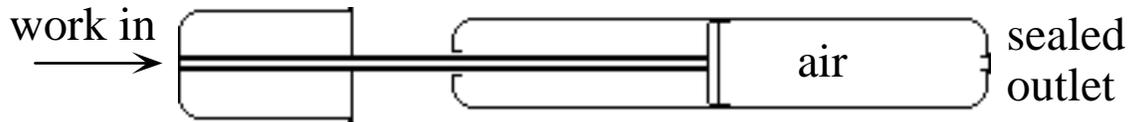


EXAMPLE: THERMAL DAMPING



A BICYCLE PUMP WITH THE OUTLET SEALED.

When the piston is depressed, a fixed mass of air is compressed.

– mechanical work is done.

The mechanical work done on the air is converted to heat.

– the air temperature rises

A temperature difference between the air and its surroundings induces heat flow.

– entropy is produced

The original work done is not recovered when the piston is withdrawn to the original piston.

– available energy is lost

MODEL THIS SYSTEM

GOAL:

**the simplest model that can describe thermal damping
(the loss of available energy)**

ELEMENTS:

TWO KEY PHENOMENA

work-to-heat transduction

**a two port capacitor represents thermo-mechanical
transduction**

entropy production

**a two port resistor represents heat transfer and
entropy production**

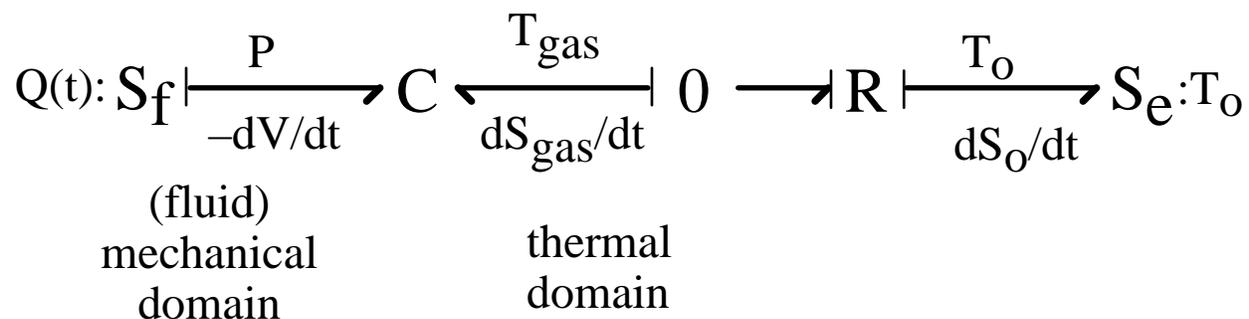
BOUNDARY CONDITIONS:

For simplicity assume

a flow source on the (fluid-)mechanical side

a constant temperature heat sink on the thermal side

A BOND GRAPH IS AS SHOWN.



CAUSAL ANALYSIS:

The integral causal form for the two-port capacitor (pressure and temperature outputs)

is consistent with the boundary conditions

and with the preferred causal form for the resistor

CONSTITUTIVE EQUATIONS:

Assume air is an ideal gas and use the constitutive equations derived above.

$$\left(\frac{T}{T_0}\right) = \left(\frac{V}{V_0}\right)^{-\frac{R}{c_v}} \exp\left(\frac{S - S_0}{mc_v}\right)$$

$$\left(\frac{P}{P_0}\right) = \left(\frac{V}{V_0}\right)^{-\left(\frac{R}{c_v} + 1\right)} \exp\left(\frac{S - S_0}{mc_v}\right)$$

Assume Fourier's law describes the heat transfer process.

$$\dot{Q} = \frac{kA}{l} (T_1 - T_2)$$

ANALYSIS:

For simplicity, linearize the capacitor equations about a nominal operating point defined by S_o and V_o

$$\left. \frac{\partial T}{\partial S} \right|_o = \frac{T_o}{mc_v}$$

$$\left. \frac{\partial T}{\partial(-V)} \right|_o = \frac{T_o R}{V_o c_v}$$

$$\left. \frac{\partial P}{\partial S} \right|_o = \frac{P_o}{mc_v}$$

$$\left. \frac{\partial P}{\partial(-V)} \right|_o = \frac{P_o}{V_o} \left(\frac{R}{c_v} + 1 \right)$$

Inverse capacitance:

$$C^{-1} = \begin{bmatrix} \frac{T_o}{mc_v} & \frac{P_o}{mc_v} \\ \frac{P_o}{mc_v} & \frac{P_o}{V_o} \left(\frac{R}{c_v} + 1 \right) \end{bmatrix}$$

equality of the off-diagonal terms (the crossed partial derivatives) is established using $P_o V_o = mRT_o$

Linearized constitutive equations

$$\begin{bmatrix} \delta T \\ \delta P \end{bmatrix} = \begin{bmatrix} \frac{T_o}{mc_v} & \frac{P_o}{mc_v} \\ \frac{P_o}{mc_v} & \frac{P_o}{V_o} \left(\frac{R}{c_v} + 1 \right) \end{bmatrix} \begin{bmatrix} \delta S \\ \delta(-V) \end{bmatrix}$$

