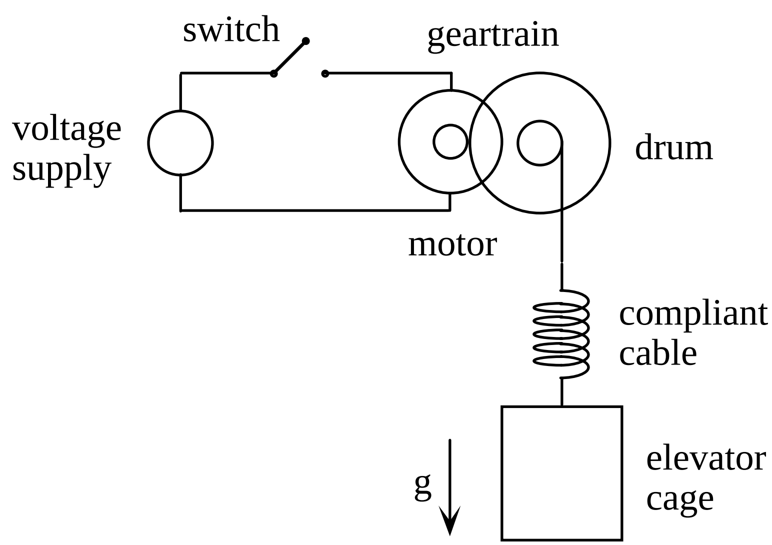
REVIEW NETWORK MODELING OF PHYSICAL SYSTEMS

a.k.a. "lumped-parameter" modeling

EXAMPLE: VIBRATION IN A CABLE HOIST

Problem

The cage of an elevator is hoisted by a long cable wound over a drum driven through a gear-set by an electric motor. The motor is relay-operated (i.e., either on or off) and the resulting abrupt transients cause the cage to oscillate on the hoisting cable. Because the cable has low internal friction, the oscillations persist for many cycles.



Even more important, the peak stress in the cable is almost double the steady-state stress required to support the weight of the cable.

Scenario

To solve this problem it has been proposed to introduce an electrical R-C filter between the relay and the motor terminals. The designer claims that this will smooth the transient, thereby reducing the oscillation amplitude to acceptable levels. Your task is to evaluate this proposal.

abruptly engaging the motor excites oscillation

does electrical filtering help?

Modeling goal

The simplest model competent to elucidate the effect of electrical filtering on the mechanical oscillation.

First reproduce the problem

To keep things simple assume that:

- variation of weight supported with length change may be ignored (i.e., consider small changes in elevation)
- weight is concentrated ("lumped") in the cage
- variation of cable compliance with length change may be ignored
- neglect cable internal damping (first, that emphasizes tendency to oscillation; second, it's small anyway; and third, it's easy to add later if necessary)
- drum and gear inertia may be neglected
- DC electric motor with constant magnetic field
- motor armature resistance & inductance may be neglected
- relay resistance may be neglected
- voltage supply "internal resistance" may be neglected

Direct approach

(i.e., just "write down" the differential equations)

Newtonian mechanics:

$m_{cage}\, \ddot{x}_{cage} := k_{cable}\,(x_{rim} - x_{cage}) - m_{cage}\, g$

Transmission

$\dot{x}_{rim} := r_{drum}\, \omega_{drum}$

$\omega_{drum} := n_{gear}\, \omega_{motor}$

Motor transduction characteristic:

$\omega_{motor} := e_{motor} / K_{motor}$

Switch:

$e_{motor} := e_{supply}$ if switch closed; 0 if switch open.

—a computable set of differential equations.

