

2.141 Modeling and Simulation of Dynamic Systems

Assignment #3

Out: Wednesday October 11, 2006

Due: Monday October 23, 2006

Problem 1

Assume two 1 cm cubes of copper are perfectly insulated from their surroundings but communicate thermally through a 1 mm thick layer of Pyrex glass. Initially one cube is at ambient (room) temperature of 20 °C while the other has just been steeped in boiling water. Assume copper has a constant heat capacity of 3.494×10^6 Joule/meter³-°C. Assume heat transfer across the glass layer follows Fourier's law with a thermal conductivity of 1.005 watt/meter-°C.

1. Develop the simplest mathematical model competent to predict the time course of the temperature and entropy of each block.
2. What is the temperature of each block when they reach equilibrium? What are initial and final entropies of each block? What is the total entropy generated by the equilibration process?

Now assume the glass is replaced by a 1 mm thick layer of PVC (PolyVinylChloride) with a thermal conductivity of 0.16 watt/meter-°C.

3. How does this affect the time to reach thermal equilibrium? How does it affect the total entropy generated by the equilibration process?
4. Simulate the time-course of entropy in each block.

Problem 2

Gas-charged accumulators are commonly used in hydraulic systems, primarily to reduce the magnitude of the pressure transients resulting from abrupt changes in flow rate. A 1984 paper by Pourmovahed & Otis¹ considers the energy-dissipating effects of work-to-heat transduction and heat transfer in a gas-charged accumulator. The authors point out that the common polytropic model ($PV^n = \text{constant}$) used to characterize the pressure-volume relation in the gas fails to account for "thermal damping" and present a nonlinear model, a linear model and experimental data to support their point.

1. Formulate a model of the thermofluid capacitive subsystem of P. & O. '84, fig. 5 and represent it by a bond graph. Assuming the flow rate Q_a as an input and that nitrogen may be modeled as an ideal gas, derive nonlinear dynamic equations suitable for simulating figs. 1, 3 & 4 of P. & O. '84. **Hint:** a careful choice of state variables can simplify your work.

P. & O. '84 devote much of their paper to a linear analysis.

2. Linearize your model about similar operating conditions.
3. Develop a bond graph corresponding to your linearized model using single-port storage and dissipative elements and idealized transducers (e.g. gyrators,

¹ A. Pourmovahed & D. R. Otis (1984) "Effects of Thermal Damping on the Dynamic Response of a Hydraulic Motor-Accumulator System", J.D.S.M.C., 106:21-26.

- transformers) and show that it yields the same linearized equations. Does your linearized model describe entropy production?
4. Using the two models developed above, determine parameters for simulations corresponding to P. & O.'s experimental results and simulate the accumulator to obtain cross-plots corresponding to P. & O. '84 figs. 1, 3 & 4.
 5. Based on this model, at what frequencies of oscillation is thermal damping significant?
 6. P. & O.'s work is based on hydraulic systems operating at (roughly) meter scales. If gas-charged accumulators were included in micro-fluidic systems built at (roughly) millimeter scales, at what frequencies of oscillation would thermal damping become significant?

Problem 3

A Van der Waals gas is described by the relation $(p+a/v^2)(v-b) - RT = 0$ where p is pressure, a and b are constants, v is specific volume, R is the universal gas constant and T is temperature.

1. Show that the integral causal form of the constitutive equations for a fixed mass of gas can be written as

$$T(s,v) = T_0 \left(\frac{v-b}{v_0-b} \right)^{\frac{-R}{c_v}} \exp\left(\frac{s-s_0}{c_v} \right)$$

$$p(s,v) = \left(p_0 + \frac{a}{v_0^2} \right) \left(\frac{v-b}{v_0-b} \right)^{\frac{-R}{c_v} - 1} \exp\left(\frac{s-s_0}{c_v} \right) - \frac{a}{v^2}$$

where s is specific entropy, subscript o denotes a reference state and c_v is the constant-volume specific heat.

2. Find the conditions for which these constitutive equations describe an intrinsically stable element.