

Hydraulic Ram

Figure 5.32 shows a sketch of a hydraulic actuator of the kind commonly used in heavy earth-moving machinery. The difference in fluid pressure acting on the two sides of the piston exerts a force on the rod. As pressure is defined as force per unit area, the power transduction phenomenon connecting the hydraulic and translational domains is that of a transformer. In general, the piston area in the two chambers need not be the same¹, and the rate of increase of the volume of one chamber, Q_{c1} , need not equal the rate of decrease of the volume of the other chamber, Q_{c2} , so two transformers may be needed to describe the piston. They are connected to a one-junction on the translational side (representing the unique piston velocity) as shown in figure 5.33(a).

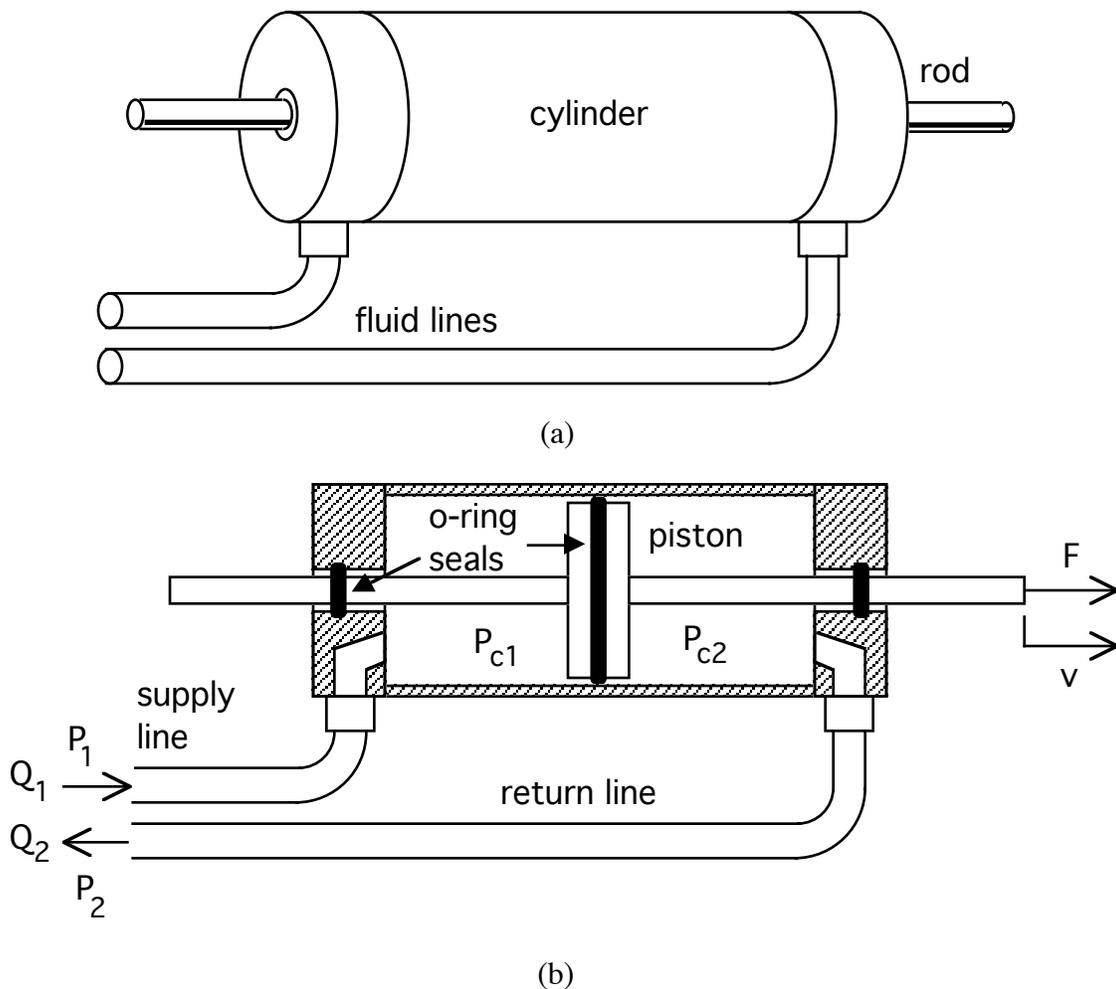


Figure 5.32: Sketch of a hydraulic actuator (a). A cross-sectional view is shown in (b).

¹ For example, the sketch shows a rod protruding from each side of the piston, but single-rod cylinders are also common; with a single rod, the area on one side of the piston exceeds that on the other side by the area of the rod.

