

CAUSAL ANALYSIS

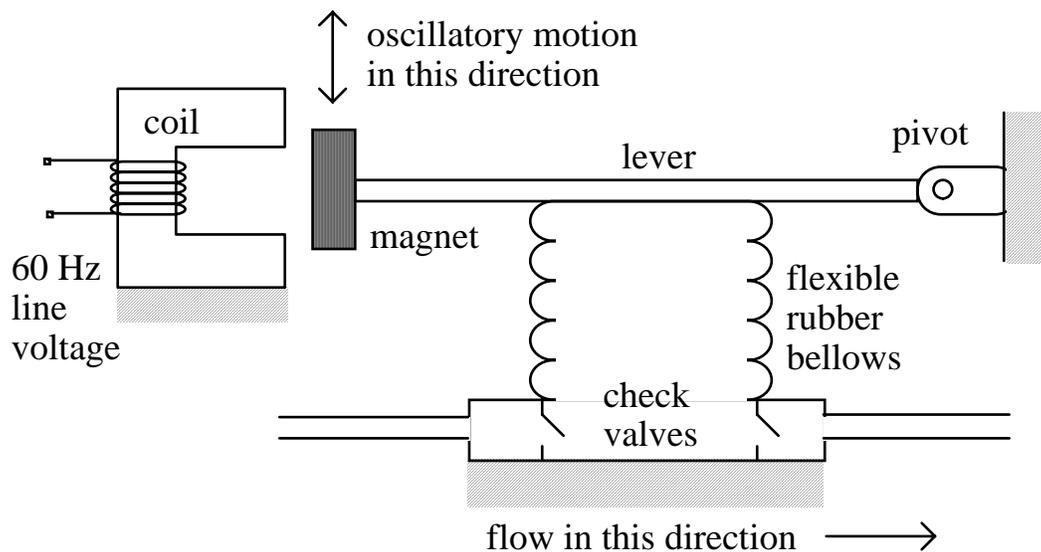
Things should be made as simple as possible – but no simpler.

Albert Einstein

How simple is "as simple as possible"?

Causal assignment provides considerable insight.

EXAMPLE: AQUARIUM AIR PUMP



Schematic of a low-cost aquarium air pump.

- Reciprocating pump
- Electromagnet driven by 60 Hz line voltage
- Drives a small permanent magnet attached to a lever
- Lever drives a flexible rubber bellows on a chamber
- Check valves prevent flow in one direction
- Small flow resistance in the other

MODEL SIMPLIFICATION WAS BASED ON THREE PROPOSALS

CLAIM #1

The magnet-lever-bellows combination should resonate near 60 Hz
– to maximize oscillation amplitude and output flow rate

CLAIM #2

Only magnet mass and bellows compliance are needed to portray resonance
(other energy-storage effects are small by comparison)

CLAIM #3

valve flow resistance is the only significant energy loss
(other energy-dissipation effects are small by comparison)

TASK: EVALUATE THESE PROPOSALS

ASSUMPTIONS

Electrical sub-system

AC power line – voltage source

(the pump's power load is unlikely to significantly affect line voltage)

Electromagnetic coil – ideal gyrator

By claims #2 & #3 all other electrical and magnetic effects are negligible.

Mechanical sub-system

Magnet – ideal translational inertia

Its velocity is the same as that of the leftmost end of the lever – one junction

Lever – ideal transformer relating two translational domains

Bellows compliance – ideal translational spring (capacitor)

(by claim #2)

