

Massachusetts Institute of Technology  
 Department of Mechanical Engineering  
 2.12 Introduction to Robotics  
**Problem Set No.6**  
 Out: October 31, 2005 Due: November 9, 2005

**Problem 1**

A robot arm is drawing a line with a ruler, as shown below. The ruler is held by another robot arm, which is not shown. Assume no friction and a quasi-static process.

- a). Obtain natural and artificial constraints, using the C-frame attached to the ruler.
- b). Sketch the block diagram of the hybrid position/force control system in accordance with the natural and artificial constraints obtained above. Also obtain the projection matrices  $\mathbf{P}_a$  and  $\mathbf{P}_c$  associated with the natural and artificial constraints.

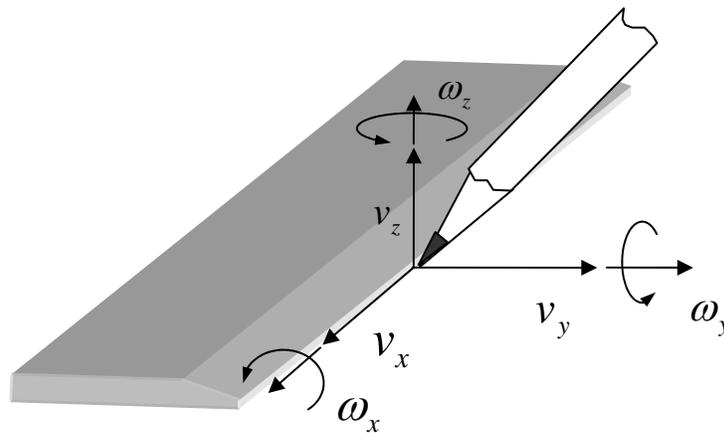


Figure 1 Ruler and C-frame

**Problem 2**

Shown below is an office robot drying ink with blotting paper attached to a semicircular roller of radius  $R$ . The roller should not slide but roll on the paper in order to avoid smearing the wet signature. Assuming that the process is quasi-static and frictionless, we want to perform the task using hybrid position/force control. Obtain natural and artificial constraints in terms of velocities and forces at the robot endpoint  $E$ . Describe the constraints with respect to the coordinate system fixed to the desk,  $O$ - $xyz$ . Note that Point  $E$  is in the middle of the top surface of the semicircular roller. Is the rolling-contact requirement a natural constraint or an artificial constraint? [The key is to differentiate constraints that physics dictates, i.e. natural constraints, from the type of trajectories that you want the robot to follow in order to accomplish a given task, i.e. artificial constraints.]

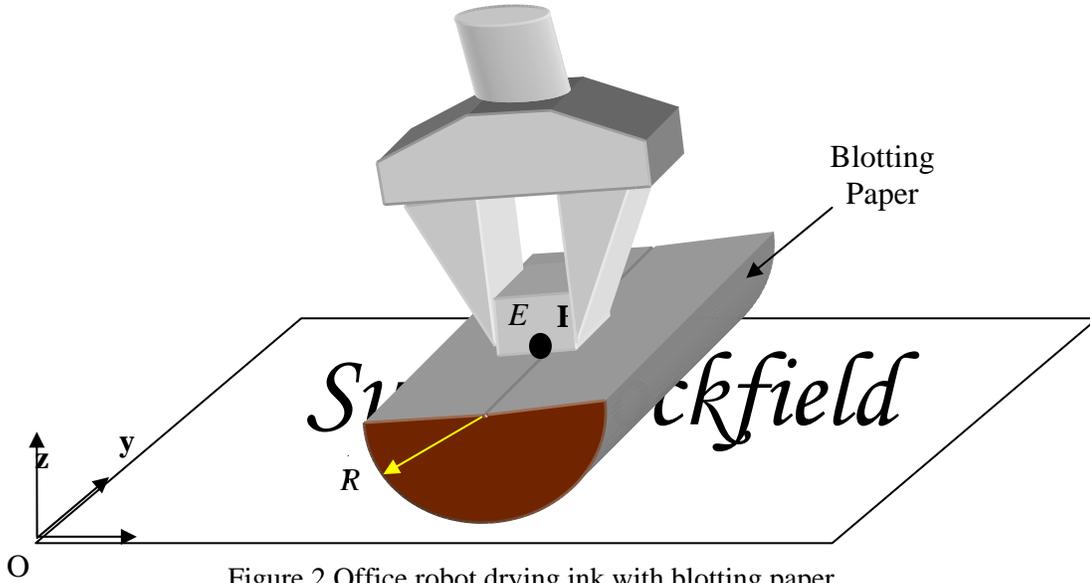


Figure 2 Office robot drying ink with blotting paper

### Problem 3

The planar 3 d.o.f. robotic leg considered in a previous problem set is shown below. Three joint angles,  $\theta_1, \theta_2, \theta_3$ , all measured from the ground, are used as an independent set of generalized coordinates uniquely locating the system. The second figure below shows the front view of the robot including actuators and transmission mechanisms. As before, Actuator 1 generates torque  $\tau_1$  between link 0 and link 1. Actuator 2 is fixed to Link 3, and its output torque  $\tau_2$  is transmitted to Joint 2, i.e. the knee joint, through the mass-less belt-pulley system with a gear ratio of 1:1. Actuator 3 is fixed to Link 3, while its output shaft is connected to Link 2. All actuator torques  $\tau_1, \tau_2, \tau_3$  are measured in a right hand sense, as shown by the arrows in the figure. Displacements of the individual actuators are denoted  $\phi_1, \phi_2, \phi_3$ , and are measured in the same direction of the torque. The location of the hip, i.e. Link 3, is represented by the coordinates of its center of mass,  $x_h, y_h$ , and angle  $\alpha$  measured from the base coordinate system fixed to Joint 1, as shown in the figure.

