summation. But this identity must hold for all values of the common effort, therefore this junction gives rise to a *flow continuity equation.*

$$\sum_{i=1}^{n} f_i = 0 \tag{3.27}$$

This flow continuity equation is a generalization of Kirchhoff's current law, encountered in electrical circuit theory. In the electrical domain, a zero junction corresponds to a parallel connection.

Sign Convention

Equation 3.27 is implicitly based on an assumption that the half arrows on all bonds point inwards as shown below.

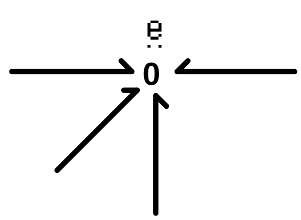

Figure 3.27: Sign convention assumed in flow continuity equation 3.27.

That is strictly in accordance with our sign convention for passive elements, but it is not always convenient. Because the junction element will be used to model the transmission and distribution of power between elements, it is more useful to allow the half arrow on any bond to point in either direction as circumstances dictate. Whatever the orientation of the half arrows, the constitutive equations for the junction require the efforts on all bonds to be identical, and that includes sign. Therefore, for a zero-junction, the half arrow denoting power sign convention also determines the sign of the flow. For any bond with an outward-pointing half arrow the corresponding flow in the continuity equation must have a negative sign, as in the following example.

$$f_1 - f_2 + f_3 - f_4 = 0 \tag{3.28}$$

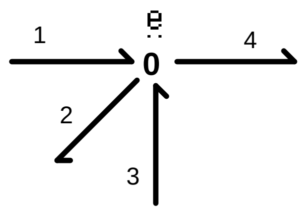

Figure 3.28: Sign convention assumed in flow continuity equation 3.28.

Causal Constraints

As the junction element has multiple interaction ports, it has multiple inputs and multiple outputs. However, the choice of what can be input and what can be output is subject to a strict constraint. By definition, the effort of the zero junction is common to all bonds, therefore one and only one bond may impress an effort on the junction element as shown in the following example.

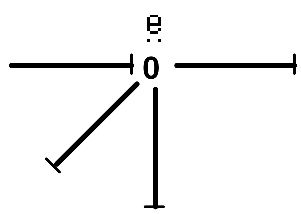

Figure 3.29: Example of a permissible causal assignment for a zero junction.

Any bond may impress the input effort, but only one may do so. Note that as soon as a single bond has been chosen to determine the effort on the junction, no further choice is available; all other bonds must provide an input flow. Conversely, that one bond must output a flow; and all others must output the common effort. This is known as a strong causal assignment.

On the other hand, provided no bond has been chosen to impress an effort, as many as n-1 bonds may be chosen to impose a flow on the junction. But if n-1 flows are imposed on the junction, the n^{th} flow is determined through the flow continuity equation. Therefore the n^{th} bond must determine the effort on the junction. This is known as a weak casual assignment.

Common Flow Junction: Type One

If you understand the common effort junction, then you also understand the common flow junction, which is simply its dual. A common-flow or type one junction element has an unique flow associated with all connected bonds. The n constitutive equations for an n-port one junction are:

$$f_i = f \text{ for } i = 1 \text{ to } n \qquad (3.29)$$

where f is the unique flow associated with the junction. The bond graph symbol for a four-port one junction is shown in figure 3.30.

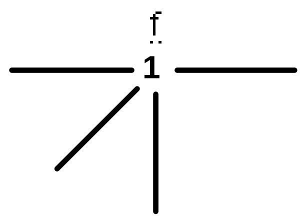

Figure 3.30: Bond graph symbol for a common flow or type zero junction.

The power balance condition for this junction element is as follows.

$$f \sum_{i=1}^{n} e_i + 0 \qquad (3.30)$$

where the common flow, f, has been brought outside the summation. Arguing as before, this identity must hold for all values of the common flow, therefore this junction gives rise to an *effort compatibility equation.*

$$\sum_{i=1}^{n} e_i = 0 \qquad (3.31)$$

This effort compatibility equation is a generalization of Kirchhoff's voltage law, encountered in electrical circuit theory. In the electrical domain, a one junction corresponds to a series connection.

Sign Convention

Equation 3.31 is implicitly based on an assumption that the half arrows on all bonds point inwards. As with the zero junction, it is more useful to let the half arrow on any bond point in either direction as circumstances dictate. Arguing as before, whatever the orientation of the half arrows, the constitutive equations for the junction require the flows on all bonds to be identical including sign. Therefore, <u>for a one-junction, the half arrow denoting power sign convention also determines the sign of the effort</u>. For any bond with an outward-pointing half arrow the corresponding effort in the compatibility equation must have a negative sign, as in the following example.

$$- e_1 - e_2 + e_3 + e_4 = 0 \qquad (3.32)$$

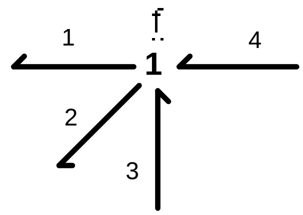

Figure 3.31: Sign convention assumed in effort compatibility equation 3.32.

Causal Constraints

As with the zero junction, the choice of what can be input to and output from a one junction is subject to a strict constraint. By definition, the flow is common to all bonds, therefore one and only one bond may impose a flow on the junction element as shown in the following example.

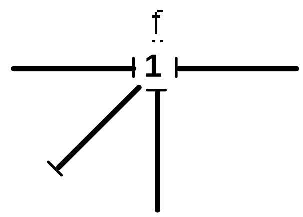

Figure 3.32: Example of a permissible causal assignment for a one junction.

The choice of any single bond to determine the flow on the junction is a strong causal assignment, and no further choice is available; all other bonds must provide an input effort. Conversely, that one bond must output a effort; and all others must output the common flow.

If no bond is selected to impose a flow, as many as n-1 bonds may be chosen to impress an effort on the junction. A weak causal assignment may be made by choosing n-1 efforts to be impressed on the junction. Because the n^{th} effort is determined through the effort compatibility equation, the n^{th} bond must determine the flow on the junction.