# 2.092/2.093

## FINITE ELEMENT ANALYSIS OF SOLIDS AND FLUIDS I

### FALL 2009

### Homework 7-solution

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|---|---|---|---|
| Instructor: | Prof. K. J. Bathe | Assigned: | Session 16 |
| TA: | Seounghyun Ham | Due: | Session 19 |

**Problem 1** (20 points):

$$\underline{K}=\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix},\ \underline{M}=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\ \underline{R}=\begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$${}^{0}\underline{U}=0\,;\quad {}^{0}\dot{\underline{U}}=0$$

Considering the eigenproblem, $\underline{K}\underline{\phi}=\omega^2\underline{M}\underline{\phi}$

$$\omega_1^2=1.7753,\quad \underline{\phi}_1=\begin{bmatrix} 0.3029 \\ 0.6739 \end{bmatrix}$$

Note: $\underline{\phi}_i^T\underline{M}\underline{\phi}_j^T=\delta_{ij}$ , $\underline{\phi}_i^T\underline{K}\underline{\phi}_j^T=\omega_i^2\delta_{ij}$

Using $\underline{U}=\underline{\Phi}\underline{X}$ where $\underline{\Phi}=\left[\underline{\phi}_1\right]=\begin{bmatrix} 0.3029 \\ 0.6739 \end{bmatrix}$

$$\ddot{x}+\omega_1^2x=\underline{\Phi}^T\begin{bmatrix} 10 \\ 0 \end{bmatrix}=3.029 \qquad (1)$$

The generalized solution for (1) is

$$x_1=A\sin\omega_1t+B\cos\omega_1t+\frac{3.029}{\omega_1^2}=A\sin\omega_1t+B\cos\omega_1t+1.7062$$

From ${}^{0}\underline{U}={}^{0}\dot{\underline{U}}=0$, $x=0$ and $\dot{x}=0$

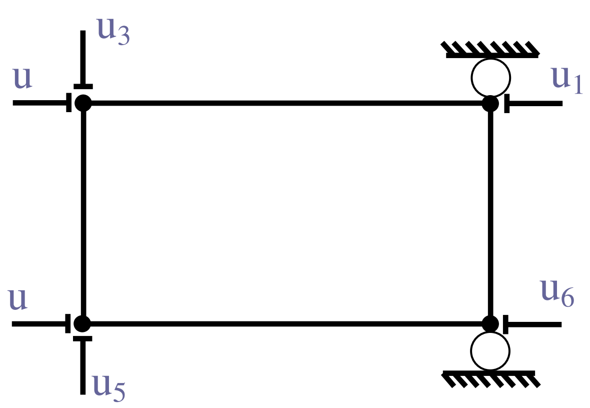
Using these initial conditions,

$$x_1=1.7062(1-\cos\omega_1t)=1.7062(1-\cos\sqrt{1.7753}t)$$

Therefore, $\underline{U}=\underline{\Phi}\underline{X}=\begin{bmatrix}0.3029\\0.6739\end{bmatrix}1.7062(1-\cos\sqrt{1.7753}t)=\begin{bmatrix}0.5168(1-\cos\sqrt{1.7753}t)\\1.1498(1-\cos\sqrt{1.7753}t)\end{bmatrix}$

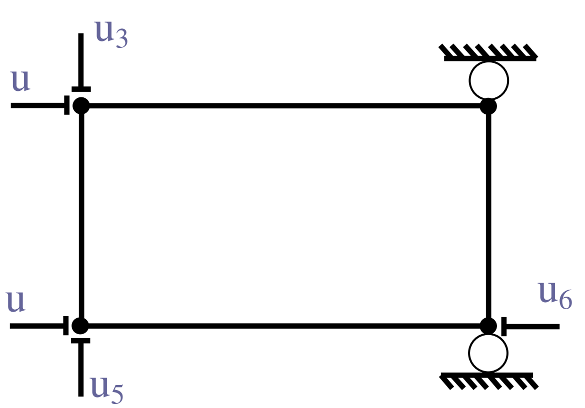
**Problem 2** (10 points):

For case 1, the structure is clearly unstable, hence a zero diagonal element will be encountered in the Gauss elimination.



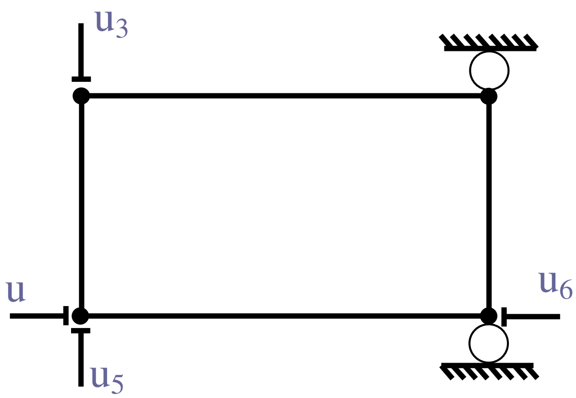

All clamps see stiffness. No rigid body motion possible.

After removing the clamp at $u_1$,



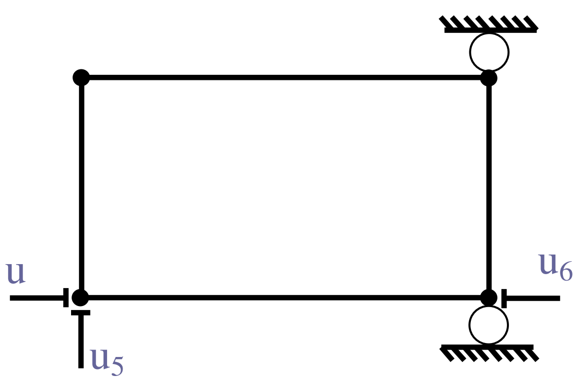

All clamps see stiffness. No rigid body motion possible.