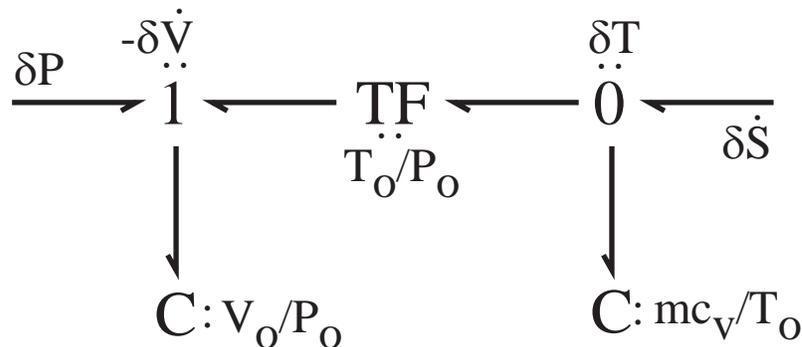
where

$$\delta S = S - S_o, \delta V = V - V_o,$$

$$\delta T = T - T_o(S_o, V_o), \delta P = P - P_o(S_o, V_o)$$

NETWORK REPRESENTATION

The linearized model may be represented using the following bond graph



This representation shows that

in the isothermal case ($\delta T = 0$) the fluid capacitance is $C_{\text{fluid}} = V_o/P_o$

in the constant-volume case ($\delta \dot{V} = 0$) the thermal capacitance is $C_{\text{thermal}} = mc_v/T_o$

the strength of thermo-fluid coupling is T_o/P_o

This uses the convention that the transformer coefficient is for the flow equation with output flow on the output power bond

$$\delta \dot{V} = \frac{T_o}{P_o} \delta \dot{S} \text{ and hence } \delta T = \frac{T_o}{P_o} \delta P$$

though causal considerations may require the inverse equations.

RESISTOR EQUATIONS

The two-port resistor constitutive equations are

$$\dot{S}_1 = \frac{\dot{Q}}{T_1} = \frac{kA}{1} \left(\frac{T_1 - T_2}{T_1} \right)$$

$$\dot{S}_2 = \frac{\dot{Q}}{T_2} = \frac{kA}{1} \left(\frac{T_1 - T_2}{T_2} \right)$$

Linearize the resistor constitutive equations about a nominal operating point defined by T_{10} and T_{20}

$$\begin{bmatrix} \delta\dot{S}_1 \\ \delta\dot{S}_2 \end{bmatrix} = \frac{kA}{1} \begin{bmatrix} \frac{T_{20}}{T_{10}^2} & -\frac{1}{T_{10}} \\ \frac{1}{T_{20}} & -\frac{T_{10}}{T_{20}^2} \end{bmatrix} \begin{bmatrix} \delta T_1 \\ \delta T_2 \end{bmatrix}$$

This is in conductance form, $\mathbf{f} = \mathbf{G}\mathbf{e}$

Note that this conductance matrix is singular:

$$|\mathbf{G}| = -\frac{T_{10}T_{20}}{T_{10}^2T_{20}^2} - \frac{1}{T_{10}T_{20}} = 0$$

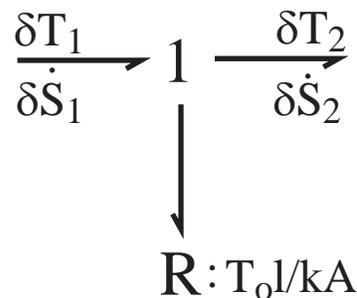
this is because both entropy flows are associated with the same heat flow

LINEARIZE ABOUT ZERO HEAT FLOW

If the two nominal operating temperatures are equal, $T_{1o} = T_{2o} = T_o$, the linearized constitutive equations are

$$\begin{bmatrix} \delta\dot{S}_1 \\ \delta\dot{S}_2 \end{bmatrix} = \frac{kA}{1} \begin{bmatrix} \frac{1}{T_o} & -\frac{1}{T_o} \\ \frac{1}{T_o} & -\frac{1}{T_o} \end{bmatrix} \begin{bmatrix} \delta T_1 \\ \delta T_2 \end{bmatrix}$$