In order to walk in rough terrain, the robot wants to make its knee, ankle, and hip joints compliant so that disturbances acting on the body may be alleviated. All the disturbance forces acting on the foot can be represented collectively with an equivalent linear force and moment acting at the hip:  $\mathbf{F} = [F_x, F_y, M]^T$ . See the figure below. Using compliance (stiffness) control, we want to support the hip with a desired stiffness defined as:

$$\begin{pmatrix} F_x \\ F_y \\ M \end{pmatrix} = \begin{pmatrix} k_x & 0 \\ & k_y \\ 0 & k_\theta \end{pmatrix} \cdot \begin{pmatrix} \Delta x_h \\ \Delta y_h \\ \Delta \alpha \end{pmatrix}$$

where  $\Delta \mathbf{p} = [\Delta x_h, \Delta y_h, \Delta \alpha]^T$  is linear and angular displacements of the hip, and the elements of the stiffness matrix,  $k_x, k_y, k_\theta$ , are appropriate positive values. For the leg configuration and the joint angles shown in the figure, obtain the joint feedback gain matrix  $\mathbf{K}$  that provides the desired stiffness given above. Also obtain the feedback gain matrix in the actuator space.

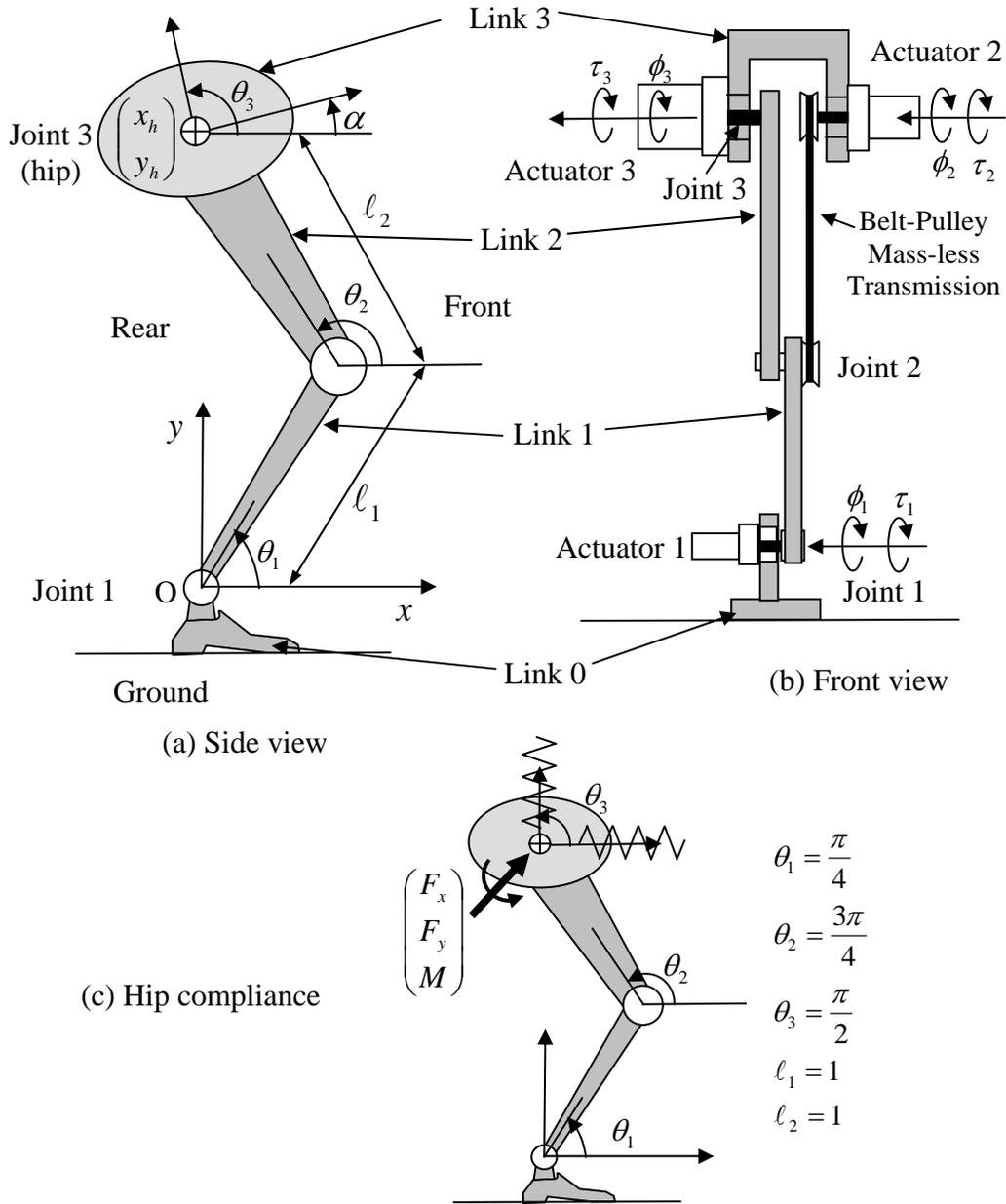


Figure 3 Leg robot revisited