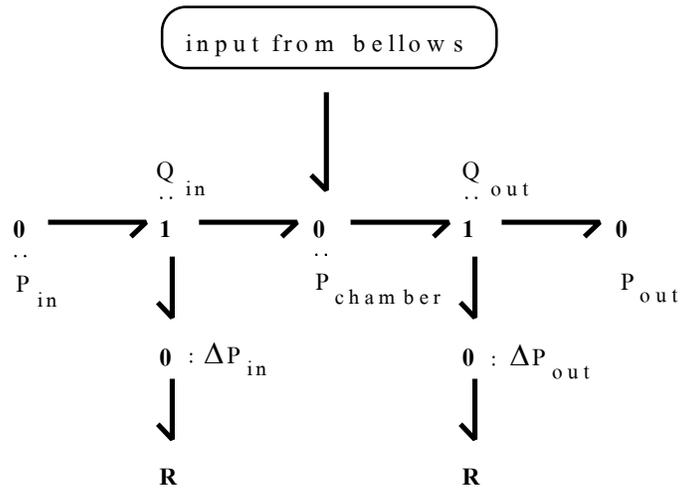
Fluid sub-system

Check valves – ideal nonlinear resistors

Chamber pressure proportional to bellows force – ideal transformer between translational and fluid domains

Deflection of bellows compliance is the motion that pumps air – one junction

FLUID SUB-ASSEMBLY



Bond graph fragment for check valves and chamber

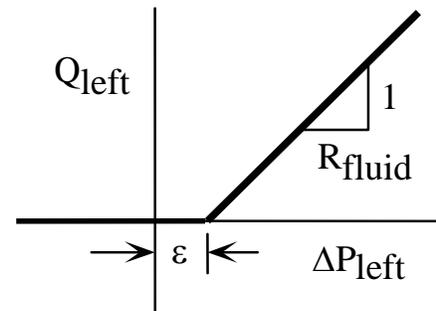
CAUSAL ANALYSIS

Check valve must have flow rate output for pressure input.

$$Q_{\text{valve}} = \begin{cases} 0 & \text{if } \Delta P_{\text{valve}} \leq \varepsilon \\ \frac{(\Delta P_{\text{valve}} - \varepsilon)}{R_{\text{valve}}} & \text{if } \Delta P_{\text{valve}} > \varepsilon \end{cases}$$

A well-defined function

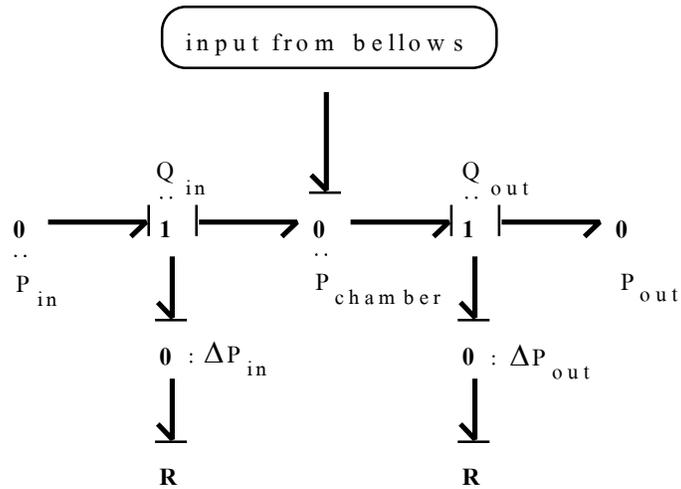
– with no definable inverse



Nonlinear check-valve resistors are *causally constrained*

– flow out, effort in

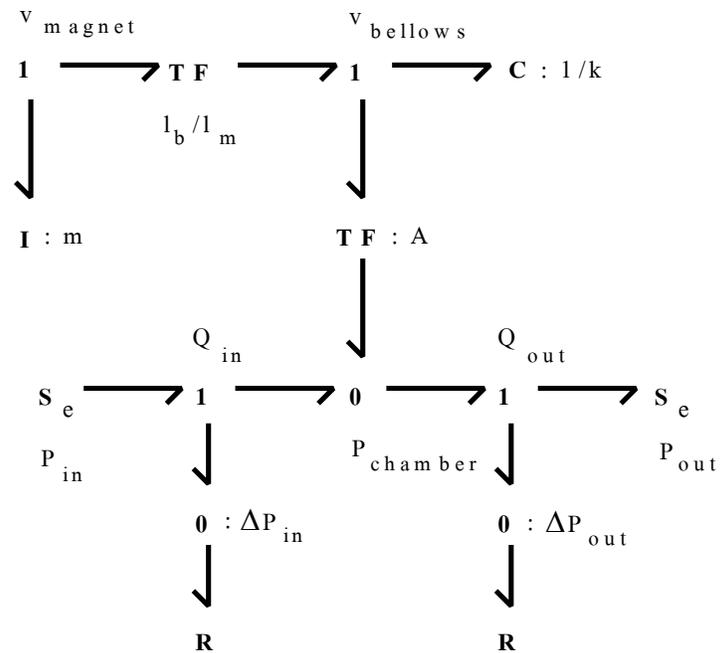
These constraints imply that the *boundary conditions* at the airflow inlet and outlet and the power input from the bellows must be efforts (pressures).