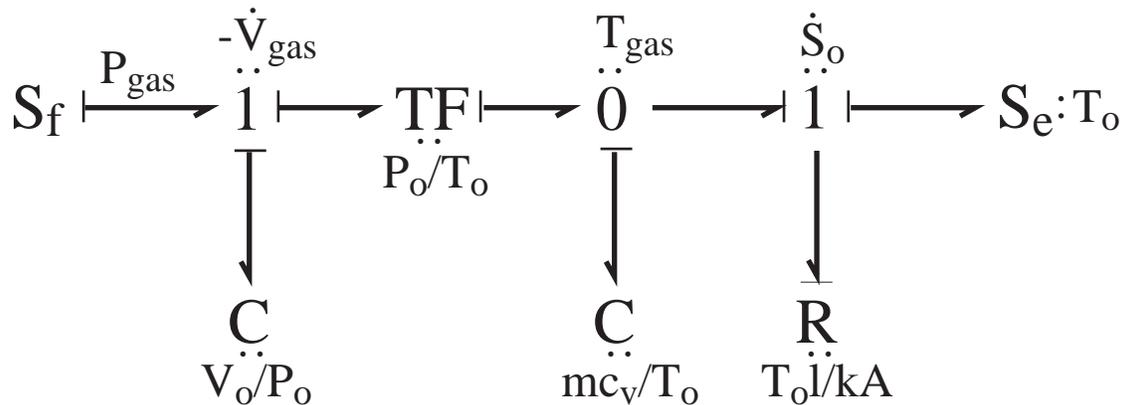
This simple form can be represented by an equally simple bond graph



This follows the usual convention of writing the resistor parameter in resistance form

ASSEMBLE THE PIECES...

LINEARIZED BOND GRAPH



Note the sign change on the capacitor thermal port (to avoid a superfluous 0-junction)

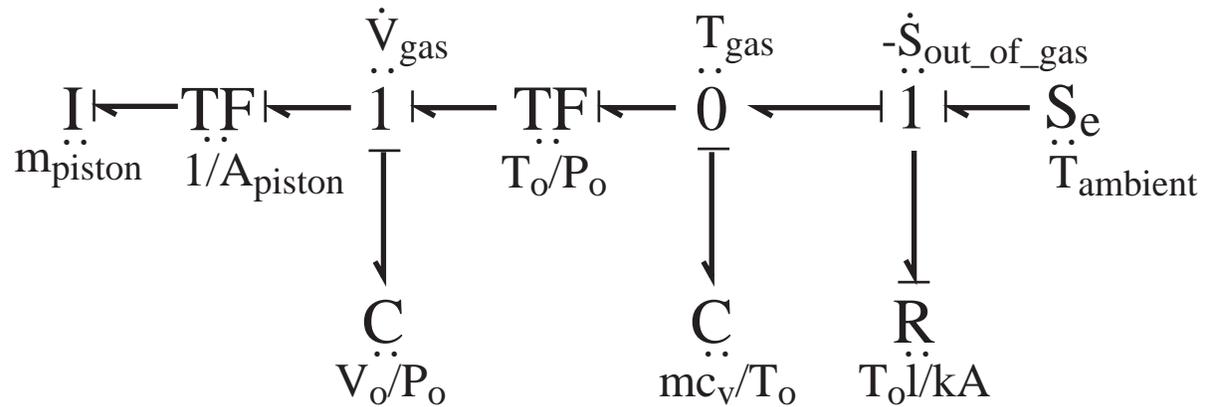
Causal assignment indicates a first-order system

Time-constant is determined by thermal (conduction) resistance and thermal capacitance

Gas pressure is determined by fluid capacitance and (reflected) thermal capacitance and resistance

INCLUDE PISTON INERTIA

BOND GRAPH



Causal analysis indicates

a third-order system

capable of resonant oscillation

In this model the only damping is in the thermal domain

heat transfer, entropy flow

SUMMARIZING

THE GAS STORES ENERGY.

It also acts as a transducer because there are two ways to store or retrieve this energy

– two interaction ports

energy can be added or removed as work or heat.

The “energy-storing transducer” behavior is modeled as a two-port capacitor.

– just like the energy-storing transducers we examined earlier.

IF POWER FLOWS VIA THE THERMAL PORT, AVAILABLE ENERGY IS REDUCED

– the system also behaves as a dissipator.

The dissipative behavior is due to heat transfer.

Gas temperature change due to compression and expansion does not dissipate available energy.

**If the walls were perfectly insulated,
no available energy would be lost,
but then, no heat would flow either.**

Without perfect insulation temperature gradients induce heat flow

Heat flow results in entropy generation.

Entropy generation means a loss of available energy.

THE SECOND LAW.

DISCUSSION

ALL MODELS ARE FALSE.

It is essential to understand what errors our models make, and when the errors should not be ignored.

It is commonly assumed that modeling errors become significant at higher frequencies.

– not so!

Compression and expansion of gases is common in mechanical systems.

Hydraulic systems typically include accumulators (to prevent over-pressure during flow transients).

The most common design uses a compressible gas.

Compression and expansion of the gas can dissipate (available) energy.

This dissipation requires heat flow, but heat flow takes time.

For sufficiently rapid compression and expansion, little or no heat will flow, and little or no dissipation will occur.

The simplest model of a gas-charged accumulator may justifiably ignore “thermal damping”.

That is an eminently reasonable modeling decision but that model will be in error

**at low frequencies
not high frequencies.**

THIS IS A GENERAL CHARACTERISTIC OF PHENOMENA DUE TO THERMODYNAMIC IRREVERSIBILITIES.