

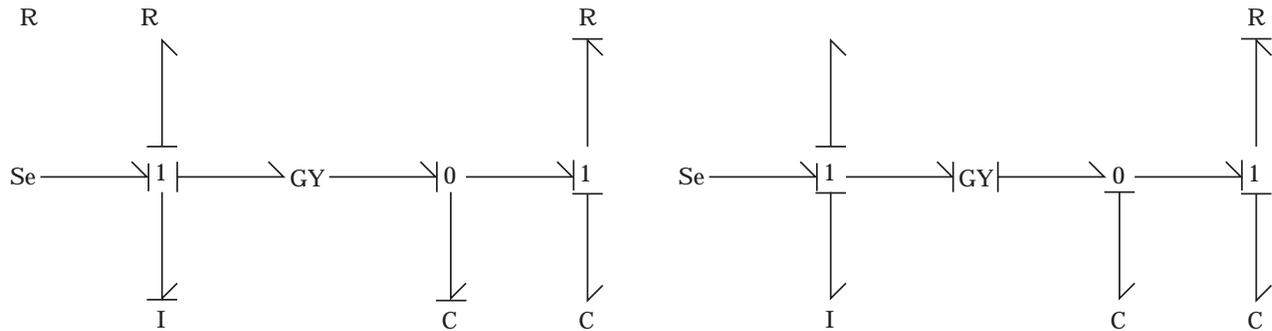
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

2.151: Advanced System Dynamics and Control –Spring 2003

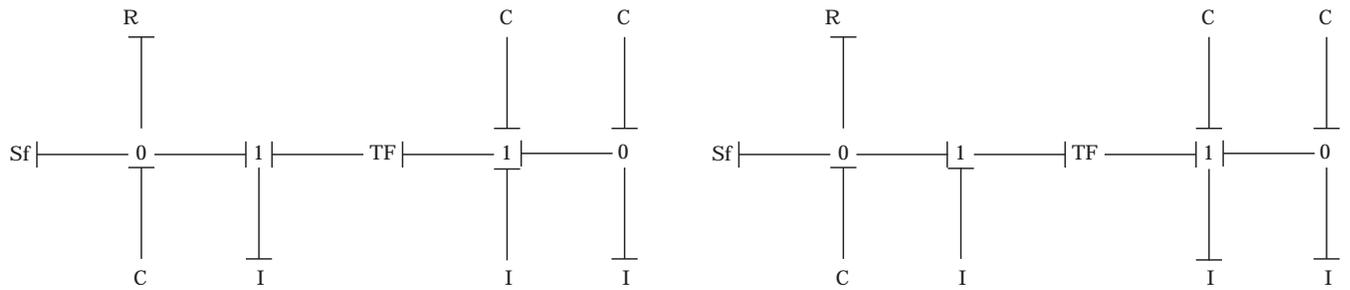
QUIZ 1

Problem 1(30pts):

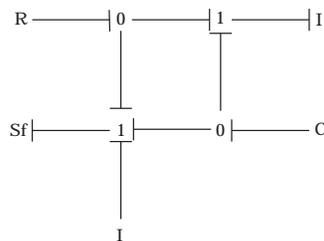
Two possible bond graphs are shown below. The minimum order is two. Though this system has a capacitance and inertance in integral causality, the gyrator does not allow oscillations.



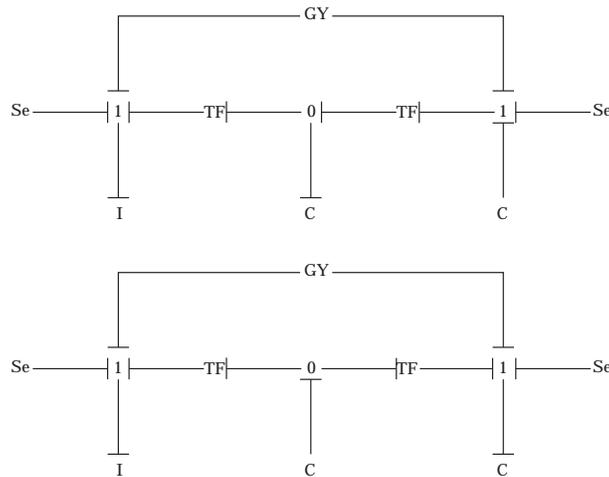
Two possible bond graphs are shown below. This system has a minimum order of five. It can oscillate.