Bond graph fragment for check valves and chamber with causal constraints

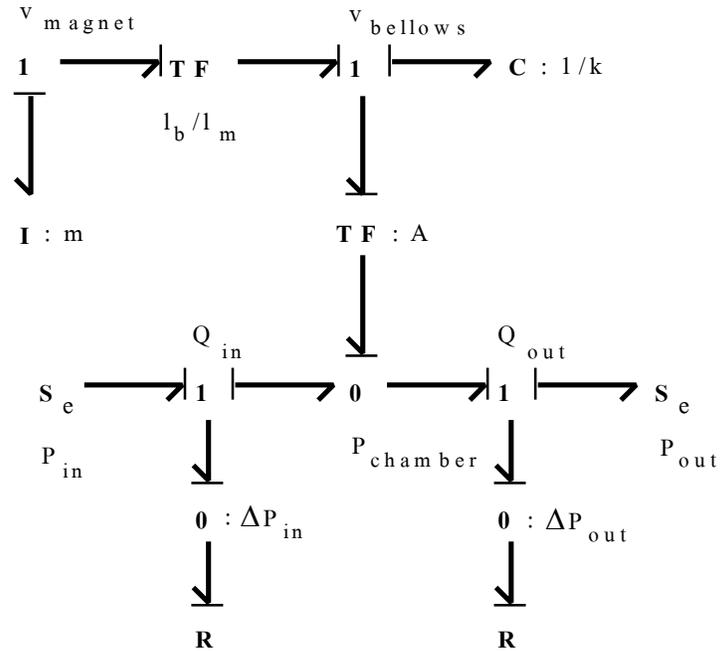
MECHANICAL & FLUID SUB-ASSEMBLY

(temporarily ignore the electromagnetic coil)



two energy storage elements
 no dissipation other than check valves
 consistent with claim #2

CAUSAL ANALYSIS



magnet inertia is a *dependent* storage element

only one independent energy storage element – bellows compliance

only one state variable

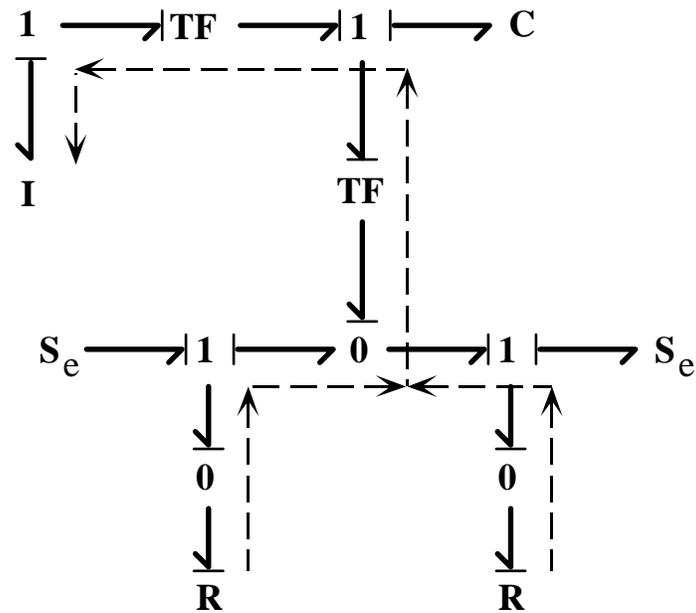
model cannot describe resonant oscillation

This is the problem we discovered previously

– revealed through causal analysis

– no need to develop equations

CAUSAL PATH



Casual path identifies which model element(s) determine magnet velocity
 – check-valve flow rates

IN PHYSICAL TERMS:

This model implies that if airflow were completely blocked, nothing could move.

