Magnetic electro-mechanical machines

Lorentz Force

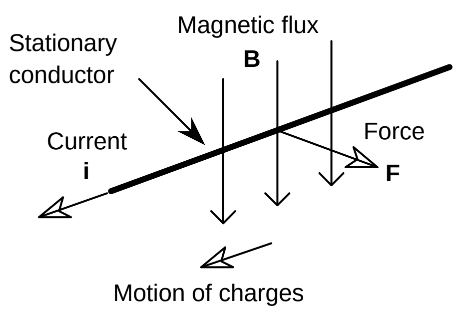
A magnetic field exerts force on a moving charge. The Lorentz equation:

$$\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

where

f: force exerted on charge q
E: electric field strength
v: velocity of the moving charge
B: magnetic flux density

Consider a stationary straight conductor perpendicular to a vertically-oriented magnetic field.



An electric field is oriented parallel to the wire. As charges move along the wire, the magnetic field makes them try to move sideways, exerting a force on the wire. The lateral force due to all the charge in the wire is:

$$\mathbf{f} = \rho A l\, (\mathbf{v} \times \mathbf{B})$$

where

ρ: density of charge in the wire (charge per unit volume)
l: length of the wire in the magnetic field
A: its cross-sectional area

The moving charges constitute a current, **i**

$$\mathbf{i} = \rho A \mathbf{v}$$

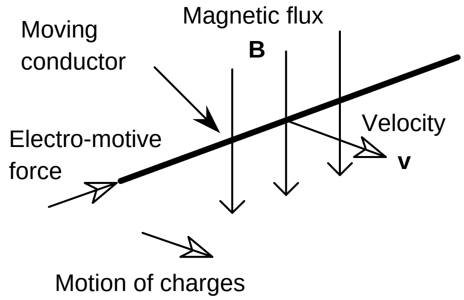
The lateral force on the wire is proportional to the current flowing in it.

$$\mathbf{f} = l\, (\mathbf{i} \times \mathbf{B})$$

For the orthogonal orientations shown in the figure, the vectors may be represented by their magnitudes.

$$f = l\, B\, i$$

This is one of a *pair* of equations that describe how electromagnetic phenomena can transfer power between mechanical and electrical systems. The *same physical phenomenon* also relates velocity and voltage. Consider the same wire perpendicular to the same magnetic field, but moving as shown



A component of charge motion is the same as the wire motion. The magnetic field makes charges try to move along the length of the wire from left to right. The resulting *electromotive force* (emf) opposes the current and is known as back-emf.

The size of the back-emf may be deduced as follows. Voltage between two points is the work required to move a unit charge from one to the other. If a unit charge moves along the wire from right to left the work done against the electromagnetic force is

$$e = v\ B\ l$$

This is the other of the pair of equations that describe how electromagnetic phenomena can transfer power between mechanical and electrical systems.

Two important points:

1. The interaction is *bi-lateral* (i.e., two-way). If an electrical current generates a mechanical force mechanical velocity generates a back-emf.
2. The interaction is *power-continuous*. Power is transferred from one domain to the other; no power is dissipated; no energy is stored; electrical power in equals mechanical power out (and v.v.).

   $$P_{electrical} = e\ i = (v\ B\ l)\ i = v\ (B\ l\ i) = v\ f = P_{mechanical}$$

Power continuity is *not* a modeling approximation. It arises from the underlying physics. The same physical quantity (magnetic flux density times wire length) is the parameter of the force-current relation and the voltage-velocity relation

D'Arsonval Galvanometer

Many electrical instruments (ammeters, voltmeters, etc.) are variants of the D'Arsonval galvanometer. A rectangular coil of wire pivots in a magnetic field as shown in figure 2.

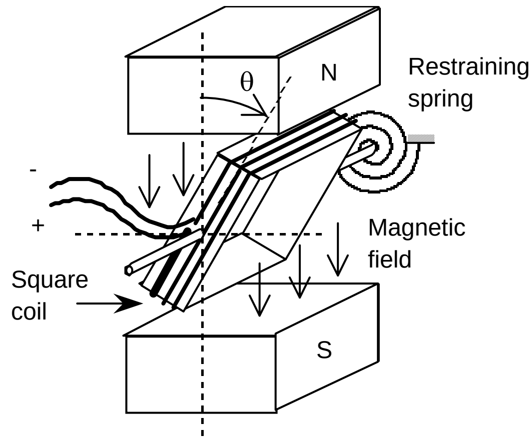


Figure 2: Sketch of a D'Arsonval galvanometer.