Given $e_{motor}(t)$ and initial conditions, $x_{cage}(t)$ is straightforward to compute. Analysis and computation is often facilitated by writing the equations in a standard state-determined form. One good choice of state variables (there are many others) yields the following.

$$\frac{d}{dt}\begin{bmatrix} x_{rim} \\ x_{cage} \\ \dot{x}_{cage} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{k_{cable}}{m_{cage}} & -\frac{k_{cable}}{m_{cage}} & 0 \end{bmatrix}\begin{bmatrix} x_{rim} \\ x_{cage} \\ \dot{x}_{cage} \end{bmatrix} + \begin{bmatrix} \frac{r_{drum}\, n_{gear}}{K_{motor}} & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e_{motor} \\ g \end{bmatrix}$$

Laplace domain analysis:

This linear model is conveniently analyzed in the Laplace domain. Standard methods (e.g., Cramer's rule (ref. Ogata text; Rosenberg & Karnopp text)) are available to transform state-determined representations into the Laplace domain.

In this case, finding the transfer function from supply voltage to cage position by direct manipulation is straightforward.

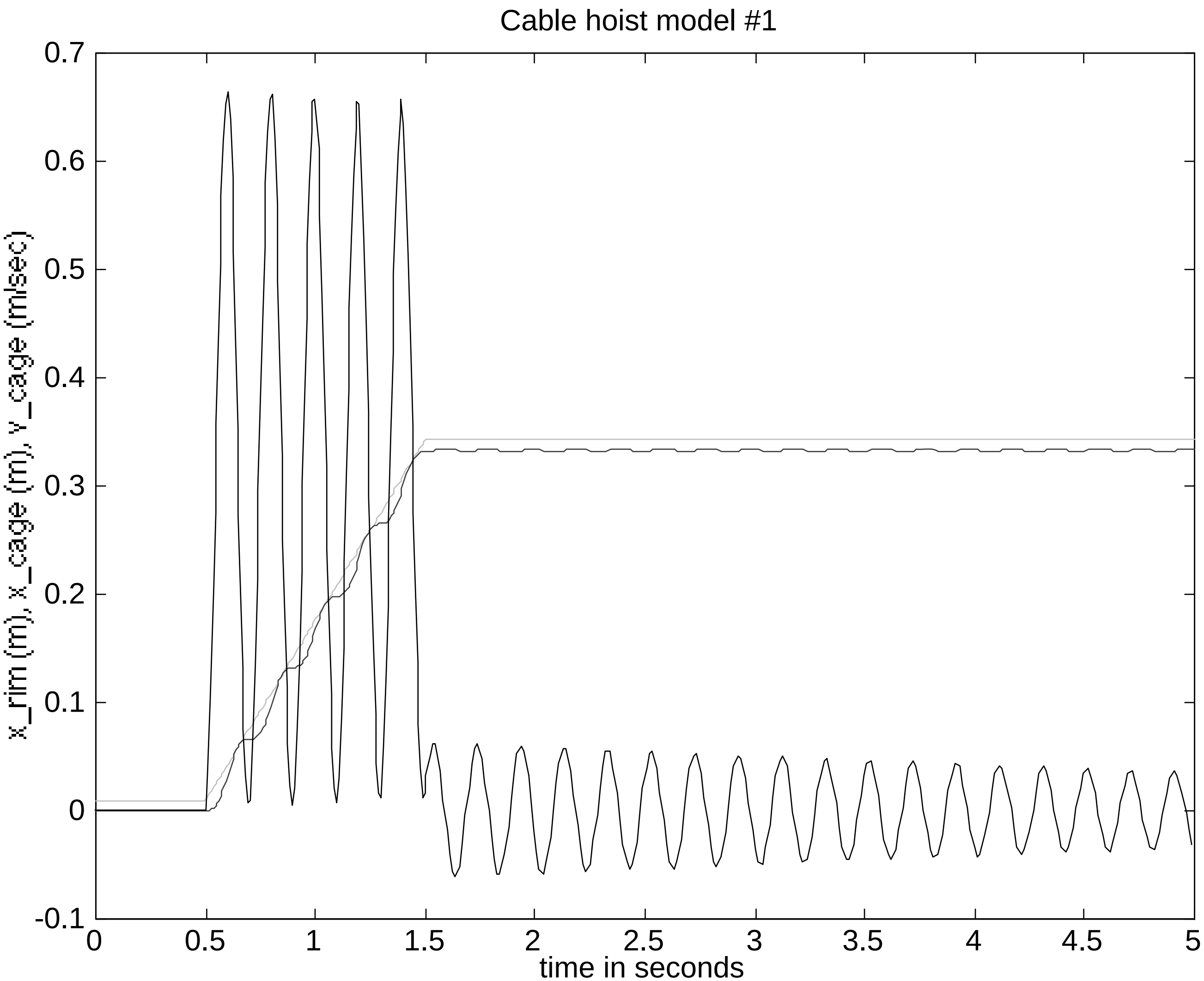
$$(m_{cage}\, s^2 + k_{cable})\, x_{cage} = k_{cable}\, x_{rim}$$