In the model, chamber pressure is determined by magnet accelerating force (amplified by lever) minus spring force.

If the chamber pressure magnitude is less than ε , both valves remain closed and the flow rate into (and through) the chamber is zero.

That, in turn, means that the lever and the magnet do not move.

As indicated by the causal analysis, the check-valve flow rates determine the magnet velocity.

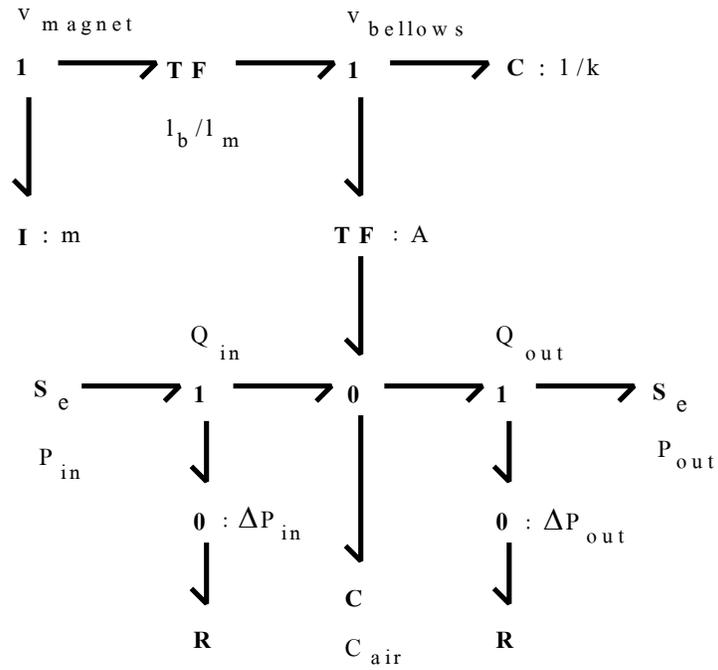
Unlikely!

– due to the assumption that air is incompressible.

REJECT CLAIM #2

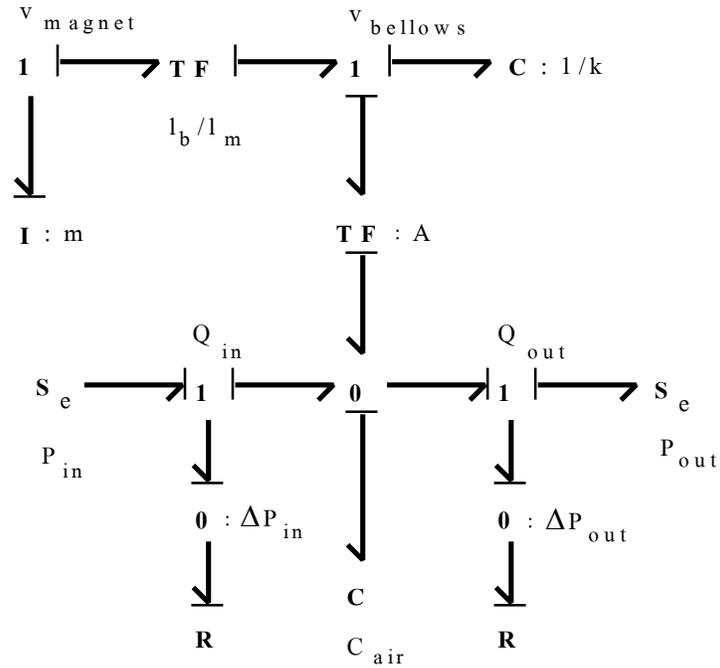
Revise the model

Include a fluid capacitor to model air compressibility in the bellows chamber.