If the piston area in each chamber is the same, then the two transformers may be replaced by a single equivalent transformer as shown in figure 5.33(b). Any increase in volume of one chamber equals the decrease in volume of the other and the transformer is connected to a one junction in the fluid domain (representing the common rate of volume change, Q_c). Constitutive equations are then as follows.

$$F = A (P_{c1} - P_{c2}) \quad (5.69)$$

$$Q_c = A v \quad (5.70)$$

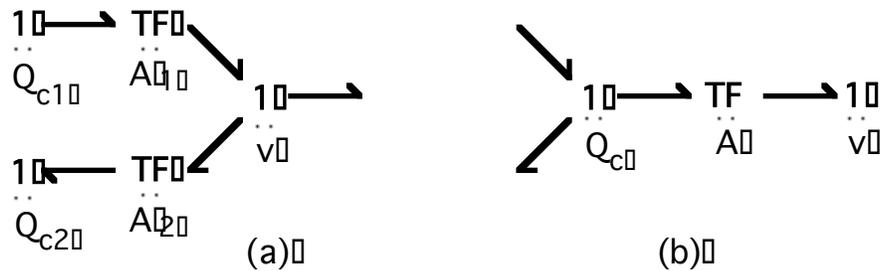


Figure 5.33: Bond graph fragments representing power transduction in the hydraulic actuator. (a): general model for unequal piston areas. (b): special case of equal piston areas.

Parasitic Dynamics

Like other power transduction devices, a hydraulic ram also embodies energy storage and dissipation effects which may need to be modeled. For simplicity, we will assume the hydraulic fluid to be incompressible (though that is often one of the prominent "parasitic" dynamic phenomena) and model only the inertial effects due to acceleration of the fluid and the power dissipation due to its motion through the lines and orifices. On the mechanical side we will model the inertia of the piston and rods (considered to be a single rigid body) and the friction due to the sliding seals, described for simplicity as a single linear friction element (though in reality, a significant component of the dissipative behavior is likely to resemble dry friction).

Friction due to flow in a pipe was considered in Chapter 4. A crude model of the inertial effects due to acceleration of the fluid may be derived as follows. Consider an incompressible fluid moving in a pipe of uniform cross-sectional area, A_p . The mass, m , of the "slug" of fluid between two cross sections separated by a distance l is determined by the dimensions of the pipe and the density, ρ .

$$m = \rho l A_p \quad (5.71)$$

Considerable simplification is obtained by assuming that the fluid flow velocity has the same value, v , at all points. That velocity is determined by the volumetric flow rate, Q , divided by the cross sectional area.

$$v = Q/A_p \quad (5.72)$$

Given these assumptions, the kinetic co-energy of the moving fluid is:

$$E_k^* = \frac{m}{2} v^2 = \frac{\rho l A_p}{2} \frac{Q^2}{A_p^2} = \frac{\rho l}{A_p} Q^2 \quad (5.73)$$

Thus, by analogy with a rigid body in translation, the energy stored in the moving fluid may be described by an ideal inertia, characterized by a linear relation between a flow, Q , and a generalized momentum, Γ , termed *pressure momentum*.

$$Q = \frac{\Gamma}{I} \quad (5.74)$$

The parameter $I = \rho l / A_p$ is the inertance of the fluid, determined by the geometric and material properties of the pipe and the fluid. From the fundamental definition of generalized momentum, the pressure difference accelerating the fluid is the time derivative of the pressure momentum.

$$\Delta P = d\Gamma/dt \quad (5.75)$$

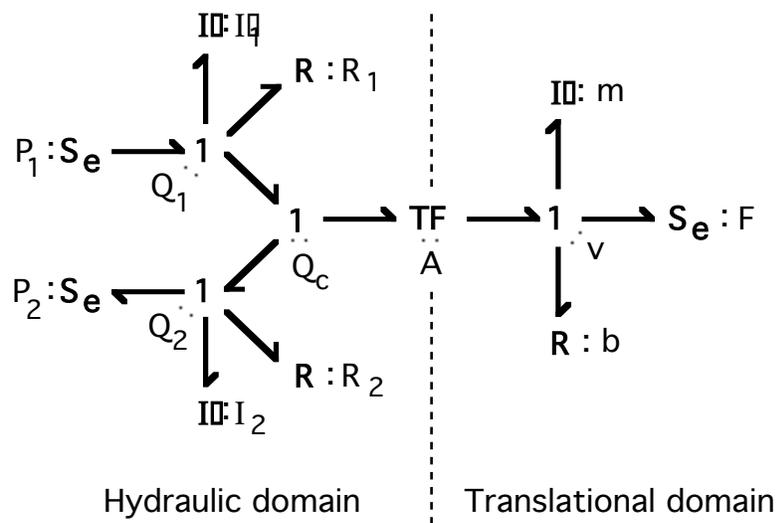
Thus pressure difference is proportional to the rate of change of fluid flow rate.

