

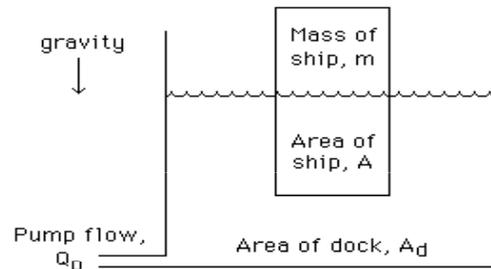
## 2.141 Modeling and Simulation of Dynamic Systems

### Assignment #2

Out: Wednesday September 20, 2006

Due: Wednesday October 4, 2006

#### Problem 1



The sketch shows a highly simplified diagram of a dry-dock used in ship maintenance. Once the ship enters the dock, water is drained out by positive-displacements pumps, an operation termed “pumping down the dock”. When work is complete, a gate to the harbor admits water, an operation termed “floating the dock”. Your task is to model the heave-mode oscillatory behavior (i.e. vertical motion only) of a ship in the dock.

For convenience, you should assume the ship and the dock are simple block shapes as illustrated. As a further simplification, assume that any motion of the water can be described as “slug flow”; that is, its velocity is constant across any given flow cross section. Finally, neglect all frictional losses.

1. Derive a steady-state force-displacement relation for vertical motion of the ship by partial differentiation of an appropriate energy (or co-energy) function for (a) pumping down the dock with the ship still floating and (b) floating the dock.
2. Derive a relation between velocity and momentum for vertical motion of the ship by partial differentiation of an appropriate energy (or co-energy) function.
3. Develop a model suitable to describe heave motion of the ship in response to pump flow input. Express it as a bond graph and as a set of state equations (no need to simulate).
4. Assuming the sea gate is closed and the pump flow is fixed at zero, derive an expression for the frequency of small oscillations about an equilibrium height. Evaluate it for two extreme cases: (i) dock surface area only slightly larger than ship horizontal cross-section area and (ii) dock surface area very much larger than ship horizontal cross-section area.

5. Given that the simplifying assumptions made above are unlikely to be true in reality, your model predictions may differ from the real system behavior. Will your model over- or under-estimate the frequency of oscillation?
  
6. It is often helpful to identify analogous behavior in different physical domains. Can you develop a model of an equivalent system involving mechanical translation only?

## Problem 2

Electro-magnetic solenoids are a very common form of linear actuator, especially in applications requiring limited linear or angular travel. (If your car has power door locks, they are almost certainly operated by solenoids.) In some applications, transient response time is especially important. The following figure shows data from a typical manufacturer's catalog ([www.electroid.com](http://www.electroid.com)). In this product (see the cutaway rendering) the solenoid squeezes two rotating plates together, serving to brake or clutch one plate relative to the other. The bottom trace shows the braking torque vs. time in response to abruptly energizing the electromagnet (i.e., applying voltage to the coils). Of greater interest is the corresponding coil current vs. time, shown in the top trace. Note the little "blip" at  $t_1$ ; what causes it?

A clearer picture is found in the following figure from another manufacturer's website ([www.solenoids.com](http://www.solenoids.com)) which depicts coil current vs. time in response to abruptly applying voltage to the coil.