Mechanical - fluid sub-model including air compressibility

CAUSAL ANALYSIS



three independent energy storage elements

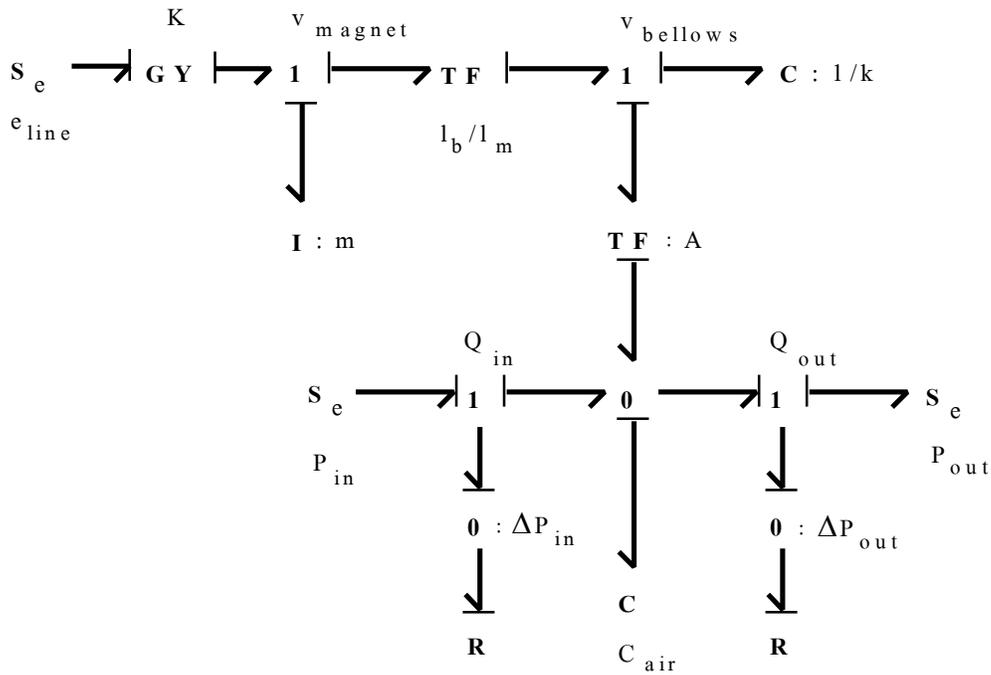
– magnet inertia, bellows compliance, air compressibility

model may describe resonant oscillation

third order system – three state variables

Surprisingly, in this system, it appears that *three* energy storage elements are required to describe resonant oscillation, not two as initially assumed.

CAUSAL ANALYSIS



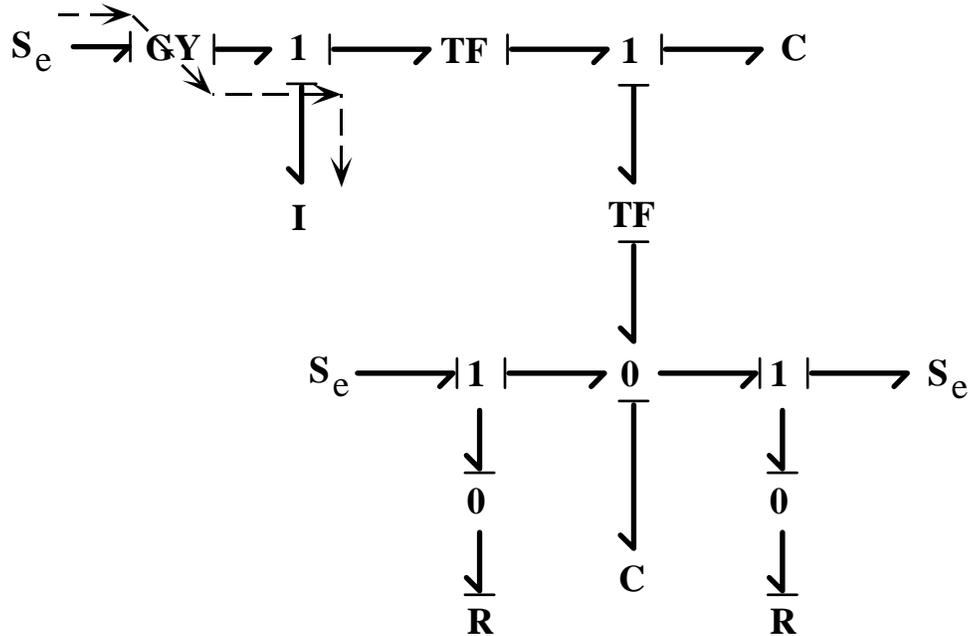
magnet inertia is again a *dependent* storage element