Current flowing parallel to the axis of rotation generates a torque to rotate the coil. Current flowing in the ends of the coil generates a force along the axis. Assuming the magnetic flux is vertical across the length and width of the coil, the total torque about the axis is:

$$\tau = 2NBlh \cos(\theta)\ i$$

where

τ: clockwise torque about the axis
N: number of turns of wire
B: magnetic flux density
l: length of the coil
h: half its height.

This torque is counteracted by a rotational spring

As the coils rotate, a back-emf is generated.

$$e = 2NBlh \cos(\theta)\ \omega$$

where

ω: angular speed of the coil.

Note that the same parameter, $2NBlh \cos(\theta)$, shows up in both equations.

Direct Current Permanent Magnet Electric Motor

With a different geometry the dependence on angle can be substantially reduced or eliminated

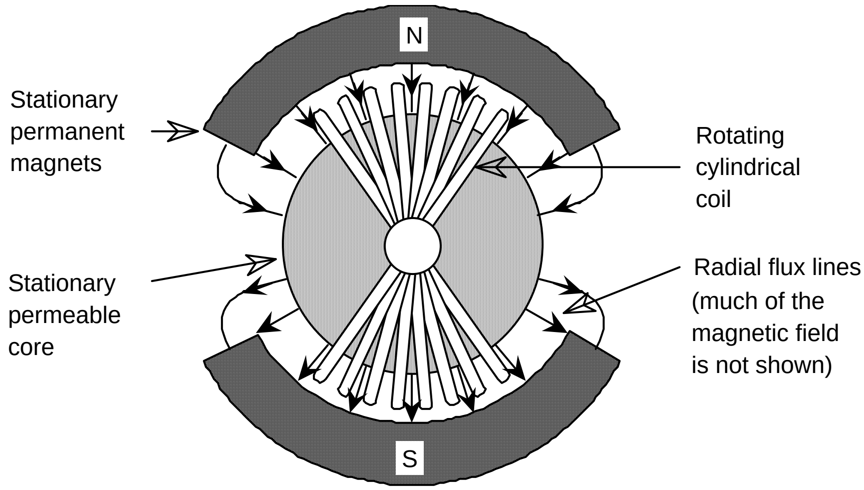


Figure 3: Schematic end-view of a D'Arsonval galvanometer modified to reduce the angle-dependence of the transduction equations.

Features:

- rotating cylindrical coil
- stationary permeable core
- shaped permanent magnets
- constant radial gap between magnets and core
- the magnetic field in the gap is oriented radially

If all turns of the coil are in the radial field the torque due to a current in the coil is independent of angle.

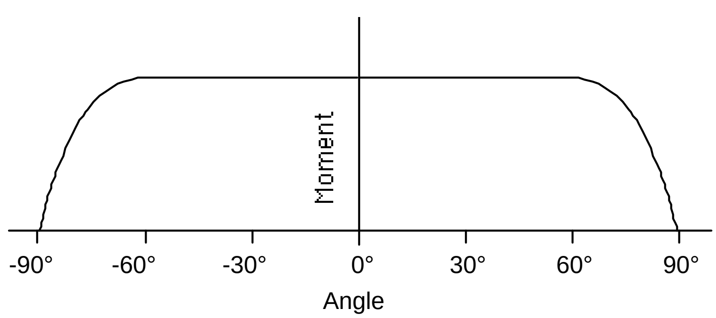


Figure 4: Sketch of half of the constant-current torque/angle relation resulting from the design of figure 3.

From symmetry the torque/angle relation for the other half of the circle is the negative of that shown above.

The reversal of torque can be eliminated by reversing the current when the angle passes through ±90°. A mechanical *commutator* is sketched in figure 5.