$$\Delta P = \frac{\rho l}{A_p} \frac{dQ}{dt} \quad (5.76)$$

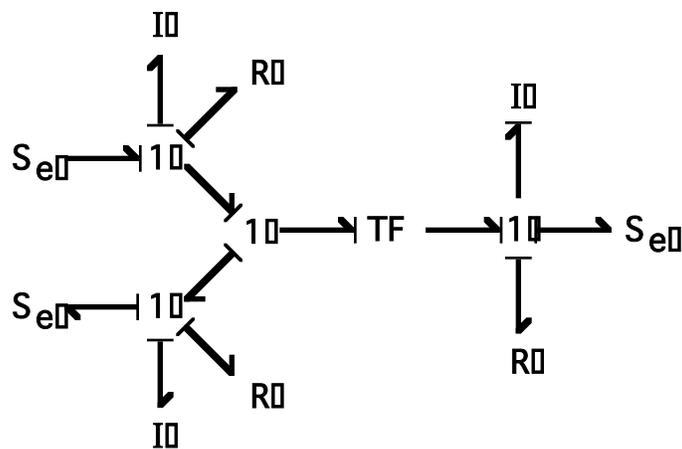
A bond graph of the complete system is shown in figure 5.34(a). For the purpose of this example, it has been assumed that the supply line is connected to a power source at constant pressure, P_1 , the return line to a sump at constant pressure, P_2 , (e.g. atmospheric pressure) and that the mechanical load is a constant force, F .

We may analyze this system by assigning causality. After assigning the required causality to the source elements, we are free to assign integral causality to one of the energy storage elements (arbitrarily chosen in figure 5.34(b) to be the translational inertia). Propagating the causal consequences of that choice through the graph completes the causality assignment.

Causality assignment reveals that two of the three inertias are dependent energy-storage elements. The velocity of the piston determines the flow rate in the fluid lines and a single variable is sufficient to characterize the kinetic energy stored in both the translating masses and the moving fluid. In effect, the piston and rods and the fluid in the lines have been described as a single "rigid body" even though the fluid in the lines would hardly be considered rigid in the usual sense of the word.



(a)



(b)

Figure 5.34: (a) Bond graph of a model of the hydraulic actuator including parasitic effects. (b) Bond graph with causality assigned.

The parasitic fluid elements may be transformed into the translational domain. The fluid inertias are equivalent to translational masses. Combining equations 5.69, 5.70 and 5.76 the effective mass due to each fluid line is

$$m_{\text{eff}} = A^2 I = A^2 \rho l / A_p \quad (5.77)$$

The fluid resistors are equivalent to linear translational friction elements. The effective friction coefficient due to each fluid line is

$$b_{\text{eff}} = A^2 R \quad (5.78)$$

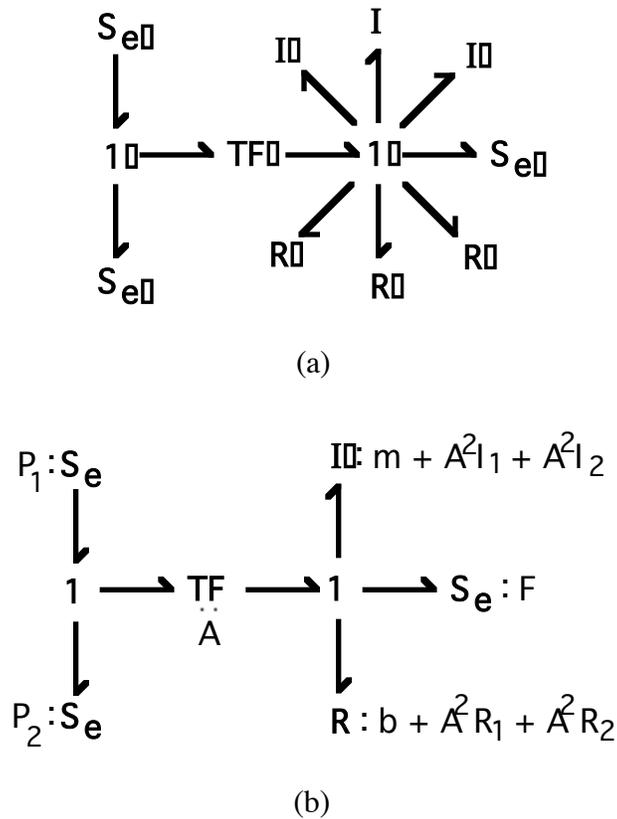


Figure 5.35: Bond graph of figure 5.34 with (a) parasitic fluid elements transformed into the translational domain and (b) inertial and dissipative elements combined.

When the parasitic fluid elements are transformed into the translational domain it becomes clear that all of the parasitic elements are connected to a single one-junction as shown in figure 5.35(a). The compatibility equation associated with the one-junction means that forces sum at the one-junction. As a result, the parasitic elements may be combined as shown in figure 5.35(b) into a single effective inertia with an apparent mass of

$$m_{app} = m + A^2 I_1 + A^2 I_2 \quad (5.79)$$

and a single effective friction element with an apparent friction coefficient of

$$b_{app} = b + A^2 R_1 + A^2 R_2 \quad (5.80)$$