two independent energy storage elements

– bellows compliance & air compressibility

but the model cannot describe resonant oscillation

CAUSAL PATH



magnet velocity depends on source voltage

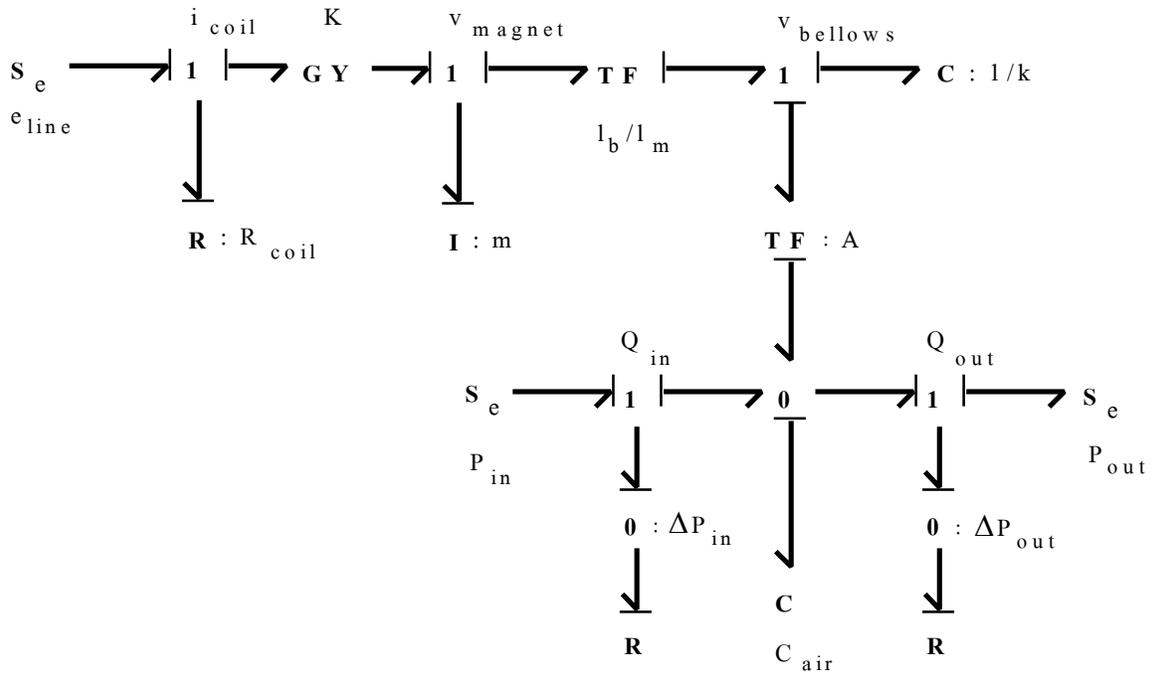
solution: add an element between these two

one obvious candidate: the electrical resistance of the coil

(same effect as the “internal resistance” of the source; doesn’t add to system order)

REJECT CLAIM #3

Revised model



Causal analysis

three independent energy storage elements

model may describe resonant oscillation

COMMENT

A bigger dissipator (resistor) usually means less oscillation

– not always true.