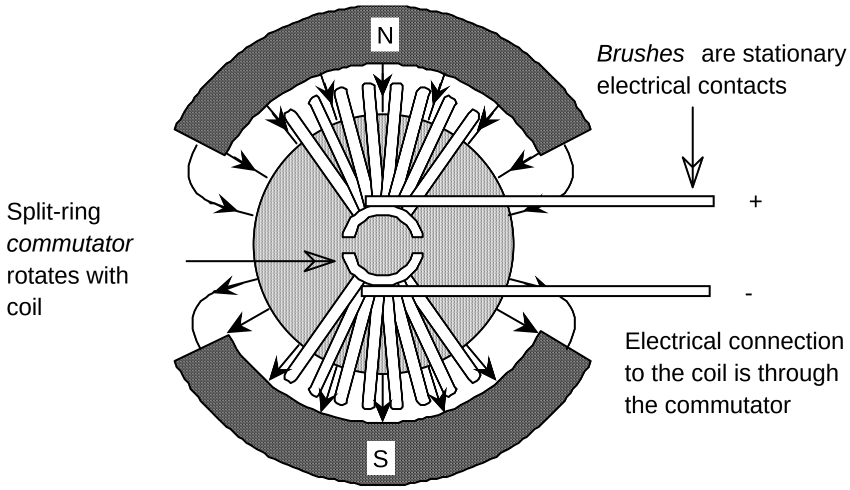


Figure 5: Schematic of the mechanical commutation system used in a direct-current permanent magnet motor.

Electrical connection is through a set of stationary conductors called *brushes*[1]. They contact a split ring called a *commutator* that rotates with the coil. This commutator design is used in a direct-current permanent-magnet motor (DCPMM). The same effect may be achieved electronically. That approach is used in a *brushless* DCPMM.

Assuming perfect commutation the relation between torque and current for a DCPMM is

$$\tau = K_t \, i$$

K_t: torque constant, a parameter determined by the mechanical, magnetic and electrical configuration of the device.

There is also a corresponding relation between voltage and rotational speed.

$$e = K_e \, \omega$$

K_e: back-emf or voltage constant.

Excerpt from a manufacturer's specification sheet for a direct-current permanent-magnet motor.

MOTOR CONSTANTS: (at 25 deg C)	**SYMBOL**	**UNITS**	
torque constant	KT	oz in/amp	5.03
back emf constant	KE	volts/krpm	3.72
terminal resistance	RT	ohms	1.400
armature resistance	RA	ohms	1.120
average friction torque	TF	oz in	3.0

[1] The reason for this terminology is historical — the earliest successful designs used wire brushes for this purpose.

viscous damping constant	KD	oz in/krpm	0.59
moment of inertia	JM	oz in sec-sec	0.0028
armature inductance	L	micro henry	<100.0
temperature coefficient of KE	C	%/deg c rise	-0.02

These specifications imply that the torque and back-emf constants are distinct parameters.

different symbols: KT, KE

different units: oz-in/amp, volts/krpm

But if we express both constants in mks units

K_t = 0.0355 N-m/amp

K_e = 0.0355 volt-sec/rad

As required by the physics of electromagnetic power transduction, the constants are in fact identical.

"Parasitic" Dynamics

The Lorentz force yields two equations describing power-continuous electro-mechanical transduction. A practical electric motor also includes energy storage and/or power dissipation in the electrical and mechanical domains. A competent model may include these effects.

Electrical side

- inductance of the coil
- resistance of the coil
- resistance of electrical connectors ("terminal resistance")

Mechanical side

- inertia of the rotating components (coil, shaft, etc.)
- friction of brushes sliding on the commutator
- viscous drag due to entrained air

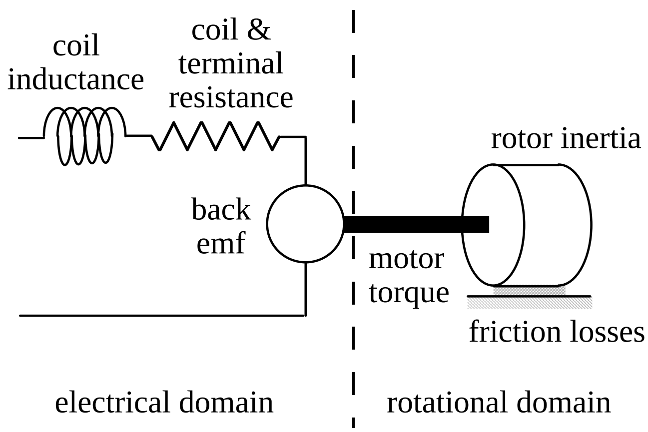
Connections:

Electrical side:

- only one distinct current—series connection of inductor, resistor & model element for back-emf due to velocity

Mechanical side:

- only one distinct speed—common-velocity connection of inertia, friction & model element for torque due to motor current



Schematic diagram of DCPMM model with "parasitic" dynamics

Motor vs. Generator

Lorentz-force electro-mechanical transduction is bi-lateral. An electric motor uses it to convert electrical power into rotational power. An electrical generator uses it to convert rotational power into electrical power.

Tachometer

Lorentz-force electro-mechanical transduction is also used for sensing. A DC permanent magnet tachometer generates voltage proportional to angular velocity as described by the velocity-voltage equation above.