$$s\, x_{rim} = \left(\frac{r_{drum}\, n_{gear}}{K_{motor}}\right) e_{motor}$$

$$\frac{x_{cage}}{e_{motor}}(s) = \frac{\dfrac{r_{drum}\, n_{gear}}{K_{motor}} \dfrac{k_{cable}}{m_{cage}}}{s\left(s^2 + \dfrac{k_{cable}}{m_{cage}}\right)}$$

A step change of motor voltage (due to switch toggling) will result in a ramp change of cage position (due to the s term in the denominator) with a superimposed sinusoidal oscillation (due to the $s^2 + k_{cable}/\, m_{cage}$ term).

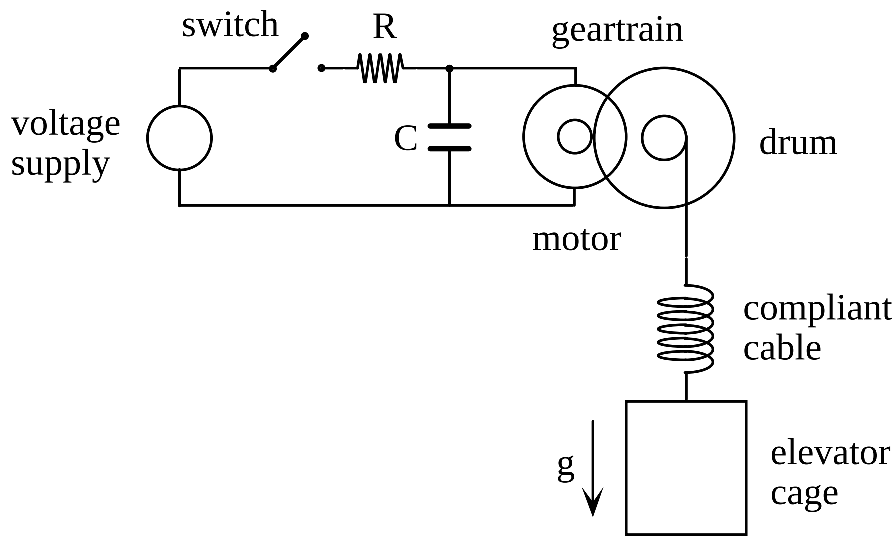
A MATLAB simulation confirms this.



These oscillations are understandably undesirable.

—This "first-pass" model reproduces the problem.

Next model the proposed solution



The differential equations are straightforward:

$C\,\dot{e}_{motor} := (e_{switch} - e_{motor}) / R$

In the Laplace domain:

$$\frac{e_{motor}}{e_{switch}}(s) = \frac{1}{RC\,s + 1}$$

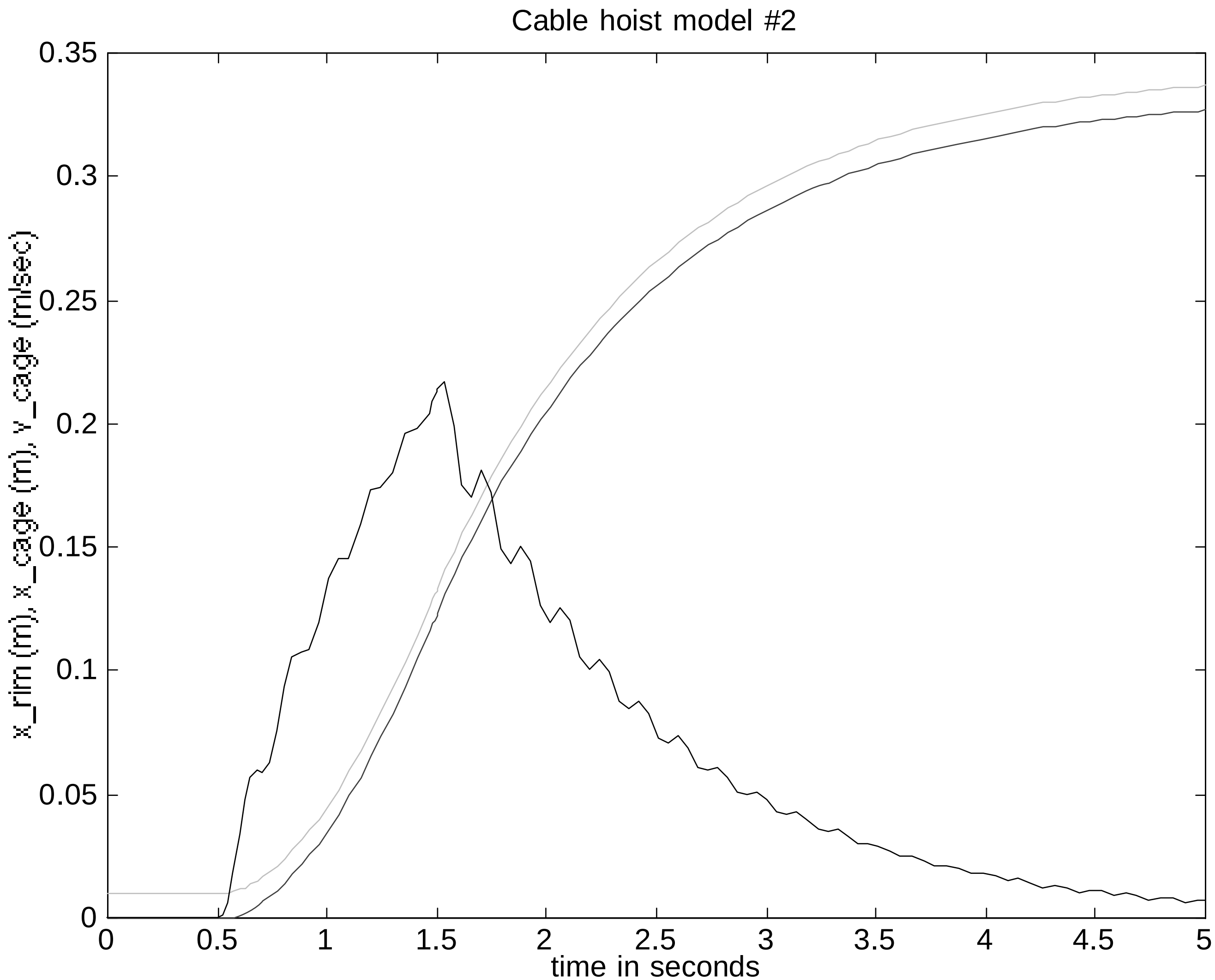
Multiplying the electro-mechanical and circuit transfer functions yields:

$$\frac{x_{cage}}{e_{switch}}(s) = \frac{\dfrac{r_{drum}\,n_{gear}}{K_{motor}}\,\dfrac{k_{cable}}{m_{cage}}}{s\left(s^2 + \dfrac{k_{cable}}{m_{cage}}\right)(RC\,s + 1)}$$

This suggests the proposed solution would work as planned.

A MATLAB simulation appears to confirm this.

The oscillations have been dramatically reduced, albeit at the cost of slower transients.



SNAG! THIS MODEL IS WRONG!

The electrical system cannot transmit power to the mechanical system without being influenced by its motion.

Multiplying two transfer functions is only meaningful if the second does not "load" the first.