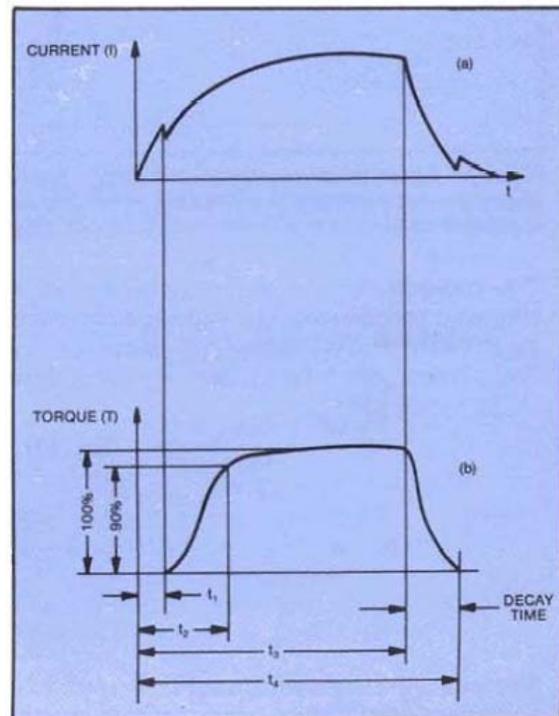
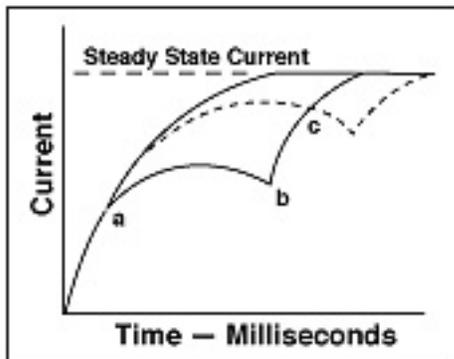


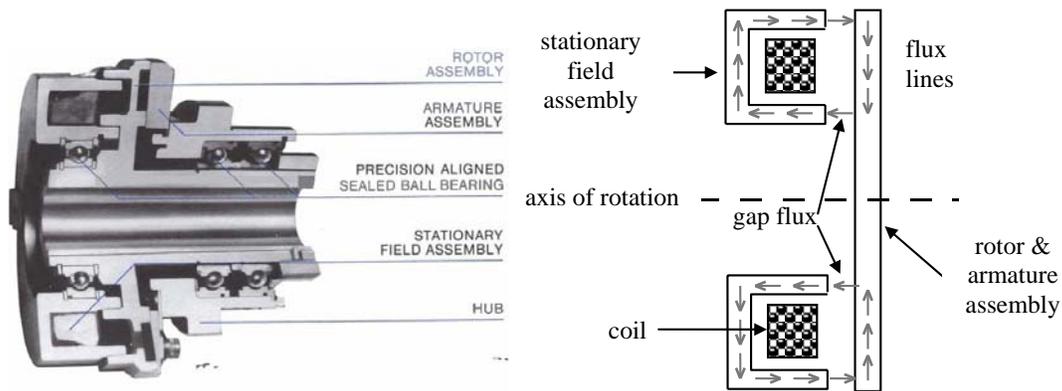
Figure 2. Response and decay time of a typical unit.

The current, instead of heading uniformly towards its steady-state value, turns back towards zero before abruptly changing direction and resuming its progress towards steady state<sup>1</sup>. These figures are obviously sketches but reflect a real phenomenon. Your task is to develop the simplest model competent to explain it.

<sup>1</sup> The three curves in this figure obviously refer to different conditions but I could find nothing in the associated material detailing what those conditions were (though I can guess).

1. What's going on? Describe qualitatively what causes (i) the smooth reversal (between a and b in the figure above) and (ii) the abrupt reversal (e.g., at b or c in the figure above).

Assume the geometry depicted in the following sketch, which is based on the design shown in the cutaway rendering.



A flat cylindrical coil is rated for a steady current of 0.35 amps at 90 VDC excitation. It has 2450 turns and an inductance of 22 henries. It is contained in a stationary housing with outside diameter 5.0 inches, wall thickness 0.3 inches; inner diameter 2.75 inches, wall thickness 0.3 inches. A disk-shaped armature, weight 2.8 lbf, may translate along the axis of rotation. It is restrained from doing so by a leaf spring characterized by  $\Delta x = cF^3$  where  $\Delta x$  is displacement from equilibrium,  $F$  is force and  $c$  is a constant,  $0.8 \times 10^{-6}$  in/lbf<sup>3</sup>. At equilibrium the gap between armature and stationary field housing is 0.03 inches. The relative permeability of the armature and housing material is 5000.

2. Develop state equations suitable to reproduce the current transient. What's the minimum system order required?

For simplicity, neglect eddy current losses, hysteresis and fringing fields.

3. As a check, develop a linearized version of your model. Can it describe (i) the smooth reversal? (ii) the abrupt reversal?
4. Develop a numerical simulation of the current transient.