Effective mechanical damping due to the electrical resistance of the coil:

$$b_{\text{effective}} = \frac{K^2}{R_{\text{coil}}}$$

In this case:

A *smaller* resistance means less oscillation.

STATE AND OUTPUT EQUATIONS

FIRST CHOOSE STATE VARIABLES.

Three independent energy storage elements,

– three independent state variables.

They define the energetic state of the system.

BEST CHOICE DEPENDS ON THE PARTICULAR MODEL

In this case, most elements are linear, so circuit variables may be appropriate.

– a.k.a. power variables:

efforts of independent capacitors,

flows of independent inertias.

In a nonlinear system, energy variables are usually a better choice.

– displacements of independent capacitors,

momenta of independent inertias.

Usually easiest to visualize position & velocity of mechanical systems.

spring displacement rather than force,

magnet velocity rather than momentum.

A REASONABLE CHOICE:

magnet velocity, v_{magnet}

bellows displacement, x_{bellows}

volume of air that has flowed into the chamber, V_{air}

STATE EQUATIONS

State equations may be written by inspection from the graph.

Note that it is *not* necessary to assemble the equations by substitution.

For numerical predictions, the un-assembled form of the equations makes for easier-reading computer code.

$$\frac{d}{dt} v_{\text{magnet}} := \frac{1}{m} \left[K i_{\text{coil}} - \frac{l_b}{l_m} F_{\text{net}} \right]$$

$$i_{\text{coil}} := \frac{1}{R_{\text{coil}}} [e_{\text{line}} - e_{\text{back}}]$$

e_{line} is the input (line) voltage

e_{back} is the back-EMF generated by the coil.

$$e_{\text{back}} := K v_{\text{magnet}}$$

F_{net} is the net force on the lever at the point of attachment to the bellows.

$$F_{\text{net}} := F_k + A P_{\text{chamber}}$$

F_k is the force in the bellows spring.

$$F_k := k x_{\text{bellows}}$$

$$P_{\text{chamber}} := \frac{1}{C_{\text{air}}} V_{\text{air}}$$

$$\frac{d}{dt} x_{\text{bellows}} := v_{\text{bellows}}$$

$$v_{\text{bellows}} := \frac{l_b}{l_m} v_{\text{magnet}}$$

$$\frac{d}{dt} V_{\text{air}} := A v_{\text{bellows}} + Q_{\text{in}} - Q_{\text{out}}$$

$$Q_{\text{in}} := \begin{cases} 0 & \text{if } \Delta P_{\text{in}} \leq \varepsilon \\ \frac{\Delta P_{\text{in}} - \varepsilon}{R_{\text{in}}} & \text{if } \Delta P_{\text{in}} > \varepsilon \end{cases}$$

$$\Delta P_{\text{in}} := P_{\text{in}} - P_{\text{chamber}}$$

ΔP_{in} is the pressure drop across the input valve and P_{in} is the pressure at the inlet port
 – one of the system boundary conditions or "inputs".

$$Q_{\text{out}} := \begin{cases} 0 & \text{if } \Delta P_{\text{out}} \leq \varepsilon \\ \frac{\Delta P_{\text{out}} - \varepsilon}{R_{\text{out}}} & \text{if } \Delta P_{\text{out}} > \varepsilon \end{cases}$$

$$\Delta P_{\text{out}} := P_{\text{chamber}} - P_{\text{out}}$$

ΔP_{out} is the pressure drop across the output valve and P_{out} is the pressure at the outlet port

– also one of the system boundary conditions or "inputs".

OUTPUT EQUATIONS

Output variables of possible interest:

input and output flows Q_{in} and Q_{out}

chamber pressure, $P_{chamber}$

magnet position and velocity, x_{magnet} and v_{magnet} .

All of the required equations are above except

$$x_{magnet} := \frac{l_m}{l_b} x_{bellows}$$

where it has been assumed that the zero value of both positions corresponds to the position shown in the figure.