Motor transduction characteristics require two equations:

$\omega_{motor} := e_{motor} \,/\, K_{motor}$

$i_{motor} := \tau_{motor} \,/\, K_{motor}$

The gear train also requires two equations:

$\omega_{drum} := n_{gear}\ \omega_{motor}$

$\tau_{motor} := n_{gear}\ \tau_{drum}$

So does the drum:

$\dot{x}_{rim} := r_{drum}\ \omega_{drum}$

$\omega_{drum} := r_{drum}\ F_{cable}$

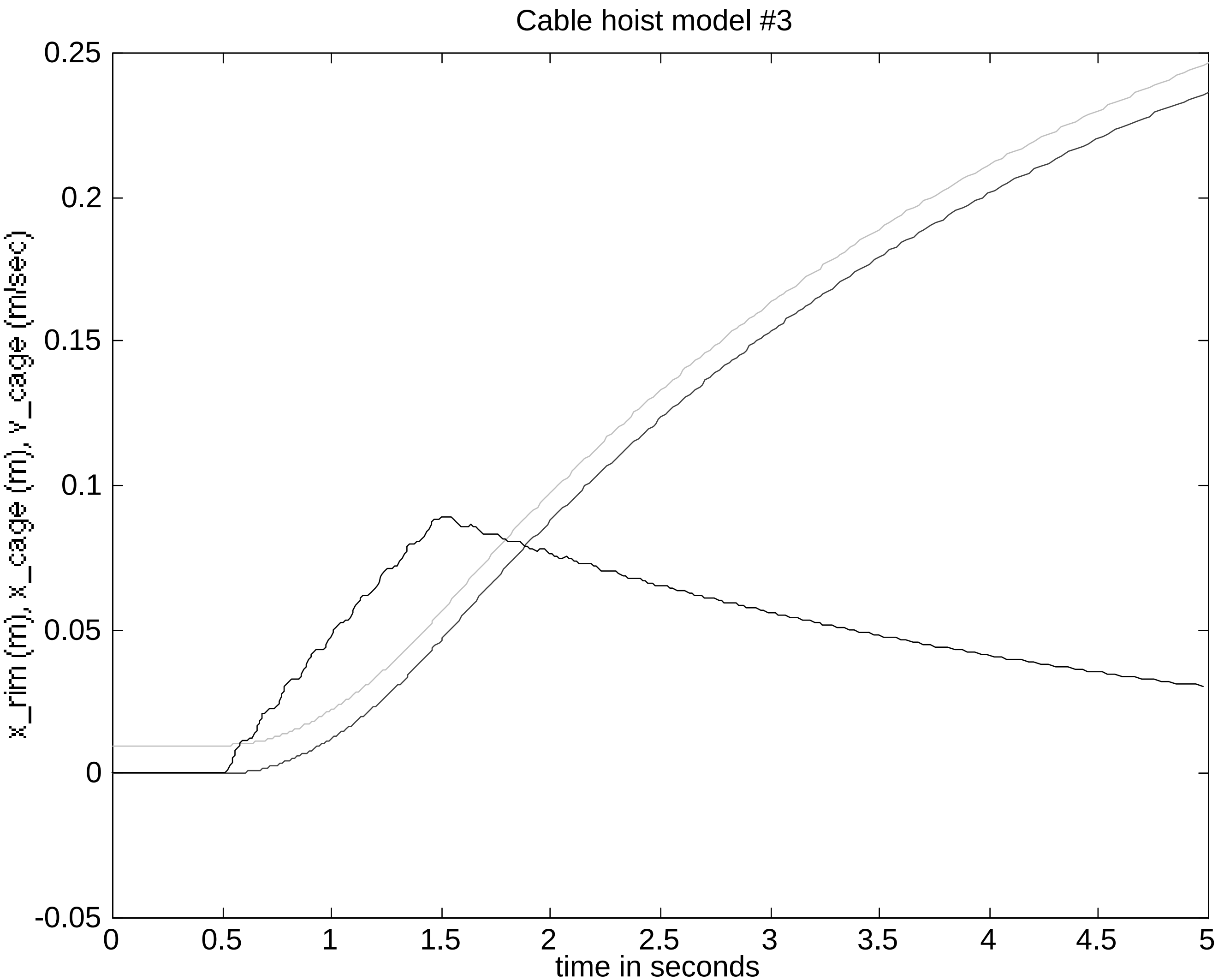
The revised electrical equations are as follows:

$$C\,\dot{e}_{motor} := \frac{(e_{switch} - e_{motor})}{R} - (r_{drum}\, n_{gear} \,/\, K_{motor})\, k_{cable}\,(x_{rim} - x_{cage})$$

Once again we have computable equations suitable for analysis. A MATLAB simulation is as follows.

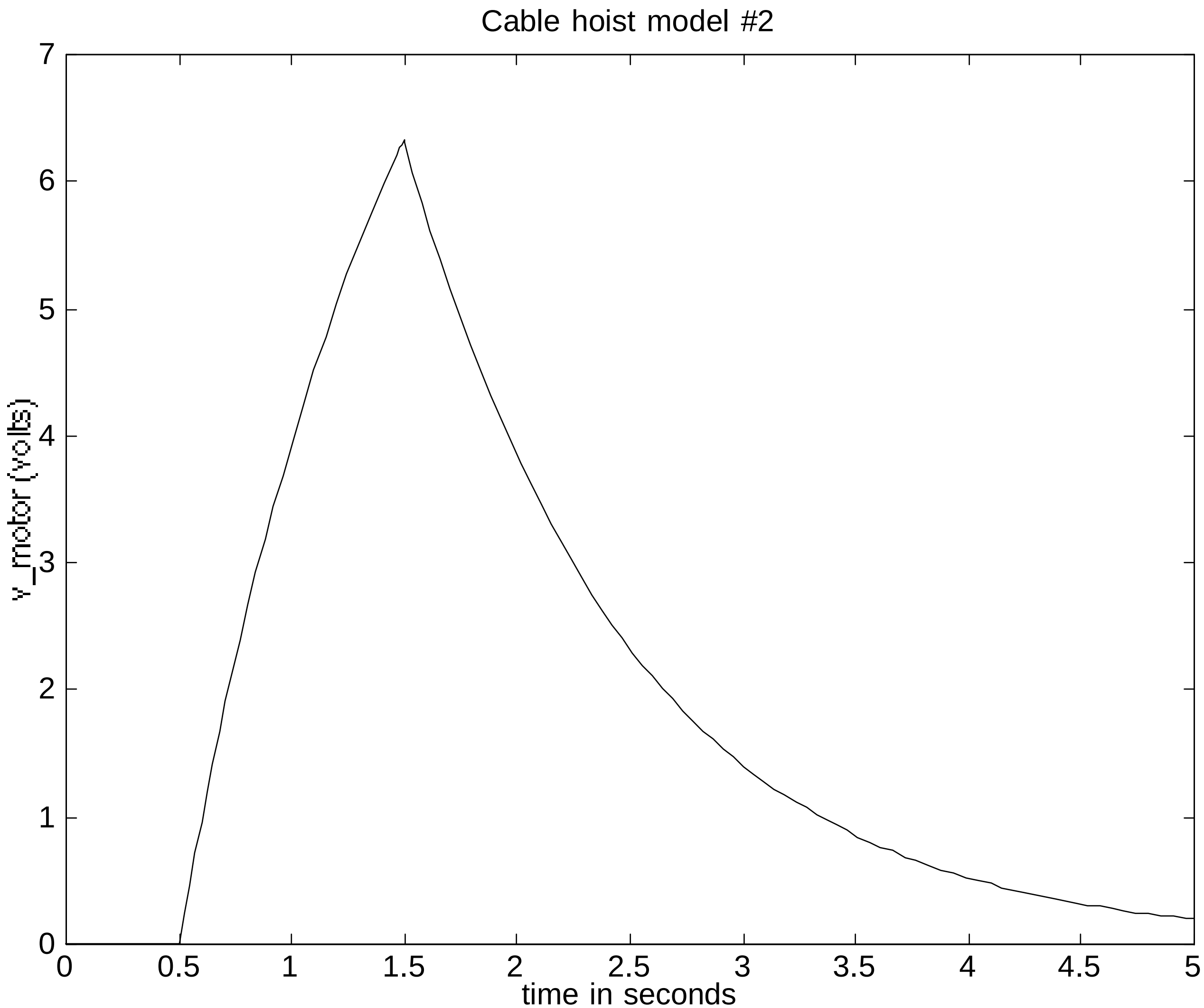
This is a substantially different response; the transient is slower.