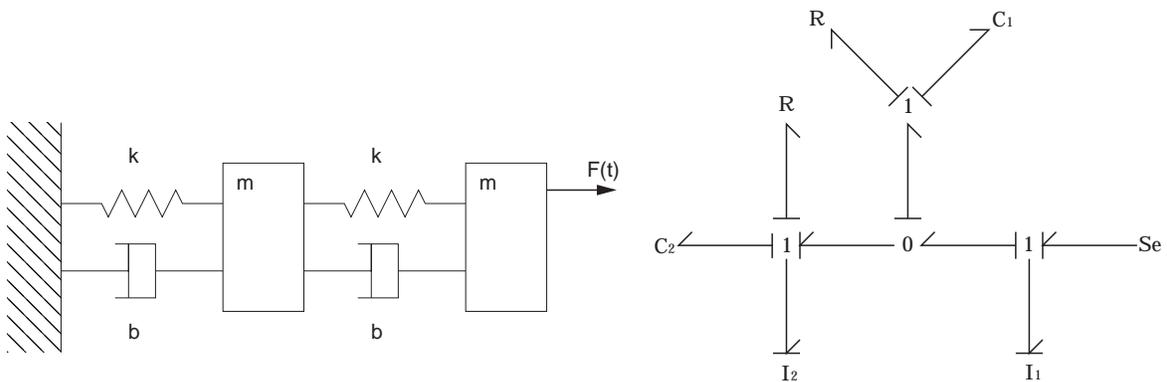
This system has a minimum order of two. It can oscillate.



This system has a minimum order of two.



Problem 2(30pts):

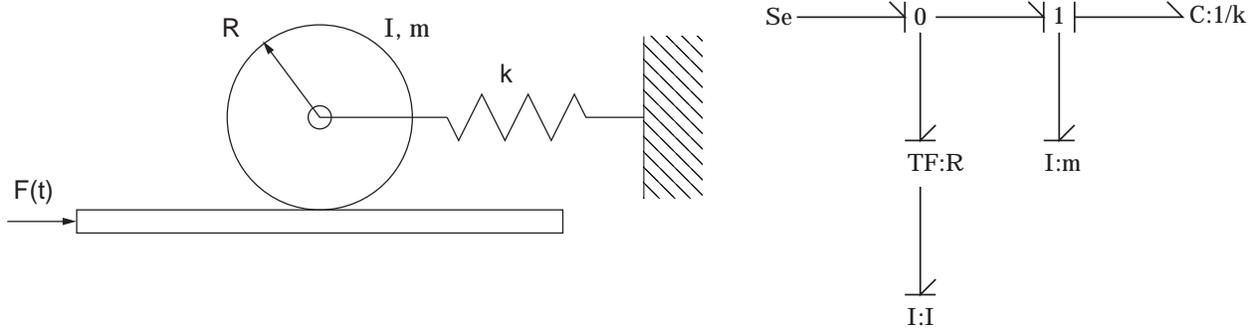


We see from the bond graph the system is at least of fourth order, with states associated with the two springs and two masses. Choosing the displacement across both springs and the momentum of the two masses, our state equations are,

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/m & -1/m \\ 0 & 0 & 0 & 1/m \\ -k & 0 & -b/m & b/m \\ k & -k & b/m & -2b/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} F(t) \quad (1)$$

If the force across the springs and the velocity of the masses is our state, then we have,

$$\frac{d}{dt} \begin{bmatrix} F_1 \\ F_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & k & -k \\ 0 & 0 & 0 & k \\ -1/m & 0 & -b/m & b/m \\ 1/m & -1/m & b/m & -2b/m \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m \\ 0 \end{bmatrix} F(t) \quad (2)$$

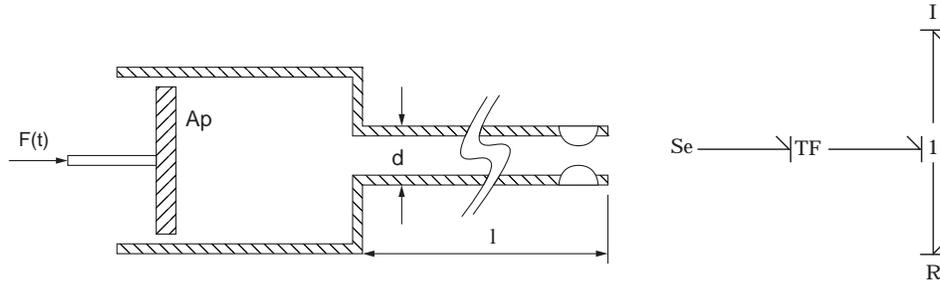


For the bond graph, note that the velocity of the cylinder can be related to the velocity of the rack, v and the angular velocity, ω , as follows,

$$v_m = v - r\omega \quad (3)$$

This relation indicates the need for two distinct flows and a zero junction. Choosing the spring displacement, the cylinder velocity and angular velocity, the state equations are,