After removing the clamp at $u_2$,




All clamps see stiffness. No rigid body motion possible.

After removing the clamp at $u_3$,




The clamp for $u_5$ "sees" no more stiffness. A rigid body rotation is possible. Therefore there will be a zero diagonal term after the third step of Gauss elimination.

For case 2, the structure is clearly stable, hence there will be no zero diagonal term in the Gauss elimination.

**Problem 3** (10 points):

$2\ddot{U}+8U=0$ (1)

${}^{0}U=10^{-12}$ , ${}^{0}\dot{U}=0$ (2)

$$\omega^2 = \sqrt{\frac{K}{M}} = \sqrt{4} = 2$$

Therefore $\omega = 2$ and $\Delta t_{cr} = \frac{T}{\pi} = \frac{2}{\omega} = 1$.

$$\Delta t = 1.01 \times 1 = 1.01$$

We are able to obtain ${}^{0}\ddot{U}$ using eq. (1) and (2)

$${}^{0}\ddot{U} = -4\,{}^{0}U = -4\times10^{-12}$$

To calculate ${}^{-\Delta t}U$, use (9.7) in textbook,

$$
\begin{aligned}
{}^{-\Delta t}U &= {}^{0}U - \Delta t\,{}^{0}\dot{U} + \frac{\Delta t^2}{2}\,{}^{0}\ddot{U} \\
&= 10^{-12} + \frac{1.01^2}{2}(-4\times10^{-12}) = -1.0402\times10^{-12}
\end{aligned}
$$

Then ${}^{t+\Delta t}U$ can be solved using the central difference method.

$${}^{t+\Delta t}U = (2 - 4\Delta t^2)\,{}^{t}U - {}^{t-\Delta t}U$$

$$\begin{bmatrix} {}^{t+\Delta t}U \\ {}^{t}U \end{bmatrix} = \begin{bmatrix} 2 - 4\Delta t^2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} {}^{t}U \\ {}^{t-\Delta t}U \end{bmatrix}$$

${}^{t+\Delta t}U$ becomes larger than $10^{30}$ after 345 time steps ( I used Matlab).

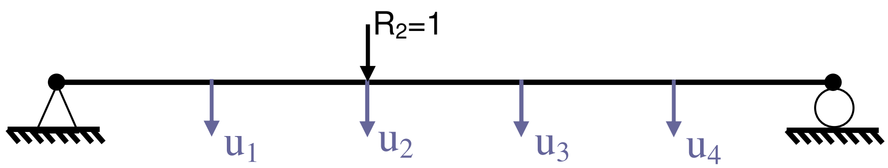
After 345 time steps

$$\begin{bmatrix} {}^{t+\Delta t}U \\ {}^{t}U \end{bmatrix} = \begin{bmatrix} 1.4630\times10^{30} \\ -1.9408\times10^{30} \end{bmatrix}$$

## **Problem 4** (10 points):

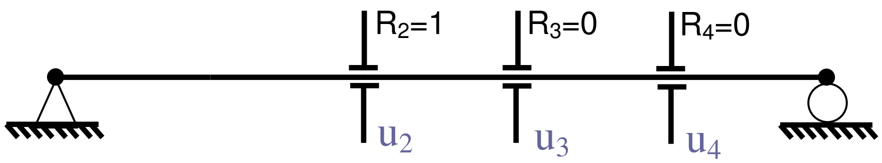
Step 1

Let a technician put an external force, $R_2=1$ and then measure the corresponding displacements, $u_1$, $u_2$, $u_3$, and $u_4$.



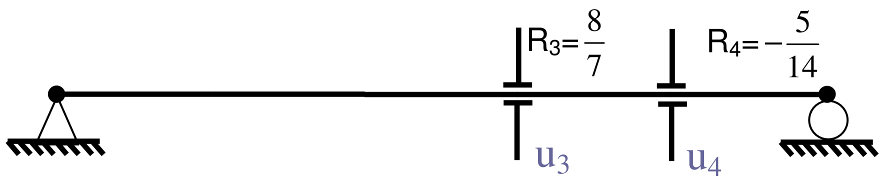

Step 2

Place clamps at $u_2$, $u_3$, and $u_4$ and then impose displacements measured in step 1 and measure the forces in the clamp.



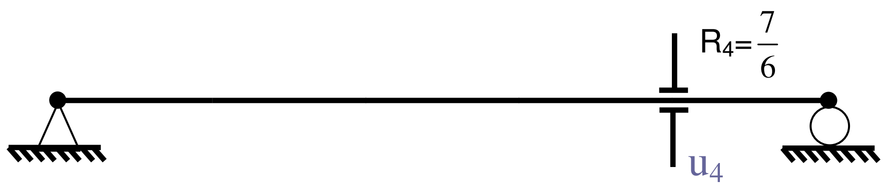

Step 3

Place clamps at $u_3$, and $u_4$ and then impose displacements measured in step 1 and measure the forces in the clamp.




Step 4

Place clamps at $u_4$ and then impose displacements measured in step 1 and measure the forces in the clamp.

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