2.094

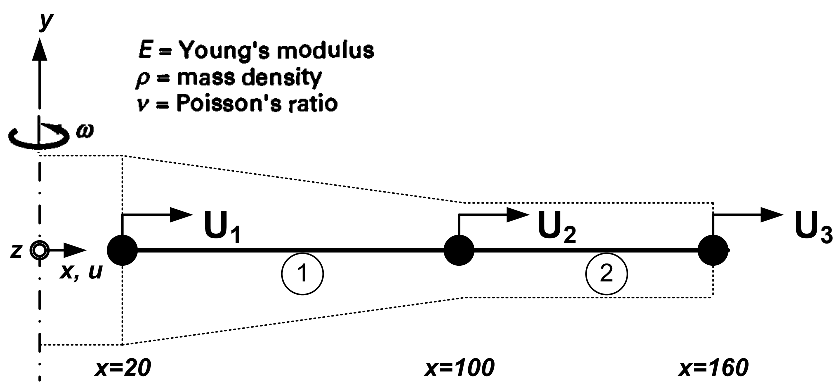
FINITE ELEMENT ANALYSIS OF SOLIDS AND FLUIDS

SPRING 2008

Homework 2 - Solution

		Assigned:	02/14/2008
Instructor:	Prof. K. J. Bathe	Due:	02/21/2008

Problem 1 (20 points):



In this problem, the global coordinate system is used.

Here, the non-zero strain components are $[\boldsymbol{\varepsilon_{xx}}, \boldsymbol{\varepsilon_{zz}}]$. Hence the stress-strain law represented by the matrix $\boldsymbol{C}$ is

$$\underline{\boldsymbol{C}} = \frac{\boldsymbol{E}}{\boldsymbol{1 - \nu^2}} \begin{bmatrix} \boldsymbol{1} & \boldsymbol{\nu} \\ \boldsymbol{\nu} & \boldsymbol{1} \end{bmatrix}$$

And

$$\underline{\boldsymbol{\varepsilon}}^T = [\boldsymbol{\varepsilon_{xx}} \quad \boldsymbol{\varepsilon_{zz}}] = \begin{bmatrix} \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} & \frac{\boldsymbol{u}}{\boldsymbol{x}} \end{bmatrix}$$

$$\underline{\boldsymbol{U}}^T = [\boldsymbol{U_1} \quad \boldsymbol{U_2} \quad \boldsymbol{U_3}]$$

The applied force is given by

$$\boldsymbol{f_x^B(x) = \rho x \omega^2}$$

The thickness of each element is given by

$$\boldsymbol{t}^{(1)} = \frac{\mathbf{1}}{\mathbf{80}}(\mathbf{180} - \boldsymbol{x})$$

$$\boldsymbol{t}^{(2)} = \mathbf{1.0}$$

For each element, the interpolation functions are

$$\underline{\boldsymbol{H}}^{(1)} = \left[\frac{1}{80}(100 - x) \quad -\frac{1}{80}(20 - x) \quad 0\right] \text{ for } 20 \leq x \leq 100$$

$$\underline{\boldsymbol{H}}^{(2)} = \left[0 \quad \frac{1}{60}(160 - x) \quad -\frac{1}{60}(100 - x)\right] \text{ for } 100 \leq x \leq 160$$

Then,

$$\underline{\boldsymbol{B}}^{T} = \left[\frac{\partial \underline{\underline{\boldsymbol{H}}}}{\partial x} \quad \frac{\underline{\underline{\boldsymbol{H}}}}{x}\right]$$

$$\underline{\boldsymbol{B}}^{(1)} = \begin{bmatrix} -\frac{1}{80} & \frac{1}{80} & 0 \\ \frac{(100 - x)}{80x} & -\frac{(20 - x)}{80x} & 0 \end{bmatrix}$$

$$\underline{\boldsymbol{B}}^{(2)} = \begin{bmatrix} 0 & -\frac{1}{60} & \frac{1}{60} \\ 0 & \frac{(160 - x)}{60x} & -\frac{(100 - x)}{60x} \end{bmatrix}$$

The stiffness matrices are

$$\begin{aligned} \underline{\boldsymbol{K}} &= \int_{20}^{100} \underline{\boldsymbol{B}}^{(1)^T} \underline{\boldsymbol{C}}\ \underline{\boldsymbol{B}}^{(1)} dV^{(1)} + \int_{100}^{160} \underline{\boldsymbol{B}}^{(2)^T} \underline{\boldsymbol{C}}\ \underline{\boldsymbol{B}}^{(2)} dV^{(2)} \\ &= \int_{20}^{100} \underline{\boldsymbol{B}}^{(1)^T} \underline{\boldsymbol{C}}\ \underline{\boldsymbol{B}}^{(1)} t^{(1)} 2\pi x dx + \int_{100}^{160} \underline{\boldsymbol{B}}^{(2)^T} \underline{\boldsymbol{C}}\ \underline{\boldsymbol{B}}^{(2)} t^{(2)} 2\pi x dx \end{aligned}$$

The external force vector is

$$\begin{aligned} \underline{\boldsymbol{R}} &= \int_{20}^{100} \underline{\boldsymbol{H}}^{(1)^T} \rho x \omega^2\, dV^{(1)} + \int_{100}^{160} \underline{\boldsymbol{H}}^{(2)^T} \rho x \omega^2\, dV^{(2)} \\ &= \int_{20}^{100} \underline{\boldsymbol{H}}^{(1)^T} \rho x \omega^2\, t^{(1)} 2\pi x dx + \int_{100}^{160} \underline{\boldsymbol{H}}^{(2)^T} \rho x \omega^2\, t^{(2)} 2\pi x dx \end{aligned}$$

Therefore we have

$$\underline{\boldsymbol{K}\boldsymbol{U}} = \underline{\boldsymbol{R}}$$

After integrations using Matlab,

$$\underline{\boldsymbol{K}} = \frac{\boldsymbol{E}}{\mathbf{1} - \boldsymbol{\nu}^2} \begin{bmatrix} \mathbf{15.2590 - 10.4704}\boldsymbol{\nu} & \mathbf{-4.0988 + 1.0470}\boldsymbol{\nu} & \mathbf{0} \\ \mathbf{-4.0988 + 1.0470}\boldsymbol{\nu} & \mathbf{24.1192 + 2.0943}\boldsymbol{\nu} & \mathbf{-13.125} \\ \mathbf{0} & \mathbf{-13.125} & \mathbf{14.4885 + 6.2820}\boldsymbol{\nu} \end{bmatrix}$$

$$\underline{\boldsymbol{R}} = \rho\omega^2 \begin{bmatrix} \mathbf{944991} \\ \mathbf{4521380} \\ \mathbf{3732212} \end{bmatrix}$$

The same results are obtained using a local coordinate system for each element (like the example 4.5 in the textbook). For your reference, a sample solution using a local coordinate system is attached.