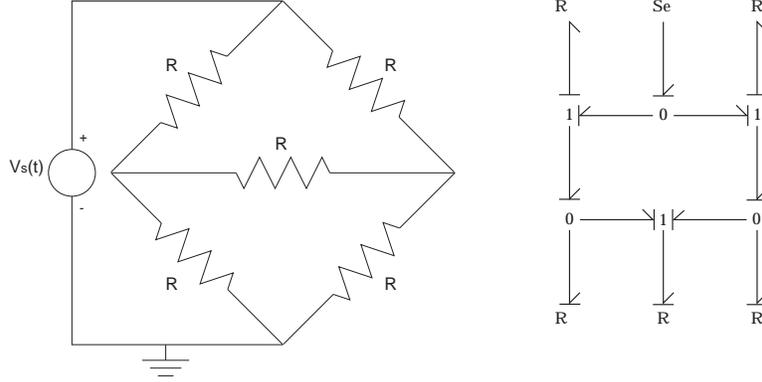
$$\frac{d}{dt} \begin{bmatrix} x_c \\ v_m \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k/m & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ v \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \\ r/I \end{bmatrix} F(t) \quad (4)$$



The bond graph above is an adequate description of the system. The fluid resistance can be a sum of the resistance through the pipe and the resistance due to the exit restriction. Choosing the volume flow rate as our state we get the following,

$$\dot{Q} = -RQ + \frac{A_p}{I} F(t) \quad \text{where } I = \frac{4\rho L}{\pi d^2} \quad \text{and } R = \frac{32\mu L}{\pi d^2} + R_{restriction} \quad (5)$$

The bond graph below indicates the system is zeroth order.



Problem 3(40pts):

a), b) A possible bond graph for the system driven with a current source is shown in the figure below. Note that including damping in the screw’s motion, or resistance in the motor’s circuit does not affect the minimum order, which is one. However, in order to describe the displacement of the nut, we need to add a second variable in our system description, raising the order to two.

c), d) A possible bond graph for the system driven with a voltage source is shown below. Again, including resistive type elements does not change the minimum order, which is now three when we include the displacement of the nut.

e) The total apparent inertia, seen through the rod’s end, is a sum of the nut’s and extension tube’s mass with the inertia of the screw and the motor armature as “seen” through the transformer. This effective inertia is,

$$I_{effective} = m_{nut} + m_{tube} + \frac{1}{T^2}(J_{screw} + J_{armature}) \tag{6}$$

where T is the transformer modulus for the screw-nut interface.

$$T = \left(\frac{1in}{5threads}\right) \left(\frac{1thread}{rev}\right) \left(\frac{1rev}{2\pi radians}\right) = \left(\frac{1ft}{120\pi}\right) \tag{7}$$

Then nut dimensions are given in the problem statement. To calculate the extension tube mass, we’ll assume a length of one foot, and radii of $3in$ and $2.9in$.

$$m_{nut} = \rho\pi(r_o^2 - r_i^2)L = (487lb/ft^3)\pi(3^2 - .5^2)in^2\left(\frac{ft^2}{144in^2}\right)3in\left(\frac{1in}{12ft}\right) = 23.2lb \tag{8}$$

$$m_{tube} = \rho\pi(r_o^2 - r_i^2)L = (487lb/ft^3)\pi(3^2 - 2.9^2)in^2\left(\frac{ft^2}{144in^2}\right)1ft = 6.3lb \tag{9}$$

$$J_{screw} = \frac{1}{2}mr^2 = \frac{1}{2}\pi(487lb/ft)18in\left(\frac{1ft}{12in}\right)(.5in)^4\left(\frac{1ft}{12in}\right)^4 = 3.5 \times 10^{-3}lbft^2 \tag{10}$$

$$J_{armature} = 0.0028ozins^2\left(\frac{1lb}{16oz}\right)\left(\frac{ft}{12in}\right)\left(\frac{32.2ft}{s^3}\right) = 4.7 \times 10^{-4}lbft^2 \tag{11}$$

$$I_{effective} = 23.2lb + 6.3lb + 497.4lb + 66.8lb = 593.6lb \tag{12}$$

Note that the inertia of the screw and extension tube combined is only 5% of the total inertia “seen” at the rod end.

