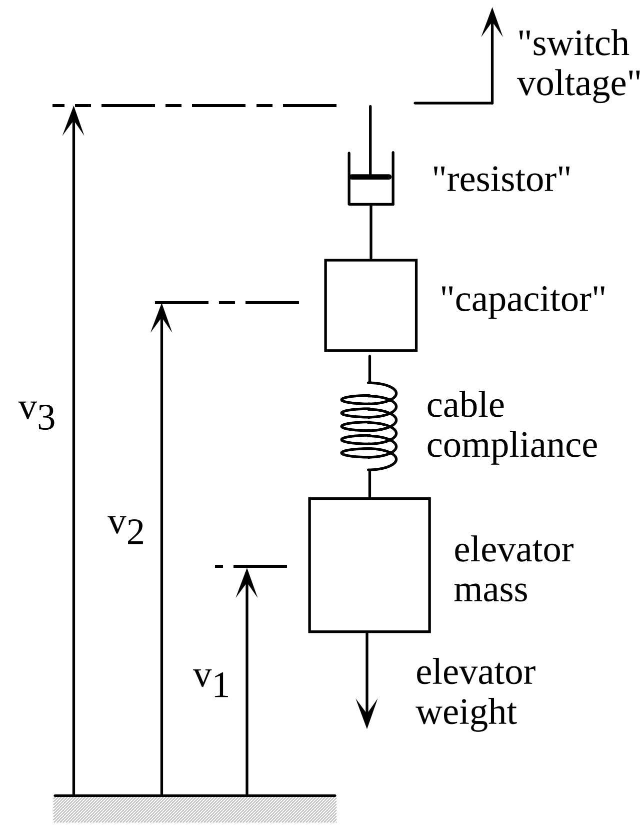
REVIEW NETWORK MODELING OF PHYSICAL SYSTEMS

EXAMPLE: VIBRATION IN A CABLE HOIST

Bond graphs of the cable hoist models help to develop insight about how the electrical R-C filter affects the mechanical system dynamics.

Equivalent mechanical system:



- **velocity source (equivalent to switch voltage)**
- **viscous damper (equivalent to electrical resistor)**
- **mass equivalent of capacitor**
- **spring (cable compliance)**
- **mass of elevator cage**
- **force source (elevator weight)**

Resonant oscillation:

due to *out-of-phase* motion of the two masses opposed by the spring

Coniser the two massess and the spring in isolation

No external forces

- **net momentum = zero**
- **the masses move in opposite directions**

$m_1 v_1 = -m_2 v_2$

(subscripts as indicated in the diagram)

$$v_2 = -\frac{m_1}{m_2} v_1$$

Kinetic energy:

$$E_k^* = m_1 \frac{v_1^2}{2} + m_2 \frac{v_2^2}{2} = m_1 \frac{v_1^2}{2} + \frac{m_1^2}{m_2} \frac{v_1^2}{2} = m_1 \left(1 + \frac{m_1}{m_2}\right) \frac{v_1^2}{2}$$

Potential energy:

$$\Delta x_2 = -\frac{m_1}{m_2}\ \Delta x_1$$

$$\Delta x_{spring} = \Delta x_1 - \Delta x_2 = \left(1 + \frac{m_1}{m_2}\right)\ \Delta x_1$$

$$E_p = \frac{k}{2}\ \Delta x^2_{spring} = \frac{k}{2}\left(1 + \frac{m_1}{m_2}\right)^2\ \Delta x_1^2$$

Undamped natural frequency:

$$\omega_n^2 = \frac{k\left(1 + \frac{m_1}{m_2}\right)^2}{m_1\left(1 + \frac{m_1}{m_2}\right)} = \frac{k}{m}\left(1 + \frac{m_1}{m_2}\right)$$

Thus the undamped natural frequency will be *increased* by the factor

$$\sqrt{\mathbf{1 + \frac{m_1}{m_2}}}$$

Check the numbers:

The parameters used in the MATLAB simulations were as follows:

R = 10 ohms

C = 0.1 farads

K_motor = 0.03 Newton-meters/amp

n_gear = 0.02

r_drum = 0.05 meters

k_cable = 200000 Newton/meter

m_cage = 200 kilograms

Undamped natural frequency *without* the R-C filter:

$$\sqrt{\frac{k_{cable}}{m_{cage}}} = 31.6 \text{ radian/second} = \mathbf{5\ Hertz}$$

This agrees with the numerical simulation.

The mass equivalent of the capacitor is

$$m_2 = \left(\frac{K_{motor}}{r_{drum}\, n_{gear}}\right)^2 C = \mathbf{90\ kilograms} \quad !!!$$

Undamped natural frequency *with* the R-C filter:

$$\sqrt{\frac{k}{m}\left(1 + \frac{m_1}{m_2}\right)} = 56.8 \text{ radians/second} = \mathbf{9\ Hertz}$$

This agrees quite well with the numerical simulation.

Decay time constant:

both masses move in unison, opposed by the damper

The viscous damping equivalent of the resistor is

$$b = \left(\frac{K_{motor}}{r_{drum}\, n_{gear}}\right)^2 \frac{1}{R} \quad \textbf{= 90 Newton-seconds/meter}$$

If the equivalent mass and equivalent damper were isolated, the decay time constant would be

$\tau_{isolated} = m_2/b = RC =$ **1 second**

which is the time constant of the electrical filter—as it should be.

In the coupled electro-mechanical system, both masses interact with the damper and the decay time constant is

$\tau_{coupled} = (m_1 + m_2)/b =$ **3.2 seconds**

This also agrees quite well with the numerical simulation.