### Problem 3

Electro-active Polymers (EAPs) have been proposed as the basis of novel actuator materials (see <http://ndea.jpl.nasa.gov/nasa-nde/lommas/eap/EAP-web.htm>). One EAP is PolyPyrrole (PPY). Immersed in an ionic solution, applying a voltage between PPY and a “counter electrode” induces an accumulation of charged ions adjacent to the material surface and a subsequent diffusion of ions into the material. Through a mechanism that is not yet fully understood<sup>2</sup>, absorption of ions induces a volumetric strain in the material, thereby providing a mechanism for electro-mechanical energy transduction. Loaded uniaxially, it serves as a translational actuator.

Your task is to develop the simplest model competent to describe an experimental EAP actuator in the form of a rectangular PPY film 12 mm long ( $x$  direction) 3.5 mm wide ( $y$  direction) and 19  $\mu\text{m}$  high ( $z$  direction) uniaxially loaded in the  $x$  direction. All shear stresses and strains may be ignored.

Mechanically, the polymer deforms under stress as an isotropic elastic material with a non-zero Poisson’s ratio. Electrically, the polymer stores ionic charge in response to applied voltage, with electrical capacitance proportional to volume  $C_V = c_V V$  where  $C_V$  (Farads) is total volumetric capacitance,  $c_V$  (Farads/ $\text{m}^3$ ) is specific volumetric capacitance, and  $V$  ( $\text{m}^3$ ) is the polymer volume. In the typical operating range of most EAPs, specific volumetric capacitance can be considered constant.

1. This EAP may be described as multiport network element. What type? How many ports? Identify appropriate power and energy variables for the mechanical and electrical domains.
2. Assuming electrical linearity and small strain, write an expression for the total energy stored in the material valid for general loading conditions (i.e., non-zero stress on any face of the film).

You may find it useful to express the mechanical behavior in terms volumetric and deviatoric components. Volumetric strain is proportional to mean stress, while deviatoric stresses deform the polymer along its principal axes without changing its volume.

$$\varepsilon_V = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

$$S_{x,y,z} = \sigma_{x,y,z} - \sigma_m$$

$$\varepsilon'_{xx,yy,zz} = \varepsilon_{xx,yy,zz} - \frac{\varepsilon_V}{3}$$

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<sup>2</sup> As far as I know.

where  $\varepsilon_v$  is volumetric strain and  $\varepsilon_{xx,yy,zz}$  are strains in the  $x$ ,  $y$  and  $z$  directions, respectively;  $\sigma_m$  is mean stress,  $\sigma_{x,y,z}$  are stresses in the  $x$ ,  $y$  and  $z$  directions, respectively;  $S_{x,y,z}$  are deviatoric stresses and  $\varepsilon'_{xx,yy,zz}$  are deviatoric strains in the  $x$ ,  $y$  and  $z$  directions, respectively.

3. Derive constitutive equations relating port variables of this multiport element.
4. Now assume uniaxial loading conditions, i.e., zero stress on the  $y$  and  $z$  faces of the film. Revise the constitutive equations to reflect these specialized conditions.
5. If they are not already linear, linearize the constitutive equations for the uniaxial loading case. Represent the result as a network of basic bond-graph elements (i.e., junctions, one-port passive elements, etc.). Is the electro-mechanical coupling described by a transformer or gyrator?
6. Evaluate the linearized constitutive equations about an operating point at 0.7 volts using the following EAP parameters:

$$x_0 = 0.012 \text{ m}$$

$$y_0 = 0.0035 \text{ m}$$

$$z_0 = 0.000019 \text{ m}$$

$$\nu = 0.426$$

$$c_V = 1.3 \times 10^8 \text{ F/m}^3$$

$$E = 0.18 \text{ GPa}$$

7. Compute the no-load strain and charge added by applying a voltage of 0.7 volts to the film. Compute the load required to double that strain and the corresponding additional charge displaced. Is transduction between the electrical and mechanical domains bilaterally symmetric? That is, does the mechanical domain affect the electrical domain as much as the electrical domain affects the mechanical?