But that's not all—the frequency of oscillation is higher.



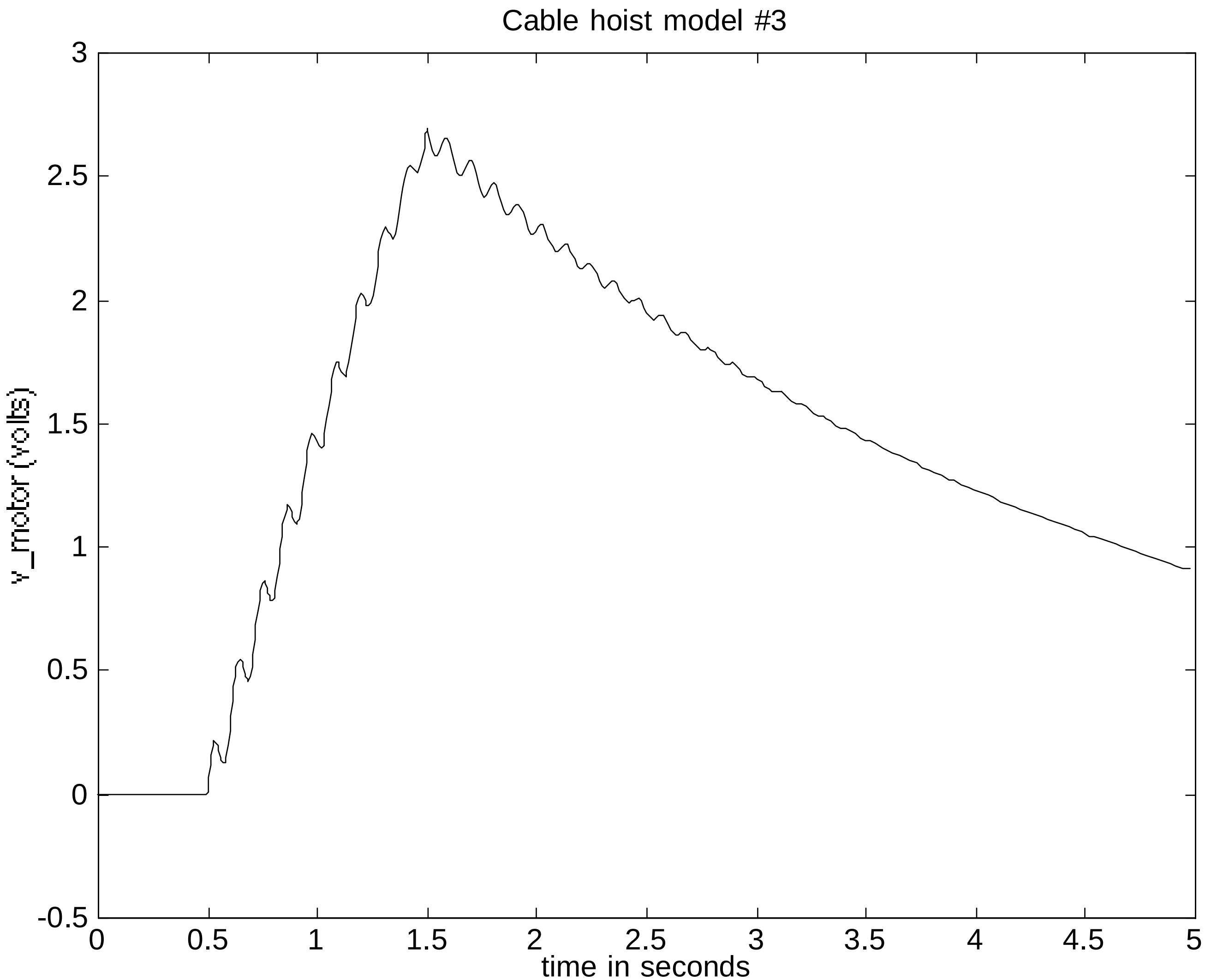
This is most evident if we plot and compare the motor voltages.

When the interaction is erroneously omitted, the voltage transient has a time constant of 1 second (as planned) and there is no oscillation.



When the interaction is included, the time constant is much longer, more than 3 seconds.

In addition, a lightly-damped oscillation at about 10 Hz is evident—about twice the frequency of oscillation of the model with no filter.



Adding a simple first-order filter results in faster oscillations (though at lower amplitude).

—why?

It also results in much slower responses than expected.

—why?

Try some further analysis ...

Laplace domain analysis:

Manipulating the equations directly to find the transfer function from e_{switch} to x_{cage} requires somewhat more algebra, some of which is shown below.

$$(RC\,s + 1)\, e_{motor} = e_{switch} - R\,(r_{drum}\, n_{gear} / K_{motor})\, k_{cable}\,(x_{rim} - x_{cage})$$

$$(RC\,s + 1)\, s\, x_{rim} = \frac{r_{drum}\, n_{gear}}{K_{motor}} \left(e_{switch} - R\, \frac{r_{drum}\, n_{gear}}{K_{motor}}\, k_{cable}\,(x_{rim} - x_{cage}) \right)$$

Shortening subscripts in an obvious way:

$$\left((RC\,s + 1)\, s + R \left(\frac{r_d\, n_g}{K_m} \right)^2 k_c \right) x_r = \frac{r_d\, n_g}{K_m}\, e_s + R \left(\frac{r_d\, n_g}{K_m} \right)^2 k_c\, x_c$$

Eliminating x_r:

$$\left(\left((RC\,s + 1)\, s + R \left(\frac{r_d\, n_g}{K_m} \right)^2 k_c \right) \left(\frac{m_c}{k_c}\, s^2 + 1 \right) - R \left(\frac{r_d\, n_g}{K_m} \right)^2 k_c \right) x_c = \frac{r_d\, n_g}{K_m}\, e_s$$

Simplifying:

$$s \left(RC\, s^3 + s^2 + R \left(\frac{r_d\, n_g}{K_m} \right)^2 k_c\, s + RC\, \frac{k_c}{m_c}\, s + \frac{k_c}{m_c} \right) = \frac{r_d\, n_g}{K_m}\, \frac{k_c}{m_c}\, e_s$$

Writing as a rational polynomial transfer function:

$$\frac{x_c}{e_s}(s) = \frac{\frac{r_d\, n_g}{K_m}\frac{k_c}{m_c}}{s\left(RC\, s^3 + s^2 + R\left(\frac{r_d\, n_g}{K_m}\right)^2 k_c\, s + RC\frac{k_c}{m_c}s + \frac{k_c}{m_c}\right)}$$

Note that this differs from the previous model only in the term $R\left(\frac{r_d\, n_g}{K_m}\right)^2 k_c\, s$.

It is due to the *two-way* interaction between the electrical and mechanical domains.

The previous model assumed a *one-way* interaction—electrical to mechanical but not vice-versa.

THIS IS A KEY FEATURE OF MULTI-DOMAIN INTERACTION

Power exchange

- **Bi-lateral interaction**

Signal transmission

- **Uni-lateral interaction**

BOND GRAPHS

The main purpose of modeling is to develop insight.

Considerable further insight is obtainable by "drawing a picture" of the models.

Block diagrams represent one-way effects.

—they will *not* serve for this purpose.

Network (circuit) diagrams are better—

but we need a form that generalizes to other physical systems.

Bond graphs are one such notation.